MATHEMATICAL MODELLING IN CAR SUSPENSION SYSTEM

PANG AIK YEN

UNIVERSITY TECHNOLOGY MALAYSIA

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To my beloved parents,

Pang Kee Yok and Lai Yook Hong,

my dearly sister and brother,

Pang Aik Niu and Pang Aik Kah,

to lovely Tiong Chan Xing,

for your everlasting love, care and support.

To my friends and coursemates,

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ABSTRACT

A vehicle suspension system is defined as a mechanical system of springs or shock absorbers connecting the wheels and axles to the chassis of a wheeled vehicle. The purpose of a car suspension system is to improve the comfort of the car subject to different road profiles. The main objective of this study is to formulate a basic quarter car suspension systems subject to a sinusoidal road profile. In this study, the suspension travel limit and magnitude of car's vertical acceleration are validated using the standard of Ford Scorpio car. The comfort of the car is studied by means of the reduction of the body acceleration caused by the car body subject to sinusoidal road profile and 'Sleeping Policeman' road profile. In order to increase the comfort of a car, a refined model is formulated by including the effects of wheel and axle in the quarter car suspension system. The results of the study is obtained and analyzed by using Fourth Order Runge-Kutte Method via Maple 12.

ABSTRAK

Suatu sistem gantungan kenderaan ditakrifkan sebagai sistem mekanikal spring atau penyerap kejutan yang bergabung dengan tayar dan gandal kepada chasis pada suatu kenderaan beroda. Tujuan sistem gantungan kereta adalah untuk menambahbaik keselesaan kereta yang dinaiki apabila melalui jalan dengan permukaan yang berbeza. Objektif utama kajian ini adalah untuk memodelkan satu per empat sistem gantungan kereta mengikut profil jalan berbentuk sinus. Dalam kajian ini, had penggantungan perjalanan dan magnitud cepatan mencangcang kereta disahkan dengan mengguna piawaian kereta Ford Scorpio. Keselesaan kenderaan bagi system gantungan kereta mengikut profil jalan 'sleeping policeman' dikaji dan dibandingkan dengan sistem gantungan kereta mengikut profil jalan berbentuk sinus. Dalam usaha meningkatkan keselesaan kereta yang dinaiki, sistem gantungan kereta diperbaiki dengan mengambil kira kesan roda dan gandal dalam satu per empat sistem gantungan kereta. Keputusan kajian diperolehi dan dianalisis menggunakan Kaedah Runge-Kutte Peringkat Keempat dengan pengaturcaraan Maple 12.

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CHAPTER 1

INTRODUCTION

1.1 Introduction

In our modern societies, most of the passengers prefer a smooth journey with advanced ride quality rather than a luxury or an excellent outlook design of vehicle. Ride quality of a vehicle has been the first priority of customer needed. A vehicle's suspension system is one of the decisive factors in determining the quality of ride. Even the smoothest roads are full of variations in height and surface texture. Combined with the car's natural shake as the engine runs and the constant buffeting of air at high speeds, suspension makes driving a comfortable and safe undertaking.

Apart from the car's tyres and seats, the suspension is the prime mechanism that separates your bum from the road. It also prevents the car from shaking itself to pieces. No matter how smooth the road is, it is a bad place to propel over a ton of metal at high speed. So we rely upon suspension. People who travel on trains wish that those vehicles relied on suspension too. But they do not and that's why the ride is so harsh. Actually it is harsh because trains have no lateral suspension to speak of. So as the rails deviate from side-to-side slightly, so does the entire trains, and its passengers. In a car, the rubber in car's tyres helps with this little problem. In it is

most basic form, suspension consists of two components: springs and shock absorbers.

A suspension system defined as a mechanical system of springs or shock absorbers connecting the wheels and axles to the chassis of a wheeled vehicle. A lot of the system's work is done by the springs. Under normal conditions, the springs support the body of the car evenly by compressing and rebounding with every up-and-down movement. This up-and-down movement, however, causes bouncing and swaying after each bump and is very uncomfortable to the passenger. These undesirable effects are reduced by the shock absorbers.

