LQG WITH FUZZY CORRECTION MECHANISM IN TILTING RAILWAY VEHICLE CONTROL DESIGN.

Hairi Zamzuri [1] Argyrios C. Zolotas Roger M. Goodall

Control Systems Group,
Department of Electronic and Electrical Engineering,
Loughborough University, Ashby Road, Loughborough,
LE11 3TU, UK

Abstract:
This paper present work on modern control approaches in titling railway vehicles. The design of the Linear Quadratic Gaussian with integral action controller provides the required tilt compensation, while the addition of the fuzzy mechanism to the controller improves the system performance both for the deterministic and stochastic track. Moreover, the use of multiobjective GA as a tuning method seeks an optimum value based on the imposed constraints in the objective functions.

Keywords: tilting trains, tilt control, fuzzy logic, genetic algorithm, Linear Quadratic Gaussian

1. INTRODUCTION

Modern control approaches such as LQG (Linear Quadratic Gaussian) are attractive methods used when multiple control objective problems need to be addressed simultaneously. The aim of the linear optimal controller design is to find the control gain that provides the best possible performance with respect to a given performance index.

The use of fuzzy logic in control design has become wider in the control fields also the use of heuristic language in fuzzy logic gives advantages to implementation as it is based on the knowledge and experience of the designer.

In this paper, the authors present work based on linear quadratic methods on the application of tilting railway vehicle control. Moreover, the use of fuzzy correction as an 'add on' in the linear quadratic controller improves the overall performance of the system.

[1] The corresponding author is also attached to Universiti Teknologi Malaysia, Kuala Lumpur, Malaysia email :h.zamzuri@lboro.ac.uk, hairi@citycampus.utm.my

2. TILTING HIGH SPEED RAILWAY VEHICLE.

The use of tilting technologies in high speed train running on conventional tracks decreases the journey time between two places. Non-tilting conventional trains operated at slower speeds on curved track due to the lateral force acting on the vehicle. By leaning the vehicle train inwards on curved sections reduces the lateral force, thus allowing an increase in speed of the train while maintaining appropriate passenger lateral accelerations.

Railway vehicles are dynamically complex systems characterised by a significant coupling between the lateral and roll motion often referred to as the 'sway modes' (see Figure 1). The mathematical model of the system is based upon the end-view of a railway vehicle, to incorporate both the lateral and roll degrees of freedom for both the body and the bogie structures. A pair of airsprings represents the secondary suspension, whilst the primary suspension is modelled via pairs of parallel spring/damper combinations. The stiffness/damping of an anti-roll bar connected between the body and the bogie is also included. Active tilting is provided by using an 'active anti-roll bar' (Pearson *et al.*, 1998).

The tilt model system can be represented in state space by :

$$\dot{x} = Ax + Bu + \Gamma w$$
$$y = Cx \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{p\times n}$ and *n*,*m* and *p* represent the number of states, control inputs and outputs respectively.

Consider ω as constant external disturbance matrix given by,

$$w = [\ \frac{1}{R}\ \theta_o\ \dot{\theta}_o\ \ddot{\theta}_o\ y_o\ \dot{y}_o]^T \tag{2}$$

where

$\frac{1}{R}$	curvature
$\theta_o\ \dot{\theta}_o\ \ddot{\theta}_o$	elavation track components
$y_v\ \ \dot{y}_v$	lateral track irregularities components

and the state vector *x*,

$$x = [\ y_v\ \theta_v\ y_b\ \theta_b\ \dot{y}_v\ \dot{\theta}_v\ \dot{y}_b\ \dot{\theta}_b\ \theta_r]^T \tag{3}$$

and $u = [\delta_a]$. For the parameters see section Notation. The output measurement of the system is the *effective cant deficiency* (e.c.d), θ_{dm} which gives 60% tilt compensation to the tilt angle given by

$$\theta_{dm} = -0.615\frac{\ddot{y}_{vm}}{g} - 0.385\theta_{2sr} \tag{4}$$

where $\ddot{y}_{vm}$ is the lateral acceleration provided from the body lateral accelerometer and θ_{sr} is the secondary suspension angle. (Note: the negative sign used for correct feedback compensation).

