Mathematical Analysis of Leakage Current Level in Correlation with Environmental Stresses for Solid Insulating Material under Tracking Test

M. Afendi M. Piah, *Member, IEEE*, and Ahmad Darus, *Member, IEEE*

***Abstract*—Environmental pollution can cause the insulator material to become progressively coated with dirt and chemicals in the long run. In the presence of wet atmospheric conditions, the leakage current flows due to the development of conducting path across the insulator surface. The level of leakage current depends on the surface wetting and the degree of electrolyte contamination as well as the environmental factors. An analytical approach based on dimensional analysis technique is applied to develop a mathematical model of leakage current in correlation with the environmental stresses. In order to verify the developed model, an inclined-plane tracking test is conducted on the polymeric insulating materials. The experimental work is carried out by measuring the magnitude of surface leakage current at different levels of contamination. Simulation results of the model have shown good agreement to the experimental results and provide useful information on describing the test condition of the tracking test procedures.**

***Index Terms*—contamination, dimensional analysis, insulators, leakage current, mathematical analysis, tracking**

I. Introduction

MOST of high voltage insulators are being used in outdoor applications. Environmental pollution can cause the insulators to become progressively coated with dirt and chemicals in the long run. This pollution coating does not have a detrimental effect when the insulator is dry. The electrostatic field determines the voltage distribution of such a dry insulator and very small capacitive leakage current (LC) flows across the entire insulator. However in the presence of wet atmospheric conditions, the contamination particles on the insulator surface will dissolve into the water and provide a continuous path between the high voltage electrode and ground.

When the insulator is wet, a resistive surface LC flows, which is generally many orders of magnitude higher than the capacitive current in the case of dry insulators [1]. This LC results in non-uniform heating of the contamination layer that eventually causes dry-bands to be formed at the narrow sections where the LC density is highest. The continuous formation of dry-bands causes the fluctuation of LC level due to the non-uniform surface resistance [2]. The development of LC, which is the main factor in surface tracking phenomena significantly affect the insulating performance of polymeric insulators. Previous works have shown many experimental results in studying the effect of electrical and environmental stresses on the polymeric materials [3-5]. The variations of LC levels and the formation of dry-bands are due to the changes of hydrophobicity loss, thus lead to dynamic changing on average electric field intensity across the insulator. This paper describes the analytical technique on the development of mathematical relationship of LC levels with the environmental stresses. Furthermore, the inclined-plane tracking test of IEC 587 is conducted to verify the model.

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II. Mathematical Analysis

The correlation of LC level with the environmental parameters under tracking test is developed by employing dimensional analysis (DA) technique. This analytical technique can be made as a contribution to model formation and has been used successfully on a very wide range of applications in all experimentally based areas of physical sciences and engineering [6-7].

From IEC 587 [8] test procedure, when the test was conducted at the fixed test voltage, it was found that the LC (I) that flows on the insulating material depends on the average electric field across the sample (E), electrolyte flow-rate (q), electrolyte resistivity (ρ), environmental pressure (p) and absolute humidity (h). Table I shows the physical quantity of parameters involved and their corresponding fundamental dimensions. M, L, T and Q are the mass, length, time and charge respectively.

TABLE I
FUNDAMENTAL DIMENSIONS

Parameters	Dimension
Leakage current (I)	$T^{-1} Q$
Average electric field (E)	$M L T^{-2} Q^{-1}$
Electrolyte flow-rate (q)	$L^{3} T^{-1}$
Electrolyte resistivity (ρ)	$M L^{3} T^{-1} Q^{-2}$
Environmental pressure (p)	$M L^{-1} T^{-2}$
Absolute humidity (h)	$M L^{-3}$

The parameters relationship can be expressed as follows;

$$I = f(E, q, \rho, p, h) \qquad (1)$$

where f is an unknown function. The dimensional matrix of the parameters is arranged according to their corresponding fundamental dimensions and can be written as;

	k_1	k_2	k_3	k_4	k_5	k_6
	I	E	q	ρ	p	h
M	0	1	0	1	1	1
L	0	1	3	3	-1	-3
T	-1	-2	-1	-1	-2	0
Q	1	-1	0	-2	0	0

where k_m is the power index of the respective parameter. The rank (r) of the dimensional matrix is 4, and the number of parameters (n) is 6. According to Buckingham-π theorem [7], the solution can be expressed in the form of $(n - r) = 2$ independent dimensionless products (π_s), which have a unity dimension. The power indices of the parameters can be determined from the following matrix, which is developed from the principle of dimensional homogeneity.

