ARTIFICIAL NEURAL NETWORK MODEL FOR RAINFALL-RUNOFF RELATIONSHIP

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Abstract.   The modelling of hydraulic and hydrological processes is important in view of the many uses of water resources such as hydropower generation, irrigation, water supply, and flood control. There are many previous works using the artificial neural network (ANN) method for modelling various complex non-linear relationships of hydrologic processes. The ANN is well known as a flexible mathematical structure and has the ability to generalize patterns in imprecise or noisy and ambiguous input and output data sets. The study area is Sungai Lui catchment (Selangor, Malaysia). This paper presents the proposed ANN model for prediction of daily runoff using the rainfall as input nodes. The method for selection of input nodes by [10] and [5] is applied. Further, the results are compared between ANN and HEC-HMS model. It has been found that the ANN models show a good generalization of rainfall-runoff relationship and is better than HEC-HMS model.

Key words: hydrologic, artificial neural network, rainfall-runoff relationship

Abstrak.   Permodelan bagi proses hidraulik dan hidrologi adalah penting apabila dilihat dari sudut kepelbagaian penggunaan sumber air seperti janakuasa hidroelektrik, pengairan, pengagihan bekalan air, dan kawalan banjir. Terdapat banyak kajian sebelum ini yang telah menggunakan kaedah rangkaian neural tiruan atau *artificial neural network* (ANN) untuk permodelan pelbagai perhubungan tak linear dan kompleks dalam proses hidrologi. Kaedah rangkaian neural tiruan ini telah diketahui bahawa ia merupakan suatu struktur matematik yang mudah ubah (*flexible*) dan berpotensi untuk menjana dan merumus set-set data masukan dan keluaran yang kurang tepat atau kabur dan tidak dihalusi dengan sempurna. Kawasan kajian adalah kawasan tadahan Sungai Lui (Selangor, Malaysia). Kertas kerja ini mengutarakan cadangan menggunakan kaedah rangkaian neural tiruan ini bagi mendapatkan jumlah air larian permukaan harian dengan menggunakan hujan sebagai nod masukan kepada model berkenaan. Terdapat dua kaedah telah digunakan dalam pemilihan bilangan nod masukan iaitu seperti yang telah dicadangkan oleh [10] dan [5]. Seterusnya, hasil keputusan yang diperolehi daripada permodelan rangkaian neural tiruan ini dibandingkan dengan hasil keputusan yang diperolehi daripada model HEC-HMS. Didapati bahawa model rangkaian neural tiruan dapat menjana dan merumus perhubungan antara air larian permukaan dan curahan hujan lebih baik berbanding dengan model HEC-HMS.

Kata kunci: hidrologi, rangkaian neural tiruan, hubungan air larian permukaan-curahan hujan

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1.0   INTRODUCTION

The rainfall-runoff model is required to ascertain the relationship between rainfall and runoff. Hydrologists are often confronted with problems of prediction and estimation of runoff using the rainfall date. In actual fact the relationship of rainfall-runoff is known to be highly non-linear and complex. The spatial and temporal precipitation patterns and the variability of watershed characteristics create a more complex hydrologic phenomena. Various well known currently available rainfall-runoff models (HEC-HMS, MIKE-11, SWMM, etc.) have been successfully applied in many problems and watersheds. However, the existing popular rainfall-runoff models can be detected as not flexible and they require many parameters for calibration. Obviously, the models have their own weaknesses, especially in the calibration processes and the ability to adopt the non-linearity of processes. Therefore, the present study was undertaken to develop rainfall-runoff models using artificial neural network method that can be used to provide reliable and accurate estimates of runoff. The artificial neural network model (ANN) is proposed to uncover the non-linear relationship between rainfall and runoff. Further, the ANN model is compared with the HEC-HMS model.

