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A NEW NEAR OPTIMAL HARMONICS ELIMINATION PWM ALGORITHM FOR AC TRACTION DRIVES

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Abstract. For AC traction system, it is necessary to develop voltage source inverters (VSI) which is compatible with existing types of signaling systems adopted by various railway operators. The varying harmonics incidents of conventional sinusoidal-type PWM VSI is not particularly suitable as it produces unwanted harmonics within the signaling frequency band. One well-known solution to ensure that the unwanted harmonics do not appear on the spectra is by using the harmonics elimination PWM (HEPWM) switching method. However, the application of HEPWM has been somewhat limited by the fact that the switching angles cannot be calculated online by a microprocessor-based waveform generator. This is due to the fact that the equations involved are non-linear and transcendental in nature. The paper presents an algorithm to calculate near optimal switching angles, which will permit a fast and efficient realization using a microprocessor. The method is based on quadratic approximation approach which is derived from the computed trajectories of angles.

Keywords: power electronics, inverter, pulse-width modulation (PWM), railway signalling, harmonics

1.0 INTRODUCTION

Induction motor is preferred over its DC counterpart for railway traction applications due to its robustness, lower maintenance, higher power to weight ratio and lower cost to power ratio. However, if AC traction drives are to be made available on a competitive basis with DC drives for rail applications, it is necessary to develop voltage source inverters (VSI) compatible with existing types of signaling systems adopted by various railway operators. For DC drive systems which use carrier-only power frequency track circuits, the signaling requirements are met by selecting one or more chopper operating frequencies that avoid signal equipment carrier frequencies [1]. However, the case for VSI induction motor drive is somewhat more complicated. The varying harmonics incident may produce unwanted harmonics within the signaling frequency band. One well-known solution to overcome such problem is by using the harmonics elimination PWM (HEPWM) switching method for VSI.

HEPWM, originally proposed by Patel and Hoft [1], is a method to eliminate selected harmonics from the PWM waveform spectra. Unfortunately, as the equations to calculate switching angles in a HEPWM scheme are non-linear and transcendental, i.e. they cannot be solved online by a microprocessor. They can be calculated off-line

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using well-known numerical techniques, but such operation requires large computational power. Furthermore, the exact switching angles for the whole frequency range, for a given ratio changing scheme, have to be pre-calculated (off-line) for a given voltage/frequency characteristics and DC link voltage. With a large number of possibilities of modulation index and modulation ratio, and the interpolation involved, the memory requirement can be very large. Despite these difficulties, HEPWM offers several advantages over the conventional PWM methods that are listed as follows:

1. For a given inverter switching frequency, the first uneliminated harmonic is almost double that for a natural or regular-sampled PWM scheme, thus resulting in a far superior pole switching waveform harmonic spectrum.
2. A much higher pole switching waveform fundamental amplitude is attainable before the minimum pulse-width limit of the inverter is reached.
3. About 50% reduction in the inverter switching frequency is achieved when comparing with the conventional carrier-modulated sine PWM scheme. The reduction in the switching frequency contributes to the reduction in the switching losses of the inverter and permits the use of high power switches in railway applications.
4. Higher voltage gain due to overmodulation is possible. This contributes to higher utilization of the power conversion process.
5. Due to the high quality of the output voltage and current, the ripple in the dc link current is also small. Thus, a reduction in the size of the dc link filter components is achieved.

Besides the off-line method suggested in [1], there exist several methods to generate on-line HEPWM waveforms. The most prominent of such work was published by Taufiq *et al*. [2] who derived a set of non-transcendental equations for near-optimal solution using sine-wave approximation approach. Using this scheme, the transcendental equations are "reduced" to a simpler form which permits on-line HEPWM computation using digital methods. Another scheme, based on regular sampled PWM technique was suggested by Bowes [3]. Other works are mostly based on pre-calculated angles which are stored in memory. These are referred to as pre-programmed harmonic elimination method. Reference [4] provides excellent review of this technique.

This paper proposes a new method of near-optimal harmonics elimination method based on quadratic-approximated equations. It is envisaged that the proposed method permits even faster and more efficient harmonic elimination PWM waveforms due to the simplicity of the algorithm.

2.0 DERIVATION OF NEAR-OPTIMAL HEPWM EQUATIONS

Figure 1 shows a generalized output waveform with M chops per half-cycle. It is assumed that the periodic waveform has half-wave symmetry and unit amplitude. The basic square wave is chopped a number of times and a fixed relationshi p between the number of chops and possible number of harmonics that can be eliminated is derived. The odd switching angles, α_1, α_3, etc., define the negative going transitions and the even switching angles, α_2, α_4, etc. define the positive going transitions.