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 3 & 3 & -1 & -3 \\ -1 & -1 & -2 & 0 \\ 0 & -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} k_3 \\ k_4 \\ k_5 \\ k_6 \end{bmatrix} = \begin{bmatrix} -k_2 \\ -k_2 \\ k_1 + 2k_2 \\ -k_1 + k_2 \end{bmatrix} \tag{2}$$

The solution of equation (2) gives;

$$k_3 = -\tfrac{3}{4}k_1 + \tfrac{1}{4}k_2 \ ; \ k_4 = \tfrac{1}{2}k_1 - \tfrac{1}{2}k_2$$
$$k_5 = -\tfrac{3}{8}k_1 - \tfrac{7}{8}k_2 \ ; \ k_6 = -\tfrac{1}{8}k_1 + \tfrac{3}{8}k_2 \tag{3}$$

The first dimensionless product (π_1) is determined by assigning $k_1 = 1$ and $k_2 = 0$. Meanwhile π_2 is determined by setting $k_1 = 0$ and $k_2 = 1$. Therefore, the complete set of dimensionless products can be developed as follows according to the calculated values of the power indexes.

	I	E	q	ρ	p	h
π_1	1	0	$-\frac{3}{4}$	$\frac{1}{2}$	$-\frac{3}{8}$	$-\frac{1}{8}$
π_2	0	1	$\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{7}{8}$	$\frac{3}{8}$

Based on the complete set of the dimensionless products above, the expressions of π_1 and π_2 are shown in equations (4) and (5).

$$\pi_1 = \frac{I \cdot \rho^{\frac{1}{2}}}{q^{\frac{3}{4}} \cdot p^{\frac{3}{8}} \cdot h^{\frac{1}{8}}} \tag{4}$$

$$\pi_2 = \frac{E \cdot q^{\frac{1}{4}} \cdot h^{\frac{3}{8}}}{\rho^{\frac{1}{2}} \cdot p^{\frac{7}{8}}} \tag{5}$$

Equations (4) and (5) can now be written in the forms of LC and average electric field as a function of environmental physical variables and are given by equations (6) and (7). The constants of D_a and D_b are called dimensional constants and must be determined from the experiment.

$$I = D_a \left(p^3 h\right)^{\frac{1}{8}} \left(\frac{q^3}{\rho^2}\right)^{\frac{1}{4}} \tag{6}$$

$$E = D_b \left(\frac{p^7}{h^3}\right)^{\frac{1}{8}} \left(\frac{\rho^2}{q}\right)^{\frac{1}{4}} \tag{7}$$

III. Experimental Procedure

The inclined-plane test (IPT) of IEC 587 under severe ambient conditions was conducted to verify the developed model. Fig. 1 shows the experimental set-up associated with the on-line leakage current and electric field monitoring system. The slab-shaped of polymeric sample with dimension of 120x50x6 mm was used. The arrangement and installation of the equipment and electrolyte used are described in [8]. A 2.0 kV test voltage at the output of the high voltage transformer was used. The test was carried out with variable contaminant flow-rates at different levels of resistivity. The value of LC and voltage across the sample were recorded every second in a period of one minute. The mean values of a set of data collected from the both parameters as well as the values of electrolyte flow-rate and resistivity for each level were used to verify the model. Table II shows the parameters values used throughout the test.