An ANN can be defined as 'a data processing system consisting of a large number of simple, highly interconnected processing elements (artificial neurons) in an architecture inspired by the structure of the cerebral cortex of the brain' [1]. The ANN models have been used successfully to model complex non-linear input-output relationships in an extremely interdisciplinary field. It behaves as a black-box model. The natural behaviour of hydrological processes is appropriate for the application of ANN method. The ANN method has been proven to be potentially useful tools in hydrological modelling such as for rainfall-runoff modeling processes [2, 3, 4, 5]; flow prediction [6, 7]; water quality predictions [8]; operation of reservoir system [9, 10]; and groundwater reclamation problems, [11]. In Malaysia, the application of neural network method is widely used in the field of mechanics, robotics, electrical, etc. In the hydrology field, it is still in nascent stages. There are only few of research have implemented the neural network approach in the hydrological study. For example, [10] has applied the neural network method to forecast of net inflows for reservoir operation.

The present study develops rainfall-runoff models using ANN method based on multilayer perceptron (MLP) and radial basis function (RBF) techniques. The modelling work is carried out using the rainfall and runoff records (1993-1997) from Sg. Lui catchment (Selangor, Malaysia) as shown in Fig. 1. It is a semi-developed area (30% urban and 70% natural) involving 68.1 km^2 of catchment area. The catchment is located in northwestern part of Petaling Jaya district of Selangor, where the latitude is 03° 10' 25" and the longitude is 101° 52' 20". The Department of Irrigation and Drainage (DID) Malaysia installed four raingauges and one water level recorder at various location with the study area (Fig. 1).
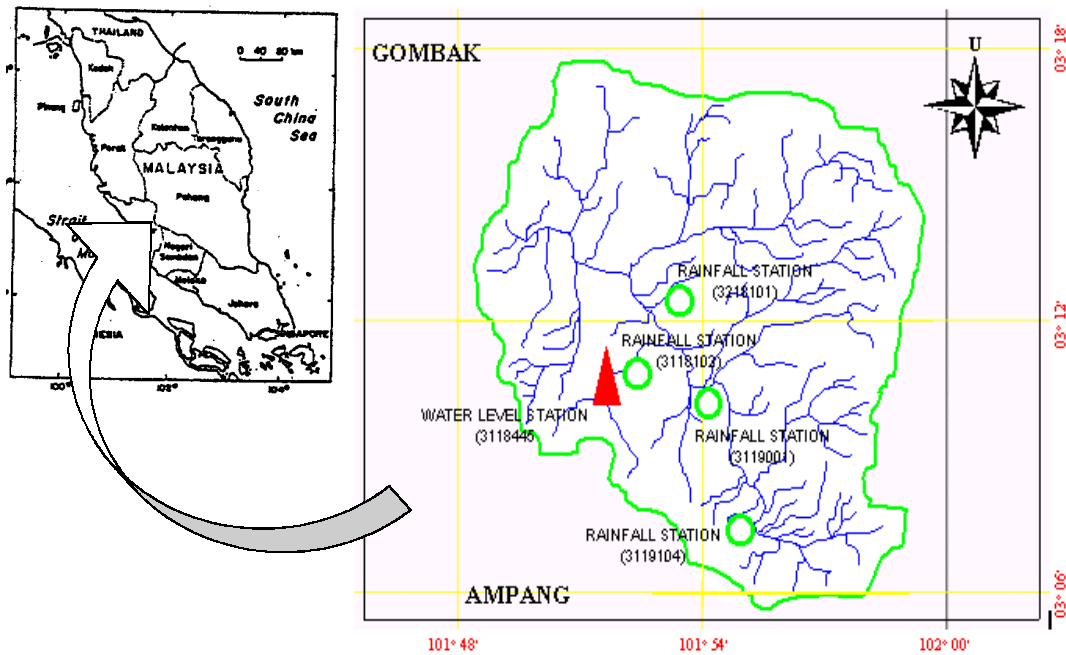
Figure 1   Sungai Lui catchment area

2.0   NEURAL NETWORK MODEL

Two types of neural network architectures, namely multilayer perceptron (MLP) and radial basis function (RBF) network are implemented. The architecture of an ANN is designed by weights between neurons, a transfer function that controls the generation of output in a neuron, and learning laws that define the relative importance of weights for input to a neuron [12]. It will process the information in a way that is previously trained, to generate satisfactory results. Neural network can learn from experience, generalize from previous examples to new ones, and abstract essential characteristics from inputs containing irrelevant data [13]. The main control parameters of ANN model are interneuron connection strengths also known as weights and the biases.