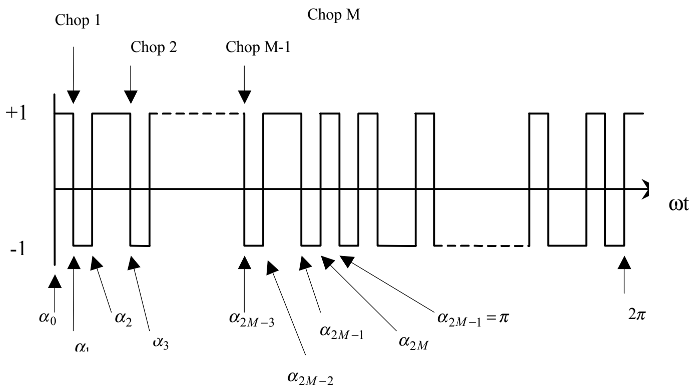


Figure 1 Generalized quarter-wave symmetric PWM waveform

As the waveform is quarter-wave symmetric, only odd harmonics exist and are given by the following Fourier series representation:

$$f(\omega t) = \sum_{n=1}^{\infty} \left[A_n \sin(n\omega t) + B_n \cos(n\omega t) \right] \quad \text{(for odd harmonics only)} \tag{1}$$

$$A_n = \frac{4}{n\pi} \left[1 + 2 \sum_{k=1}^{m} (-1)^k \cos n\alpha_k \right] \tag{2}$$

$$B_n = 0 \tag{3}$$

Equation (2) has m variables (α_1 to α_m) and a set of solutions is obtainable by equating any m-1 harmonics to zero and assigning a value to the fundamental. Thus both the harmonics incidents and the fundamental components can be independently controlled. These equations are nonlinear as well as transcendental in nature and can be solved using numerical method such as the Newton-Raphson iteration. This method produces accurate solutions and good convergence provided that the initial guess for the switching angles is near the local minima. Using proper programming techniques, the trajectories for the switching angles (α_1, α_3, ... α_m) versus the amplitude of the fundamental component of the pole switching waveform (*NP*1) for odd number of switching per quarter cycle can be obtained. Figures 2(a)-(d) show some trajectories calculated for m=3,5, 7 and 13, respectively.