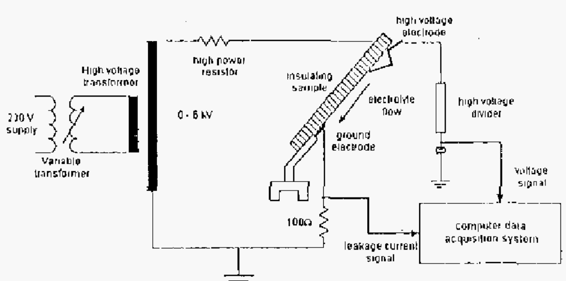

Fig. 1. Experimental set-up

TABLE II
TEST PARAMETER CONDITIONS

Test voltage = 2 kV	Average electrolyte flow-rate (mm/min)	Average electrolyte resistivity (kΩ-cm)
Electrode distance = 5 cm	0.07	0.990
	0.19	0.719
	0.31	0.529
	0.43	0.373
	0.56	0.308

IV. RESULTS AND DISCUSSION

The experiment was conducted at the stable atmospheric condition, which is considered to have no significant change on the environmental pressure and humidity. Assuming both parameters (p and h) are constant, equation (6) is simplified as follows;

$$I = f(q,\rho) = A \cdot \left[\frac{q^3}{\rho^2}\right]^{\frac{1}{4}} \tag{8}$$

The value of the factor A is determined from the experimental results and is dependent on the sample material and atmospheric conditions. This factor was calculated using Mathcad software by manipulating the experimental data into the equation (8). Equation (8) shows that the level of LC increases when the electrolyte flow-rate increases and also when the electrolyte resistivity decreases. Tests on the different compositions of linear low-density polyethylene-natural rubber blends show that the calculated values of factor A were in the range of 8.0 to 10.0. The analytical results obtained from the model and the experimental results are plotted together for comparison, which are illustrated in Fig. 2 and Fig. 3. It is observed that the results calculated from the model are in good agreement with the experimental results.

However, the results at the highest electrolyte resistivity as shown in Fig. 2 indicate higher deviation. It is believed that at a further increase of electrolyte resistivity, the higher resistivity could not affect much on the LC level since the measured LC from the experiment is very small.

In addition, the results shown in Fig. 3 indicate the same observation. At the lowest electrolyte flow-rate, a higher deviation of the results is noticed. This can be explained by the fact that at this condition, the distribution of electrolyte on the surface is not uniform which leads to non-uniform resistance of insulator surface. Observations from the experiment also show that at a very low flow rate, the water film tends to dry out before it reaches the ground electrode. Consequently, a dry region is found on the insulator surface for most of the time.

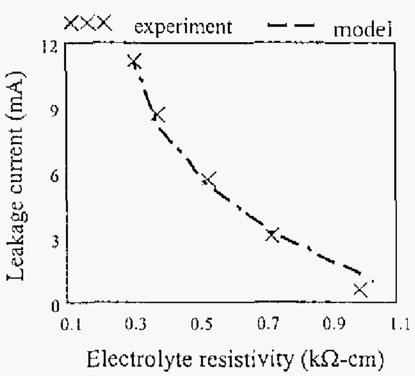

Fig. 2. Leakage current levels at different electrolyte resistivity (Test voltage = 2 kV; Factor A = 9.5)

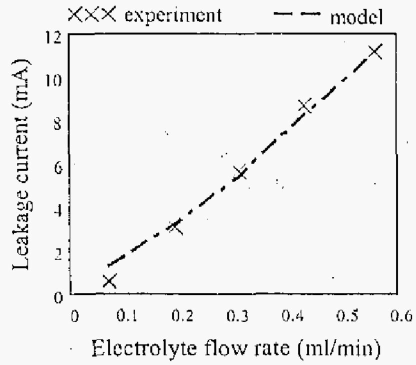

Fig. 3. Leakage current levels at different electrolyte flow-rate (Test voltage = 2 kV; Factor A = 9.5)

Variations of LC levels of equation (8) against any value of electrolyte resistivity and flow rate are depicted in Fig. 4. This contour plot can be used to predict the value of surface LC at any required electrolyte resistivity and flow-rate. It is observed that at very low electrolyte resistivity, the LC cannot be properly defined.

For the tracking test to be successfully conducted, a scintillation that developed the tracking was observed at the LC levels of 9 to 10 mA. Based on these levels and the proposed range of electrolyte resistivity from IEC 587 procedures, it was found that the suitable range of electrolyte flow-rate was 0.15 to 0.9 ml/min. These values of flow-rate are found to be in agreement with the range specified by the IEC 587 test procedures.