2.1   Multilayer Perceptron

The first technique of neural network modelling is the MLP model. The MLP is the most commonly used neural computing technique. The architecture of a typical neuron is shown in Fig. 2. Basically the MLP consists of three layers: the input layer, where the data are introduced to the network; the hidden layer, where the data are processed (that can be one or more) and the output layer, where the results for given inputs are produced.

Each layer is made up of several nodes, and layers are interconnected by sets of correlation weights. Each input node unit ($i = 1,...,m$) in input layer broadcasts the
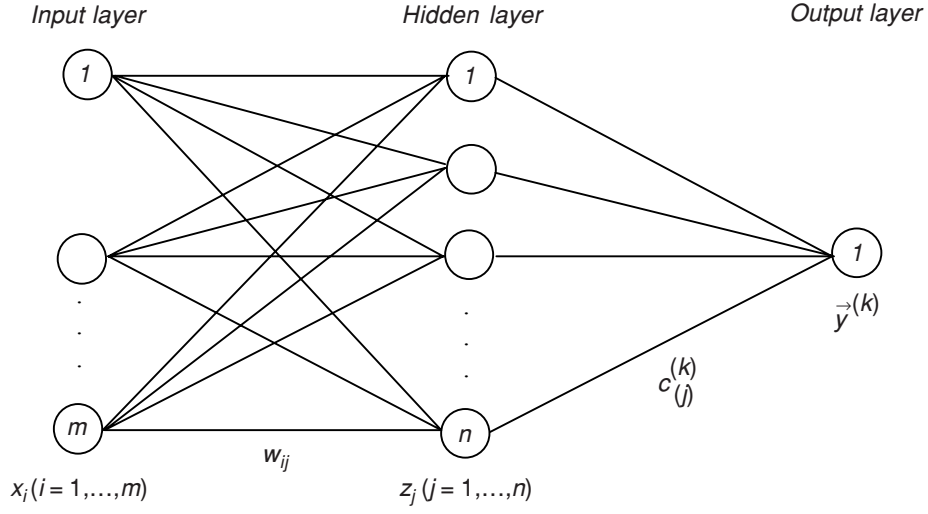
Figure 2 Structure of a MLP model

input signal to the hidden layer. Each hidden node ($j = 1,...,n$) sums its weighted input signals according to [13],

$$z\_in_j = w_{0j} + \sum_{i=1}^{m} x_i\, w_{ij} \tag{1}$$

applies its activation function to compute its output signal from the input data as

$$z_j = f(z\_in_j) \tag{2}$$

and sends this signal to all units in the hidden layer. Note that w_{ij} is the weight between the input layer and the hidden layer, w_{0j} is the weight for the bias; and x_i is the input rainfall signal. In this study, a sigmoid function used is hyperbolic-tangent as proposed by [5]. This function is continuous, differentiable everywhere, monotonically increasing, and it is the most commonly used function in the backpropagation networks. The tangent sigmoid (tansig) activation function will process the signal that passes from each node by

$$f(z\_in_j) = \frac{2}{1 + e^{-2z\_in_j}} - 1 \tag{3}$$

Then, from the second layer, the signal is transmitted to the third layer. The output unit ($k = 1$) sums its weighted input signals as

$$x\_in_k = c_0^{(k)} + \sum_{j=1}^{n} z_j\, c_j^{(k)} \tag{4}$$

and applies its activation function to compute its output signal,

$$\overline{y}^{(k)} = f(x\_in_k) \tag{5}$$

where $c_j^{(k)}$ is the weight between the second layer and the third layer, and $c_0^{(k)}$ is the weight for the bias. The output node ($k = 1$) receives a target pattern corresponding to the input training pattern, computes its error information, calculates its weight correction (used to update $c_j^{(k)}$ later), and its bias correction (used to update $c_0^{(k)}$ later) term.