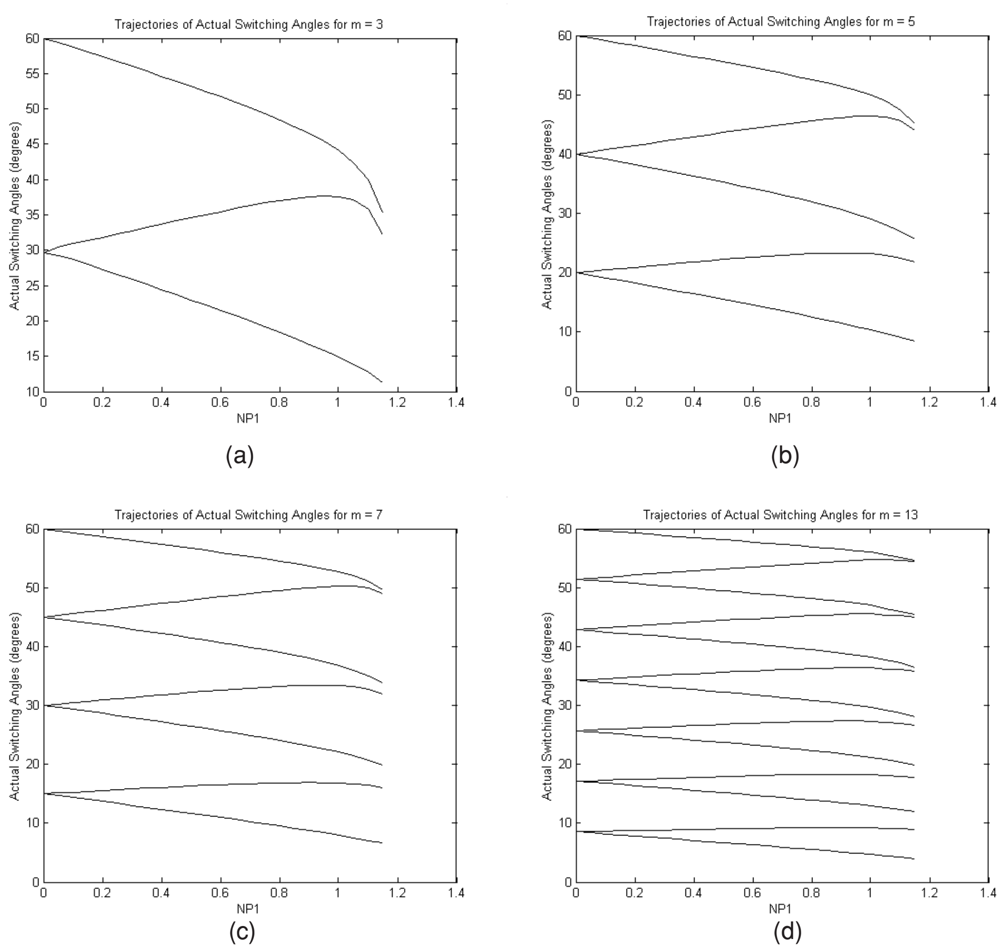


Figure 2 Trajectories of switching angles for an odd number of switchings per quarter cycle: (a)m=3, (b)m=5, (c)m=7, (d)m=13

From, the trajectories in Figures 2(a)-(d) the highest intersection point for the trajectories with the y-axis is 60^o. Furthermore, the switching angle α_m is equal to when $NP1$ is equal to zero, for all values of m. From Figure 3, the angular separation of the trajectories at the y-axis is can be defined as:

$$\text{Angular separation } \frac{2 \times 60^\circ}{\text{m}+1}, \text{ m is odd} \tag{4}$$

It can also be seen from Figure 3, for $NP1$ ranges from 0 to 0.8, the trajectories approximates a straight line. Hence a straight-line approximation of the trajectories could be used. However, for $NP1$ greater than 0.8, the trajectories are no longer straight lines. Nevertheless, the straight-line approximation could still be used with an error correction scheme for the region of $NP1$ greater than 0.8.

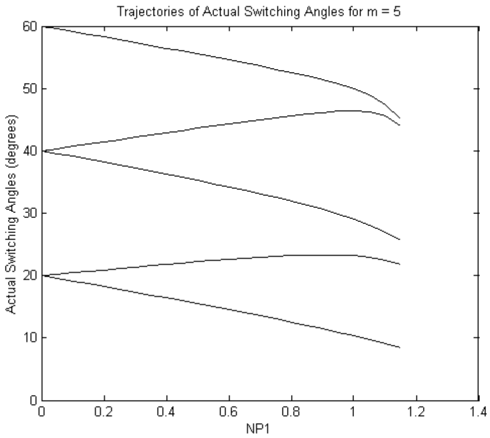


Figure 3 Trajectories for m = 5 to illustrate angular separation

Observing Figures 2(a)-(d), the trajectories of the odd and the even switching angles are apparently parallel lines over most of the range of *NP*1. Also the slopes of the trajectories reduce with increasing values of m. Therefore, to obtain a relationship between the slopes of the trajectories for the different values of m, the slopes should first be normalized towards the angular separation of the trajectories. At *NP*1 = 0.8,

$$\text{Slope of the trajectories} = \frac{\dfrac{60^\circ(k+1)}{(m+1)} - k}{0.8}, \quad \text{for k odd} \tag{5}$$

$$\text{Slope of the trajectories} = \frac{\alpha_k - \dfrac{60^\circ(k)}{(m+1)}}{0.8}, \quad \text{for k even} \tag{6}$$

Let Δ_k = normalized slope $\times 0.8$. Then

$$\Delta_k = \frac{\dfrac{60^\circ(k+1)}{(m+1)} - \alpha_k}{\dfrac{2\times 60^\circ}{m+1}}, \quad \text{for k odd} \tag{7}$$

$$\Delta_k = \frac{\alpha_k - \dfrac{60^\circ(k)}{(m+1)}}{\dfrac{2\times 60^\circ}{m+1}}, \quad \text{for k even} \tag{8}$$

Equations (7) and (8) are used to plot out versus k in separate graphs for odd k and even k as shown in Figure 4 and 5, respectively.