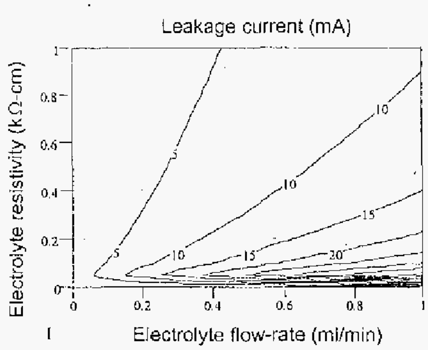

Fig. 4. Leakage current plots at different electrolyte resistivity and flow-rate

The magnitude of the average electric field stress across the sample is influenced by the variations of LC. At higher electrolyte flow-rate, experiment results show that the average electric field was decreased due to the decreased surface resistivity. This can be explained by the probable existence of additional branching of conducting film routes, which consequently reduce the field strength [9]. Analytical results from equation (7) show that the average electric fields tend to decrease when the flow-rate increases.

The results obtained from the experiment are used to verify the model of equation (7). The minimum measured voltage has been recorded as 0.625 kV, which gives 0.125 kV/cm minimum electric field for 5 cm electrodes distance. According to equation (7), it seems that the average electric field stress could not be determined if the electrolyte flow-rate is set to zero, or in other words, no electrolyte exists in the test. But in the real situation and affirmed by the observation from the experiment conducted, the average electric field existed across the insulator even though the experiment was conducted without electrolyte under the high voltage stress. For this reason, it is believed that there must be a limit value for the average electric field. Equation (7) is modified by adding a constant E_{min}, to make the analysis valid as shown in equation (9). E_{min} can be stated as a minimum measured of electric field across insulator during dry condition or the value of average electric field recorded whenever the LC was initially detected.

$$E = B \cdot \left[\frac{\rho^2}{q} \right]^{\frac{1}{4}} + E_{min} \qquad (9)$$

Taking E_{min} as 0.125 kV/cm, the value of the factor B is found to be 0.04 from the experimental results. Fig. 5 illustrates the variations of average electric field stress at any value of electrolyte flow-rate and resistivity. It is observed that the average electric field cannot be properly defined at the flow-rate below than 0.1 ml/min. In fact, at a very low electrolyte resistivity (less than 0.1 kΩ-cm), the average electric field stress across the sample tends to be constant with further increase in electrolyte flow-rate.

Based on the contour plots of the LC (Fig. 4) and the average electric field (Fig. 5), the minimum electrolyte resistivity has to be 0.1 kΩ-cm, while the minimum flow-rate is 0.15 ml/min. Below these values, the validity of the model could not be accepted. This is due to the existence of anomalous circumstances such as discharge phenomena and electrolyte evaporation, which is not considered in the analysis when developing the models.

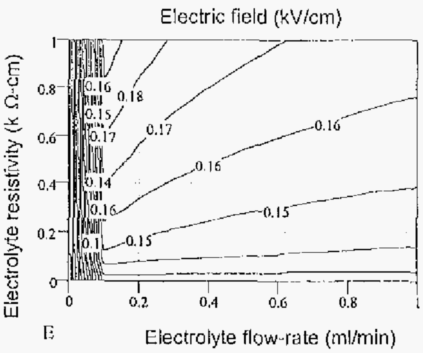

Fig. 5. Average electric field plots at different electrolyte flow-rate and resistivity

V. Conclusion

An analytical method based on dimensional analysis technique has been described on modeling the leakage current and average electric field of insulator under environmental stresses. The proposed method is very practical and useful on a very wide range of applications in all experimentally based area. Curves plotting of the developed models have shown good agreement to the experimental results. It is found that the surface wetting and the degree of electrolyte contamination affect both leakage current and average electric field of the insulating material. Moreover, the results of the models could provide useful information on describing the test conditions of the inclined-plane tracking procedures. The method can be extended by considering other physical variables in order to give reliable predictions of the model.

VI. References

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VII. Biographies

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