Note that $\overline{y}^{(k)}$ is the neural network output. The error information is transferred from the output layer back to earlier layers. This is known as the backpropagation of the output error to the input nodes to correct the weights.

2.2   Radial Basis Function

The second technique of the neural network modelling is the RBF. RBF is supervised and feed forward neural network. Fig. 3 illustrates the designed architecture of the RBF that can be considered as a three layer network. The hidden layer of RBF network consists of a number of nodes and a parameter vector called a 'center' which can be considered as the weight vector. The standard Euclidean distance is used to measure how far an input vector from the center is. In the RBF, the design of neural networks is a curve-fitting problem in a high dimensional space [14]. Training the RBF network implies finding the set of basis nodes and weights. Therefore, the learning process is to find the best fit to the training data.

The transfer function of a RBF network is mostly built up of Gaussian (e^{-d^2/β^2}) rather than sigmoid, where d is the distance from the center and β is a parameter. The Gaussian functions decrease with distance from the center. The transfer functions of the nodes are governed by nonlinear functions that is assumed to be an approximation of the influence that data points have at the center. The Euclidean length is represented by r_j that measures the radial distance between the datum vecto $\underline{y}(y_1, y_2, \cdots y_m)$; and the radial center $\underline{Y}^{(j)} = (w_{1j}, w_{2j}, \ldots w_{mj})$; can be written as,
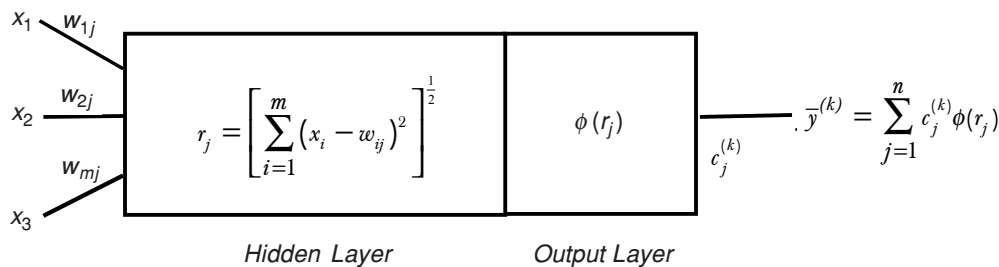
Figure 3   Structure of a RBF model

$$r_j = \left\| \underline{y} - \underline{Y}^{(j)} \right\| = \left[\sum_{i=1}^{m} (y_i - w_{ij})^2 \right]^{\frac{1}{2}} \tag{6}$$

A suitable transfer function is then applied to r_j to give,

$$\phi(r_j) = \phi \left[\left\| \underline{y} - \underline{Y}^{(k)} \right\| \right] \tag{7}$$

Finally the output layer ($k = 1$) receives a weighted linear combination of $\phi(r_j)$,

$$\overline{y}^{(k)} = \sum_{j=1}^{n} c_j^{(k)} \phi(r_j) = \sum_{j=1}^{n} c_j^{(k)} \phi \left(\left\| \underline{y} - \underline{Y}^{(j)} \right\| \right) \tag{8}$$

3.0   MULTIPLE LINEAR REGRESSION (MLR)

Regression analyses are among the oldest statistical techniques used in hydrology. Multiple regression method has been used in various scientific and engineering disciplines. There are many problems in hydrology that may be solved by multiple regression procedures. Multiple Linear Regression (MLP) represents a mathematical equation expressing one random variable as being correlatively related to another random variable, or to several random variables. The regression equation may be any function that can be fitted to a set of points of observed variables. A commonly used criterion for the 'best' fit is to select the equation yielding the largest value of correlation of coefficient (R^2). One of the most commonly used procedures for selecting the best regression equation is stepwise regression. The general equation of multiple regression model is [15],