Moreover, the variety of track inputs also contribute to the complex system which can be categorised into deterministic and stochastic. Deterministic inputs refer to the curved track which is carefully designed by civil engineers to meet the requirement of the passenger comfort index. The curved track is leaned inward(elevated) around 6^o rising linearly over a period of 2-3 seconds at the transition of the start and end of the curve. The stochastic track input represents the deviations of the actual track from the intended alignment, irregularities which occur in the vertical, lateral and cross-level directions. The secondary suspension of the vehicle is designed to reduce the effect of track irregularities, expressed in RMS acceleration levels in the body of the vehicle. In principal, the design of the tilt controller is to provide a fast response related to the transition to and from the curves, but at the same time does not affect the responses on track irregularities, i.e the ride quality on straight track.

Current tilting trains now use 'precedence' tilt control strategies (Goodall, 1999), whereby a bogie-mounted accelerometer is used to developed a tilt command

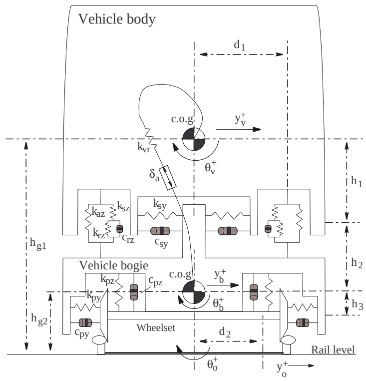


Fig. 1. End-view of the vehicle model

signal by measuring the curving acceleration on the non-tilting part of the vehicle. However, because the accelerometer also measures high frequency movements associated with lateral track irregularities, it is necessary to filter the signal. This filtering action (time delay) creates a detrimental performance on the transition from straight track to curved section. The usual solution is to use an accelerometer signal from the vehicle in front to provide "precedence", carefully designed so that the delay introduced by the filter compensates for the preview time corresponding to a vehicle length.

Research reported (Zolotas and Goodall, 2000) used local-per-vehicle nulling-type tilt control using modern approaches while, (Zolotas, 2002) reveal the disadvantages of using local feedback PI controller. Researchers in their paper (Zamzuri *et al.*, 2005) investigated the capability of using a fuzzy controller in local loop control schemes; they also introduced a fuzzy mechanism, as a correction in PID controller, to improve the performance on curved track while minimizing the effect of straight track irregularities. In this paper, a linear quadratic control scheme with a Kalman observer is used with fuzzy correction mechanism, to further improve the response both on curved track and track irregularities.

3. LQG TILT CONTROLLER DESIGN WITH INTEGRAL ACTION.

The difficulty of designing the tilt controller for railway vehicles is to minimize the effect of track irregularities to the vehicle while providing the fast response on curved track. This is more critical when the train is traveling at high speed (in this case, assumed 210 km/h; the conventional speed is 30 % slower). The trade-off between both deterministic and stochastic must be a compromise in order to achieve the desired performance.

Figure 2 illustrates the combination of control law and Kalman filter estimator forming the LQG compensator. This solution is based on the separation principle where the full state feedback controller LQR

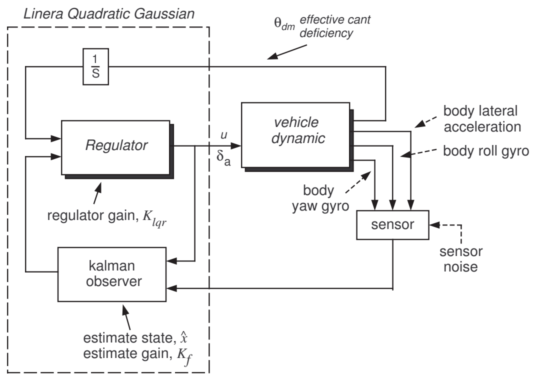


Fig. 2. LQG control scheme with integral action

and Kalman filter are designed independently and then combined to form the LQG compensator.

In practical terms, the integral state needed for regulation purpose, $\int \theta_{dm}$ is generated by the output measurement of *effective cant deficiency*, θ_{dm} while the states , *x* are estimated by the Kalman filter i.e $\hat{x}$.

