ABSTRACT

The study of viscoelastic fluids gives many opportunities to mathematicians, numerical analysts and simulationists to introduce suitable algorithms for computing the flow due to fulfil many application of it such as in paints, coating, inks, and jet fuels. In this thesis, the mixed convection boundary layer flow of a viscoelastic fluid past a sphere subjected to constant temperature has been studied. The constitutive equations of viscoelastic fluids usually generate a higher-order derivative term in the momentum equation than equations of Newtonian fluid. Thus, we are facing the problem where the boundary conditions insufficient to solve the problems of viscoelastic fluid completely. Therefore, we need an extra boundary condition by augmenting an extra boundary condition at infinity. The governing boundary layer equations are first transformed into a non-dimensional form, and then, into a set of non similar boundary layer equations, which are solved numerically using an efficient implicit finite-difference method known as Keller-box method. Numerical results are presented for different values of the viscoelastic and mixed convection parameters K and λ, respectively, and with the Prandtl number Pr = 0.7, 1 and 7. It is found that both skin friction C_f and heat transfer Q_w coefficients decrease as K is increased. Further, for cases of cooling sphere $(\lambda < 0)$ and heating sphere $(\lambda > 0)$, the boundary layer separates from the sphere. It is worth mentioning that the result obtained in viscoelastic fluid when we set the value of $K = 0$ (Newtonian fluid), are in excellent agreement with those obtained in viscous fluid.

ABSTRAK

Kajian tentang bendalir likat-kenyal memberi banyak peluang kepada ahli-ahli matematik, ahli-ahli analisis berangka dan ahli pakar penyelaku memperkenalkan algoritma-algoritma yang sesuai dalam aliran ini untuk memenuhi banyak aplikasi seperti pembuatan cat, penyaduran, dakwat dan minyak untuk jet. Dalam tesis ini, olakkan campuran aliran lapisan sempadan dalam bendalir likat-kenyal melepasi sfera bergantung kepada suhu yang tetap di kaji. Persamaan-persamaan juzuk bendalir likat-kenyal ini terjana dengan sebutan terbitan peringkat tinggi di dalam persamaan momentumnya berbanding persamaan bendalir Newtonan. Oleh itu, permasalahan yang di hadapi ialah ketidakcukupan syarat-syarat sempadan untuk menyelesaikan masalah bendalir likat-kenyal ini. Oleh yang demikian, kita memerlukan tambahan syarat sempadan dengan menambahkan syarat sempadan di infiniti. Persamaan-persamaan lapisan sempadan menakluk, pada mulanya diubah kepada bentuk tak bermatra, selepas itu, diubah kepada set persamaan lapisan sempadan tak serupa dimana penyelesaian secara berangka dengan menggunakan skim beza terhingga tersirat yang efektif dikenali sebagai kaedah kotak-Keller. Keputusan-keputusan berangka yang meliputi nilai-nilai parameter likat-kenyal dan parameter olakkan campuran masing-masing dipaparkan dengan nombor Prandtl yang berbeza iaitu Pr = 0.7,1 and 7. Dapati kedua-dua pekali geseran kulit C_f dan pekali pemindahan haba Q_w menurun apabila nilai K meningkat. Untuk kes penyejukan dan pemanasan, lapisan sempadan terpisah dari sfera. Dengan kata lain, di dalam bendalir likat-kenyal, keputusan yang diperolehi apabila menetapkan nilai K = 0 (bendalir Newtonan) menunjukkan keputusan yang memuaskan setanding dengan keputusan yang diperolehi menerusi bendalir likat.

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LIST OF SYMBOL/NOTATIONS

a - radius of sphere

C_f - local skin friction coefficient

f - dimensionless stream function

K - viscoelastic coefficient

m - velocity exponent parameter

n - temperature exponent parameter

g - gravitational acceleration

Gr - Grasshof number

Pr - Prandtl number

Re - Reynolds number

Q_w - heat transfer coefficient

T - fluid temperature

u - velocity component in x-direction

$u_e(x)$ - non-dimensional velocity outside boundary layer

U_0 - dimensional constant

U_∞ - free stream velocity

v - velocity component in y-direction

x - coordinate in direction of surface motion

y - coordinate in direction normal to surface motion

Greek symbols

α - thermal diffusivity

β - thermal expansion coefficient

μ - dynamic viscosity

λ - mixed convection parameter

η - dimensionless similarity variable

θ - dimensionless temperature

ν - kinematic viscosity

ψ - stream function

ρ - fluid density

Subscripts

w - condition at the surface

∞ - condition at ambient medium

Superscripts

‘ - differentiation with respect to η

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CHAPTER 1

INTRODUCTION

1.1 Research Background

It is well known that the Newtonian equations do not adequately describe the flow properties of some naturally occurring fluids such as animal blood. Therefore, non-Newtonian fluids have become more important industrially such as polymer solution, polymer melts, blood, paints and certain oils.