In this research, a basic model of car suspension system is formulated when a car is moving over a 'sinusoid' road surface profile. In this model, the shock absorber is modelled as a simple spring-dashpot system with spring stiffness and dashpot constant. Most car suspension systems use springs in the form of a coil or a series of leaves and are usually made of steel, although rubber and plastic composites are possible. The dashpot which commonly known as damper is usually a hydraulic device which is effectively a piston moving inside a housing containing fluid.

Next, a model refinement will be constructed by including wheel and axle in the model with the same road surface profile to improve the car suspension system. Aside of this, a model of car suspension will be formulated when a car moves over a series of 'sleeping policemen' road surface profile. All the mathematical problems will solve by analytical or by Maple programming.

1.2 Background of the Problem

A suspension system plays a vital role to smooth out the ride while maintaining excellent control. The suspension system has two basic functions, to keep the car's wheels in firm contact with the road and to provide a comfortable ride for the passengers. This may sound like a simple job, but with acceleration comes force, and force translates into raw energy. When a vehicle accelerates down a road, bumps cause forward energy to be converted into vertical energy, which travels through the frame of the vehicle. Without coil and leaf springs to absorb this, the vertical energy would cause the vehicle to jump up off the road, reducing tire friction and control. The car would then come bounding back down with even greater force, making for a very uncomfortable and dangerous ride. As a consequence, the car suspension system should be investigated and suspensions for car also have their potential to improve vehicle performance.

1.3 Statement of Problem

This research is concerning on the following questions. How do formulate a basic quarter car suspension model by a ‘sinusiod’ road profile? How do solve this basic model by analytical methods? How do refine the basic quarter car suspension model by including wheel and axle behavior? How do construct a mathematical formulation of the Fourier series when a car moves over a series of ‘sleeping policemen’ road profile? How do solve the mathematical by constructing Maple 12 programming? How do plot the graphs by Maple 12 software? How do interpretation and validation of the graphical results of each model which obtain by Maple 12 Programming?

1.4 Objectives of the Research

The objectives of this research are:

(i) To formulate a basic quarter car suspension model by a 'sinusiod' road profile.

(ii) To refine the basic quarter car suspension model of 'sinusiod' road profile by including wheels and axle behavior.

(iii) To formulate a car suspension model based on the situation when a car moves over a series of 'sleeping policeman' road profile.

1.5 Significance of the Research

Nowadays, the automotive industries still put great efforts in producing highly developed vehicles' suspension system as it is one of the decisive factors in determining the quality of ride. This research will give benefits to Mathematicians, Physicists, Engineers and others field indirectly to improve the vehicles' suspension system and also improve vehicles' performance.

The advantage that we gain is we will able to understand the application of Mathematics in our real life. In this case the application of Mathematics is specific to a car suspension system by different road profile. In addition, we can learn how to plot graphs by using Maple software. We can also learn how to construct a Maple programming to solve the problems of different equations in this research. As a result, the problems can be solved to obtain approximate answers. By using Maple software, it save time and the graph can be plotted immediately. Although it takes

time to familiar oneself with Maple computer programming system, but this time is well-spent. After carry out this research, we gain a greater understanding and appreciation of the power of mathematics.

1.6 Scope of the Research

The car suspension system is ideal to have a general understanding of the functions of car suspension system. There are some engineers design good car suspension systems which can be used in our real life to provide customers comfortable and safe ride. However, this research construct a basic model for a car suspension system where the shock absorber is modeled as a simple spring-dashpot system with spring stiffness and dashpot constant. The model is obtained from the application of Newton's Second Law of Motion in a vertical direction of car body.

This research will focus on differential equation approach and linear ordinary differential equation approach by applying on the car suspension system. Mathematical formulations based on a quarter car suspension system and different road surface profiles will be modeled. These mathematical models will be solved by Maple 12 programming and the graphical results will be obtained. Maple will be used to graphically and numerically simulate the solution to obtain approximate answers. The assembly of the complete model and the general analysis of the smoothness of solutions benefits greatly from Maple programming. The numeric and graphical tools provided by Maple are well suited for solving the system for specific sets of parameter values. Therefore, the car suspension system problem is ideally suited for analysis with Maple.

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