$$y = a + b_1 \, x_1 + b_2 \, x_2 + \ldots + b_k \, x_k + e \tag{9}$$

where, y, the dependent variable, and several other variables x_1, x_2, x_3, …, x_k, the independent variables, and in which the objective requires the relationship between the variables y and the variables x_1, x_2, x_3, …, x_k to be investigated. a is constant, b_1, b_2, …, b_k are the coefficients and e is the residual or error. For any i-th set of observations, the model can be written more conveniently as,

$$y_i = \alpha + \beta_1 (x_{1i} - \overline{x}_1) + \beta_2 (x_{2i} - \overline{x}_2) + \cdots + \beta_k (x_{ki} - \overline{x}_k) + e_i \tag{10}$$

where, x_{ki} is the value of independent variable x_k at i-th set of observations totalling n; and $\overline{x}_k = 1/n \sum_{i=1}^{n} x_{ki}$. Using the criterion of minimization of the sum-squared error, it is possible to show that estimate of,

$$\alpha = \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \tag{11}$$

where n is the total number of observation sets. One needs to estimates β using,

$$\beta = S_{xx}^{-1} S_{xy} \tag{12}$$

where,

$$\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_k \end{bmatrix}$$

The equations may now be condensed into,

$$\begin{bmatrix} S_{x1y} \\ S_{x2y} \\ \vdots \\ S_{xky} \end{bmatrix} = \begin{bmatrix} S_{x1x1} & S_{x1x2} & \cdots & S_{x1xk} \\ S_{x1x2} & S_{x2x2} & \cdots & S_{x2xk} \\ \vdots & \vdots & & \vdots \\ S_{x1xk} & \cdots & \cdots & S_{xkxk} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_k \end{bmatrix}$$

or,

$$S_{xy} = S_{xx} \beta$$

where,

$$S_{xjy} = \sum_{i=1}^{n} (x_{ji} - \bar{x}_j)(y_i - \bar{y}) \qquad \text{(for } j = 1, 2, \ldots, k)$$

$$S_{xj\,x1} = \sum_{i=1}^{n} (x_{ji} - \bar{x}_j)(x_{li} - \bar{x}_l) \qquad \text{(for } j = 1, 2, \ldots, k \text{ and } l = 1, 2, \ldots, k)$$

The multiple linear regression equation for Sg. Lui catchment has been derived and as follows:

$$\begin{aligned} y(t) = {} & 0.881 + 0.204x_t + 0.336x_{t-1} + 0.167x_{t-2} + 0.131x_{t-3} + 0.096x_{t-4} + \\ & 0.123x_{t-5} + 0.082x_{t-6} + 0.077x_{t-7} + 0.067x_{t-8} + 0.064x_{t-9} + \\ & 0.055x_{t-11} + 0.072x_{t-12} + 0.063x_{t-13} + 0.063x_{t-15} + 0.059x_{t-17} \end{aligned} \tag{13}$$

where $y(t)$ is predicted runoff; x_t, x_{t-1}, …, x_{t-15}, x_{t-17} are the rainfall input for the corresponding time.

4.0   MODEL APPLICATION

The steps involved in the identification of a nonlinear model of a system are selection of input-output data suitable for calibration and verification; selection of a model structure and estimation of its parameters; and validation of the identified models.

The selection of training data that represents the characteristics of a watershed and meteorological patterns is extremely important in modelling [16]. Input variable (rainfall) is selected to describe the physical phenomena of the rainfall-runoff process, in order to forecast runoff. Record of 5 years of daily rainfall-runoff series of Sungai Lui catchments, Selangor is selected to evaluate the performance of the neural network model. The data used consist of two sets: the first three years of data (1993-1995) are used for model calibration (training) in the case of ANN, and the remaining two years of data (1996-1997) are used for model validation (testing). Increasing the number of training data in the training phase, with no change in neural network structure, will improve performance on the training and testing phase. Thus, it is dependant on providing an adequate number of training data.