3.1 The LQR design.

Referring to states equation (3) and Figure 2, the state model is augmented with the integral of the *effective cant deficiency* $\int \theta_{dm}$ to enhance the controller for regulation purposes, i.e achieve required tilt compensation without steady state errors. This gives an optimal PI controller (Dorato *et al.*, 1995). The overall system model with $\int \theta_{dm}$ is,

$$\begin{bmatrix} \dot{x}' \\ \dot{x} \end{bmatrix} = \begin{bmatrix} C' & 0 \\ A & 0 \end{bmatrix} \begin{bmatrix} x' \\ x \end{bmatrix} + \begin{bmatrix} 0 \\ B \end{bmatrix} u \qquad (5)$$

where $x' = \int \theta_{dm}$ and C' is the selector matrix for the output of *effective cant deficiency*, θ_{dm}. The control law is given by

$$u(t) = -K_I x'(t) - K_P x(t) \qquad (6)$$

where the initial condition of integral is set to zero and the quadratic performance index for state regulator is given by

$$J = \int_0^\infty [\bar{x}(t)^T Q \bar{x}(t) + u^T(t) R u(t)] dt \qquad (7)$$

where

$$\bar{x} = [\overbrace{\int \theta_{dm}}^{x'} \; \overbrace{y_v \; \theta_v \; y_b \; \theta_b \; \dot{y}_v \; \dot{\theta}_v \; \dot{y}_b \; \dot{\theta}_b \; \theta_r}^{x}]^T \qquad (8)$$

and *u* is angle of the tilt actuator δ_a, while matrix *Q* is chosen as 10×10 diagonal matrix and *R* is the control weight matrix.

3.2 Kalman state observer design.

Ideally, the original state space expression can be used for the design of the Kalman filter. However, to provide an accurate estimation model, the elements of curved track state should be included into the state vector rather than as a disturbance vector. Therefore, the extended model is given by,

$$\dot{x}_k = A_k x_k + B_k u + \Gamma_k \omega_k \qquad (9)$$

where

$$x_k = \begin{bmatrix} x & \tilde{w} \end{bmatrix}^T \qquad (10)$$

and the output equation is given by

$$y_k = C_k x_k + D_k u + v \qquad (11)$$

where v is sensor noise corruption and C_k, D_k are based on the relative rows of A_k and B_k. The reformulated state x_k becomes

$$x_k = [\, y_v \; \theta_v \; y_b \; \theta_b \; \dot{y}_v \; \dot{\theta}_v \; \dot{y}_b \; \dot{\theta}_b \; \theta_r \quad \overbrace{\theta_o \; \dot{\theta}_o \; R^{-1}}^{deterministic\, track} \,]^T \qquad (12)$$

while

$$w_k = \begin{bmatrix} R^{-1} & \ddot{\theta}_o \end{bmatrix}^T \qquad (13)$$

and

$$A_k = \begin{bmatrix} A & \tilde{\Gamma} \\ 0 & \Delta_{LP} \end{bmatrix} \quad B_l = \begin{bmatrix} B \\ 0_{3\times 1} \end{bmatrix}$$

$$\Gamma_l = \begin{bmatrix} \Gamma_{\omega k} & (0\ 0) & (0\ 1) & (1\ 0) \end{bmatrix}^T \qquad (14)$$

$$\Delta_{LP} = \begin{bmatrix} 0 & 1 & 0 \\ -\epsilon_1^2 & -2\epsilon_1 & 0 \\ 0 & 0 & -\epsilon_2 \end{bmatrix}$$

Δ_{LP} is a ”filter” matrix to force the pair (A_k, C_k) observable, $\epsilon_{1,2}$ chosen sufficiently small time constant (i.e 0.001,0.002). The state estimate Kalman-Bucy filter can be calculated by the following differential equation,

$$\dot{x}_e = A_k x_e + B_k u + K_f (Y_k - C_k x_e D_k u) \qquad (15)$$

where x_e is the state estimate vector of the reformulated state and K_f is the Kalman filter gain. The performance of the Kalman-Bucy filter was tuned based on the variance matrix of noise track Q_{kf} where Q_{kf}=diag($Q^{kf}_{\ddot{\theta}_o}$,$Q^{kf}_{\frac{1}{R}}$)= diag(10^{-6},0.85^{-3}). Figure 3 shows the passive system responses between actual and estimate output using the Kalman-Bucy filter. For more details on the chosen parameters Q_{kf} and R_{kf} for Kalman-Bucy filter, refer to (Zolotas, 2002).