Amongst the several models of non-Newtonian fluids, the viscoelastic fluids have attracted much attention from researchers. The flow of viscoelastic fluids gives many opportunities to mathematicians, numerical analysts and simulationists to introduce suitable algorithms for computing the flow. There are two special categories of viscoelastic fluids, second-order fluid and Walters' fluid, which have particularly attracted the attention of researchers during the last two decades.

The boundary value problem (BVP) characteristic feature's of these fluids is that the presence of viscoelasticity of the fluid hikes the order of differential equation. However, there is no consequent increase in the number of boundary conditions. This is proved by Rivlin and Ericksen (1955), where they introduced classification of the viscoelastic fluids in simple ways. However, the main difficulty which arises in the solution of the flow problems of these fluids is that the constitutive equations of viscoelastic fluids usually generate a higher-order derivative term in the momentum equations than equations of Newtonian fluids. Thus, we are facing the problem where the boundary conditions insufficient to solve the problems of this viscoelastic fluid completely. Thus, we need an extra boundary condition to the usual obedience boundary conditions. Ariel (2002) introduced on extra boundary condition in the stagnation point flow of a second grade fluid due to work out the solution on this flow.

Motivated by the work above, this study aims to obtain mixed convection boundary layer flow of a viscoelastic fluid over a sphere subjected to a constant surface temperature by augmenting the extra boundary condition introduced by Ariel (2002). The coupled non-linear partial differential equations governing the flow have been solved numerically. The effects of the mixed convection and viscoelastic parameters on the skin friction and heat transfer around the sphere are studied. The results have also been compared with Nazar *et al*. (2003). We also wish to mention to this end that to our best knowledge this classical very important problem has not been studied before for a viscoelastic fluid so that the results are new for these fluids.

1.2 Problem Statement

The study will investigate the following questions. How the viscoelastic mathematical models of mixed convection boundary layer flow past a sphere? What are the effects of viscoelastic fluids parameter on the skin friction and heat transfer coefficients?

1.3 Objectives of the Study

The main objectives of this study are to carry out the mathematical formulation of the governing equations of the boundary layer flow of a viscoelastic fluid and to study the effects of skin friction and heat transfer coefficients of mixed convection boundary layer flow of a viscoelastic fluid past a sphere. No experiments will be conducted, but we are going to compare our result with Nazar *et al*. (2003) to validate the results.

1.4 Scope of the Study

This study will take into consideration of two-dimensional incompressible viscoelastic fluid model. Problem will be narrow down to boundary layer flow over a sphere subjected to constant temperature. We just considered the mixed convection problem.

1.5 Significance of the Study

The boundary layer problem of viscoelastic fluids theory has generated a lot of interest, and become important in recent years because of their applications in several industrial-manufacturing processes concerning petroleum drilling, manufacturing of foods and paper. In engineering problem, viscoelastic fluids possible to reduce frictional drag on the hulls of ships and submarines. Some typical applications for viscoelastic boundary layer flow over a stretching sheet are polymer sheet extrusion from a dye, glass fiber and paper production, and drawing of plastic films. There are also many applications involving atomization of viscoelastic fluids such as paints, coating, inks, and jet fuels. The relationship between viscoelasticity and drop formation aimed at the production of mono-disperse colloidal sized droplets used the same approach as ink jet printing and particle production.

More to the point, the mixed convection (combined forced and free convection) flow with and without mass transfer occurs in many technological and industrial applications such as solar central receivers exposed to wind currents, nuclear reactors cooled during emergency shutdown, heat exchangers placed in low-velocity environments, boundary-layer control on airfoil, lubrication of ceramic machine parts and food processing. Mixed convection flows arise when the free stream, inertial and near wall buoyant forces have strong effects on the resulting convective heat transport.

Thus, the study of mixed convection of viscoelastic boundary layer flow problems is important due to the strong applications in real life. The result or output of this research will enhanced the understanding of the fluids flow phenomena and improved the development of related industries, for example the manufacturing industries. Besides that, the generation of efficient algorithm of the viscoelastic problem will help in solving the problem of Computational Fluid Dynamics in future

1.6 Outline of Dissertation

This dissertation consists of six chapters; Chapter 1 discusses the background of research, the problem statement, objectives, scope and significance of the study. The literature review for the research problem is given in Chapter 2.

Next, in Chapter 3 we will discuss the constitutive equation of the mixed convection boundary layer flow of a viscoelastic fluid in sphere. The important of viscoelastic term also discuss in this chapter. Then, we proceed to our mathematical formulation in Chapter 4.

Further, Chapter 5 includes the result and discussion of a problem. The results are presented both in the form of tables and graphs. Lastly, Chapter 6 contains a summary of the dissertation and recommendation for future research.