In this particular study, the structure of ANN model is designed based on methods proposed by [5] and [10]. The first method proposed by [5] treat the rainfall as directly related to runoff by using the following equation,

$$y(t) = f\{x(t)\} \tag{14}$$

This model treats the rainfall as directly related to runoff at the present time t. The goodness-of-fit statistics are computed for both training and testing for each ANN architecture. The input node at $(t-1)$ is added as an additional input variable to the model. During training and testing the goodness of fit statistics is used to evaluate the suitability of input variable $(t-1)$. This procedure is repeated by adding rainfall at previous time periods as input variable until there is no significant change in model training and testing accuracy. The second ANN structure proposed by [10] employed the multiple linear regression (MLP) input variables as input nodes. The MLR model is a relationship as an equation connecting the response or dependent variable, y (runoff) and independent variables, x_1, x_2, …, x_n (present and previous rainfall series).

There are no fixed rules about the number of nodes in the hidden layer. A trial and error procedure is generally applied in selecting the number of hidden layers and in assigning the number of nodes to each of these layers. [17] proposed that normally, neural networks were developed using 15, 30, 45, 60 and 100 hidden nodes. This procedure is also looked over the performance of neural network model with different number of hidden nodes.

5.0   MODEL PERFORMANCE CRITERIA

Both MLP and RBF models are developed and compared with the Hydrologic Modelling System (HEC-HMS) model. The HEC-HMS model [18] is designed to simulate the rainfall-runoff processes of watersheds systems. Because there was no definitive test to evaluate the success of each models, a multi-criteria assessment was carried out. The prediction of each model is evaluated using the coefficient of efficiency (R^2), mean square error (MSE), mean absolute error (MAE), and mean relative error (MRE). A MSE is one of the most commonly used performance measures in hydrological modeling. The other is to try to fill some of the gaps left by considering only MSE and stated that the MSE, MAE, and MRE provide different types of information about model prediction capabilities. Formulas for calculating MSE, MAE, and MRE are given as follows:

$$MSE = \left[\frac{1}{n} \sum_{i=1}^{n} (y_{pi} - y_{oi})^2 \right]^{\frac{1}{2}} \tag{15}$$

$$MAE = \frac{1}{n} \sum_{i=1}^{n} \left| y_{pi} - y_{oi} \right| \tag{16}$$

$$MRE = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{(y_{pi} - y_{oi})}{y_{oi}} \right| \tag{17}$$

where, y_{pi} and y_{oi} are the observed and predicted values of output; n is the number of observations or time periods over which the errors are computed. A model with the minimum error is considered the best choice.

6.0   RESULTS AND DISCUSSION

Table 1 presents the efficiency and the errors (MSE, MAE, and MRE) resulting from MLP, RBF and HEC-HMS models.

The proposed ANN models encompass the multilayer perceptron (MLP) and radial basis function (RBF). Record of daily rainfall and runoff of Sungai Lui catchment, Selangor has been used for the ANN modelling. Initially, the number of input node should be decided to construct the best architecture of the ANN model. It has been computed that the input node selection offered by Tokar and Johnson (1999) yields 18 input nodes. Harun (1999) procedure of selection reduces the input nodes to 15. Table 1 illustrates the performance of neural network and HEC-HMS models, indicated by the coefficient of efficiency (R^2), mean square error (MSE), mean absolute error (MAE), and mean relative error (MRE).

Table 1   Results of the prediction models

MODEL STRUCTURE		TRAINING/CALIBRATION PHASE			
		COE R^2	MSE (cumecs)	MAE (cumecs)	MRE (%)
	*18-20-1	0.745	0.965	0.716	49.20
MLP	*18-20-4-1	0.677	0.961	0.719	49.07
MLR	15 input	0.604	0.910	0.677	53.37
	18 input nodes	0.936	0.927	0.654	47.73
RBF	15 input nodes	0.902	0.884	0.627	54.54
HEC-HMS		0.483	3.421	2.564	98.65
MODEL STRUCTURE		TESTING/VERIFICATION PHASE			
		COE R^2	MSE (cumecs)	MAE (cumecs)	MRE (%)
MLP	*18-20-1	0.723	1.164	0.860	53.04
	*18-20-4-1	0.674	1.126	0.840	52.12
MLR	15 input	0.587	0.963	0.748	55.61
RBF	18 input nodes	0.811	1.260	0.930	58.45
	15 input nodes	0.782	1.122	0.848	65.28
HEC-HMS		0.037	4.822	3.657	256.87