3.3 LQG tuning design : multiobjective GA.

In the LQG design, the choice of the weighting factors *Q* and *R* from equation (7) influence the overall performance of the closed loop system.

Multiobjective genetic algorithms have been successful tools of automatic design of controllers in various areas of control engineering (Chipperfield and

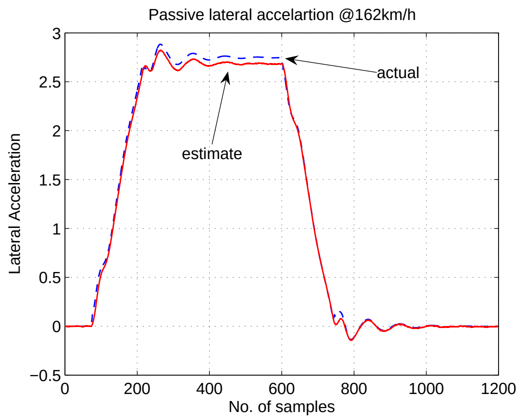


Fig. 3. Kalman-Bucy Filter estimate for passive lateral acceleration output responses.

Fleming, 1996; Popov *et al.*, 2005). For multiobjective problems there may not exist one solution which is the best with respect to all objectives. Typical for multiobjective problems, the existence of a set of solutions which are superior compared to the other solutions in the search space. In this study, multiobjective genetic algorithms (MOGA) methods proposed by Fleming in (Fonseca and Fleming, 1993) are used to obtain the optimum values for the weighting matrix parameter Q while parameter R is fixed to $\frac{1}{0.215^2}$ (This is actually based on taking the value of the control weight as $\frac{1}{(max.controlsignal)^2}$). LQ is a diagonal matrix for relaxing the computational burden. The objective functions consists of 6 functions. These are (referring to Figure 4),

(1) settling time at the steady state curve , t_s not more than 4.5 seconds.
(2) optimize the lateral acceleration percentage overshoot,(%OS) when the train transverses into(f_2) and out from the curve(f_3).
(3) optimize the lateral acceleration responses to a system change based on the difference between the actual and ideal lateral acceleration profile. The calculation is based on the root mean square of absolute lateral acceleration error from the start (f_4), and end of the curved (f_5) transitions.
(4) stochastic ride quality,(f_6): constrain the degradation of the straight track ride quality within the allowance of 7.5% worst taken between active and passive system at high speed.

4. LQG DESIGN WITH FUZZY CORRECTION.

The overall output performance based on the controller scheme proposed in section 3 give an acceptable response. However, adding fuzzy correction proposed in this section shows improvement to the output responses. The objective of designing the control scheme is to minimise the overshoot and oscillation on curved track while reducing the effect of track irregularities on straight track.

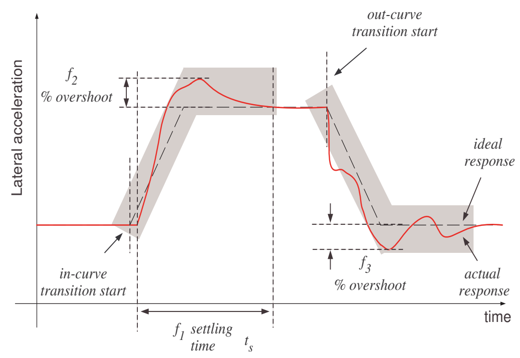


Fig. 4. calculation of the overall performance on the curved track.

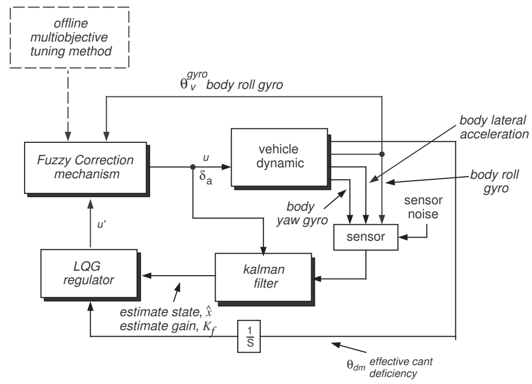


Fig. 5. PI LQG controller scheme with fuzzy correction.

Figure 5 shows the controller scheme. The design is based on the procedure of section 3.1, however the output regulator is fed to the fuzzy corrector to further accommodate for curved transition and straight track performance. The fuzzy correction block is also fed by the signal of body roll gyro as an additional decision making variable.

The design of the controller can be divided into two stages :

(1) design LQG controller with integral action to give fast response on curved track (see section 3.1 and 3.2).
(2) tune fuzzy correction using multiobjective GA (MOGA method) aimed at minimizing straight track irregularities effect and preventing large overshoot and oscillations on curved track.

The fuzzy correction mechanism, both inputs are shown in Figure 6, consist of three equally distributed gaussian Membership Functions with 50% overlap for each signal. Furthermore, the *Center of Area* (COA) defuzzification procedure with well known max-min inference method.

The linguistic variables for each membership function represent the condition for each value. For example, the regulator output *u'* is represented by the linguistic variables *Neg*, *Zero* and *Pos*. For the body roll gyro input $\dot{\theta}_v^{gyro}$, the linguistic variable are also *Neg*, *Pos*

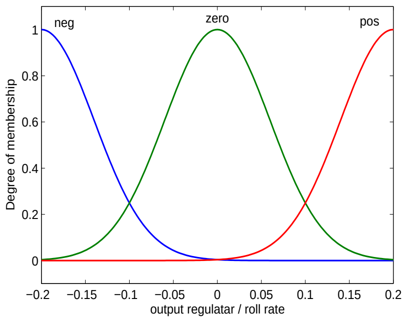


Fig. 6. output from regulator (u') and roll gyro ($\dot{\theta}_v^{gyro}$)

and *zero*, while for the fuzzy correction output *u"* , the linguistic variables represent the tilting direction of the car body as *tilt the car body clockwise maximum* represented by *TiltClkwM*, and tilt car body medium anticlockwise represented by *TiltAclkwm* etc. Clockwise and Anticlockwise characterize the direction of tilt based on the curve direction (i.e inwards and outwards of the curve respectively). Note that the membership function ranges represent the required operating range of the variables. The development of fuzzy rules were based on :

- stabilizing the system:
  if u' is **changing fast** *and the* $\dot{\theta}_v^{gyro}$ *is* **zero** *then* **apply maximum tilt effort u"**
- preventing overshoot and oscillation.
  if u' **changes** *and* $\dot{\theta}_v^{gyro}$ **changes** *then* **maintain medium till effort u"**

Detail on the rules is shown in Table 1,

Table 1. LQG-Fuzzy Correction Rule Base

$\dot{\theta}_v^{gyro}$/*u"*	Neg	Zero	Pos
Neg	TiltClkwm	TiltClkwm	TiltClkwM
zero	TiltAclkwM	NoChange	TiltClkwM
Pos	TiltAclkwM	TiltAclkwm	TiltAclkwm

4.1 Tuning the fuzzy correction mechanism: MOGA method.

In this paper, the fuzzy correction is also tuned using MOGA method proposed in section 3.1. The position and width of the output fuzzy membership functions (see Figure 5 off-line GA) are tuned based on 5 real coded GA variables. The upper and lower limits on the parameters are established based on the previous control setup, see (Zamzuri *et al.*, 2006) for more details. The initial choice of the parameters and the limits clearly reduce the computational time.

The output membership function consists of three *triangular* membership functions and two *trapezoidal* membership functions located at each end of the fuzzy set . Figure 7 illustrates the concept of coding the membership functions. The genetic algorithm seeks the optimal profile (based on position and width of the membership functions), except for the position of the

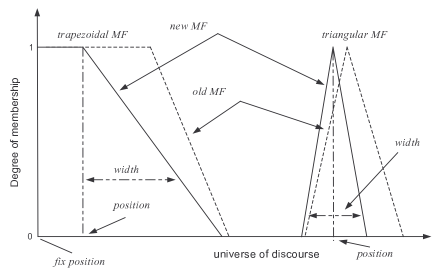


Fig. 7. Tuning of position and width of membership functions

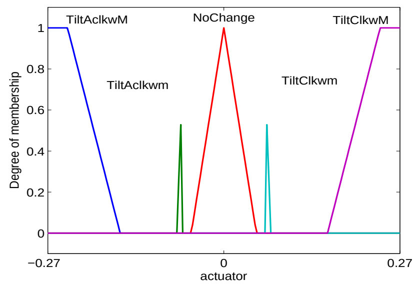


Fig. 8. Output membership function after tuned by MOGA method.

membership function *NoChange* and those at the limit of the range (*TiltClkwM* , *TiltAclkwm*).