*input nodes-hidden nodes-output nodes; cumecs-meter cubic second

The performance of neural network model is better than the HEC-HMS model. For the MLP model, the results of computed errors are quite close although the number of hidden layer was increased from one to two. It shows that by increased the number of layer in the hidden layer were not improved the performance of the model. Meanwhile, the RBF type displays a better performance than the MLP and the time taken for RBF training process is much shorter. Table 1 shows the performance of neural network and MLR models. According to the coefficient of efficiency of the model, it shows that the performance of the neural network model is better than the MLR model in the training and testing phase. For both training and testing processes, the application of input node selection by Harun (1999) considerably can reduce the MSE and MAE but not R^2 and MRE. The advantage of Harun (1999) compared to Tokar and Johnson (1999) is that Harun (1999) offers a reduction in the time taken for neural network training process. Figure 4a–4c shows the results for the MLP and RBF models training and testing.
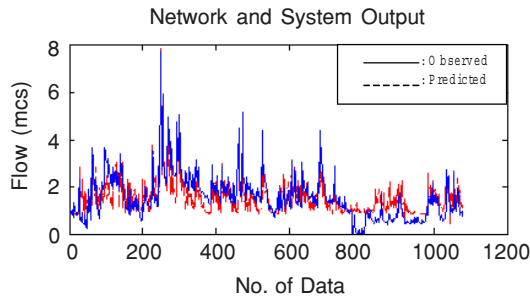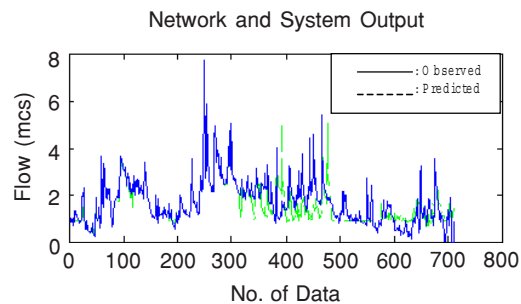
Figure 4a   MLP training (18-20-1)
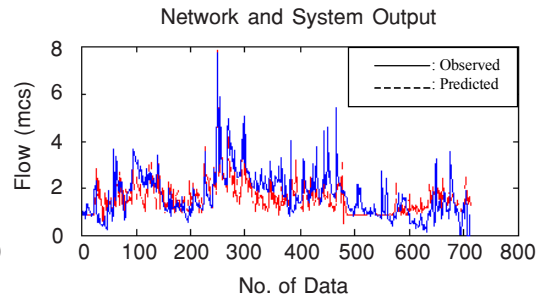
Figure 4b   MLP testing (18-20-1)
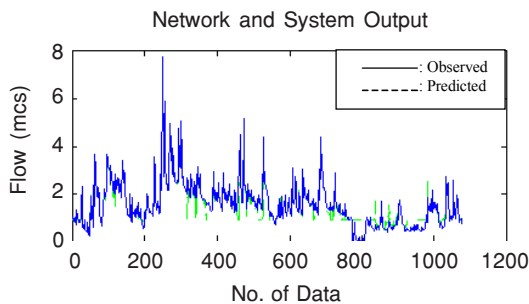
Figure 4c   RBF training (18 input nodes)

Figure 4d   RBF testing (18 input nodes)

The ANN performance is influenced by the level of non-linearity and the selection of training data. A large number of training data sets are required to perform successful training. The number of hidden layer neurons significantly influences the performance of a network. If this number is small, the network can suffer from under fit of the data and may not achieve the desired level of accuracy, while with too many nodes it will take a long time to be adequately trained and may some times over fit the data.

7.0   CONCLUSION

The nonlinear nature of the relationship of rainfall-runoff processes is appropriate for the application of ANN methods. Results of ANN models reflect that the performance of neural network model is better than HEC-HMS and MLR models, for modelling the rainfall-runoff relationship. Apparently, the neural network has the ability to predict runoff accurately using the rainfall data as input variable.

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