In order to illustrate the advantages of using the fuzzy correction mechanism, the objective function for this scheme will be used as in the section 3.3. Figure 8 shows the output membership functions after tuning by multiobjective genetic algorithms (MOGA) method.

5. RESULTS AND ANALYSIS.

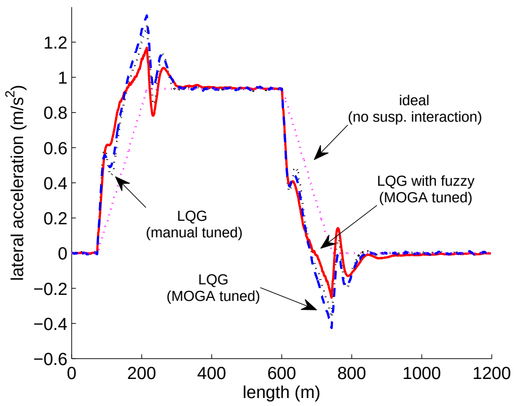
The system model was implemented using SIMULINK while the GAs process is implemented in MATLAB. A random initial population of size 30 and *shuffle crossover* with *binary* mutation were used with crossover and mutation probability of 0.9 and 0.02 respectively. Moreover, the system was simulated at a speed of 210 km/h with 1000 m curve radius and 6^o cant angle (the track profile included 145 m transition length at each end of the curved).

To highlight the advantages of using the fuzzy correction, the scheme proposed in section 4 was compared with the control scheme proposed in section 3 (manual and multi objective GA tuned). It is worth mentioning that the only difference between these two control schemes is the use of fuzzy correction mechanism. Figure 5 shows the output responses on curved track using LQG with difference tuning approaches (manual and MOGA). The figure also illustrates the advantages of using fuzzy mechanism as a correction factor on the LQG design. The improvement is also shown for the body lateral acceleration on straight track in Table

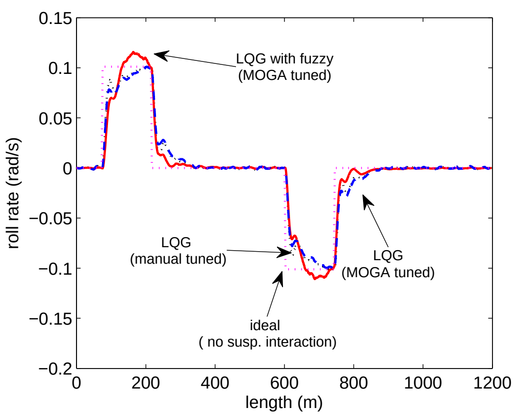
2. The ride quality degradation shown in the table is the difference in rms values taken between the passive (without controller: rms value= 0.381 %/g) and active system traveling at high speed (58 m/s, 30% increase compared to non-tilting speed).

Table 2. Body lateral acceleration on straight track.

	RMS values (%/g)	Ride quality degradation (%)
LQG (manual tuned)	0.394	3.357
LQG (MOGA tuned)	0.377	-1.072
LQG with fuzzy correction	0.352	-7.546



(a) Lateral acceleration.



(b) Body roll rate

Fig. 9. Output responses for two different LQG control schemes on curved track.

6. CONCLUSIONS.

This paper considered the design of LQG controller design via tilt local feedback scheme. It was illustrated that the use of LQG with integral action approaches shown improved tilt performance both on straight and curved track. The use of Kalman filter helps estimating both the vehicle states and track disturbance states.

The use of GA tuning (MOGA approach) of the LQG controller provided some improvement in ride quality compared to the manual tuned LQG. However, the incorporation of a fuzzy correction mechanism with the LQG controller based on the use of GA tuning (MOGA) provided a noticeable improvement both on deterministic and stochastic track input excitations. In particular, the fuzzy correction provided an extra degree of freedom in the control design, while the MOGA tuning allows for multiple objective to simultaneously being optimised (both deterministic and stochastic considerations).

NOTATION

y_v,y_b,y_o	Lat. displacement of body, bogie and track.
θ_v,θ_b,δ_a	Roll displacement of body, bogie and anti-roll bar actuator.
θ_r	airspring reservoir roll deflection.
θ_o,R	Track cant, curve radius.

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