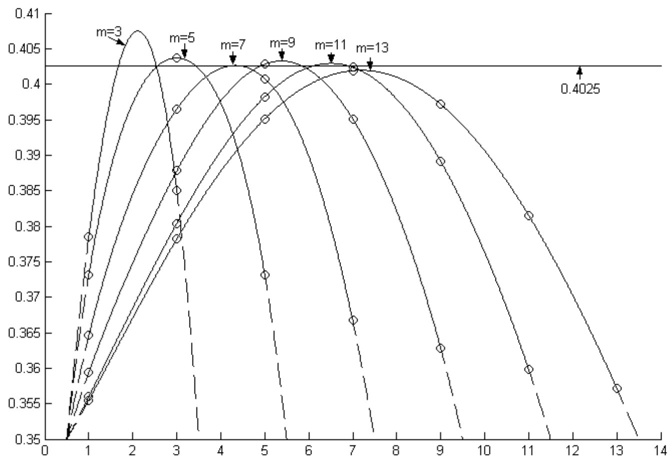


Figure 4 Variation of Δ_k towards k for several values of m (for odd k)

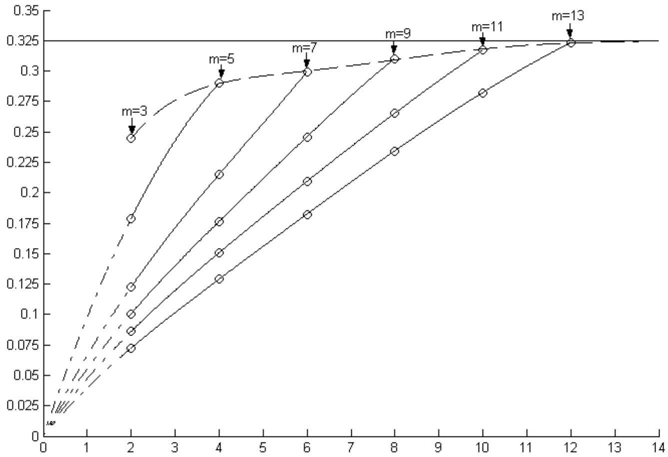


Figure 5 Variation of Δ_k towards k for several values of m (for even k)

Figure 4 shows the variation of Δ_k, for odd and even values of k, for several values of m. The graphs suggest that for odd values of k, the function Δ_k is rather like a set of quadratic curves with nearly constant amplitude. Therefore, the obvious solution would be to apply a quadratic fit to the curves in Figure 4. This leads the generalized equation for the odd values of Δ_k which will be of the form,

$$\Delta_k = -\frac{0.21}{m^2}\left(k - \frac{m+1}{2}\right)^2 + 0.4025, \quad \text{for k odd} \tag{9}$$

The complete derivation of equations for the quadratic-approximated curves of the variation of Δ_k with k for odd values of k is shown in appendix A. The results is a generalized equation for the odd switching angles, for any value of m and $NP1$, is given by the equation below,

$$\alpha_k = \frac{60^\circ(k+1)}{m+1} - \left[\frac{2\times 60^\circ}{m+1} \times \frac{\Delta_k \times NP1}{0.8}\right], \quad \text{for k odd} \tag{10}$$

Figure 5 shows the variation of Δ_k for the even switching angles. Extrapolation shows that all the curves pass through the origin. For increasing values of m, the values of Δ_{m-1} appear to be asymptotic to a line drawn parallel to the x-axis and intersecting the y-axis at 0.325. Taking this into account, the generalized equation for the even values of Δ_k will be of the form given in the equation below,

$$\Delta_k = -\frac{0.082}{(m-1)^2}\left[k - 2.482(m-1)\right]^2 + 0.505 - \frac{k}{m^3}, \quad \text{for k even} \tag{11}$$

The derivation of equations for the quadratic-approximated curves of the variation of with k for even values of k is as shown in Appendix B. The generalized algorithm for the even switching angles for any value of m and $NP1$, is then given by:

$$\alpha_k = \frac{60^\circ \times k}{m+1} + \left[\frac{2\times 60^\circ}{m+1} \times \frac{\Delta_k \times NP1}{0.8}\right], \quad \text{for even k} \tag{12}$$

Equations (9), (10), (11) and (12) can now be used to calculate the approximate switching angles for any value of m and $NP1$. These simple equations can be implemented easily on a 16-bit microprocessor which has the multiplication command in its instruction set thus, allowing very fast and efficient generation of the PWM waveform online. The accuracy of the algorithm is investigated in the following section.

3.0 ACCURACY OF THE GENERALIZED ALGORITHM

The accuracy of the derived equations is evaluated by calculating the difference between the approximate switching angles from the proposed method and the exact switching angles from the trajectories. The absolute difference is termed as the angle error. The angle errors relationship with *NP*1 and the kth angle for values of m=3, 5, 7, 9, 11 and 13, are shown in Figures 6(a) through (f), respectively. For each of the six

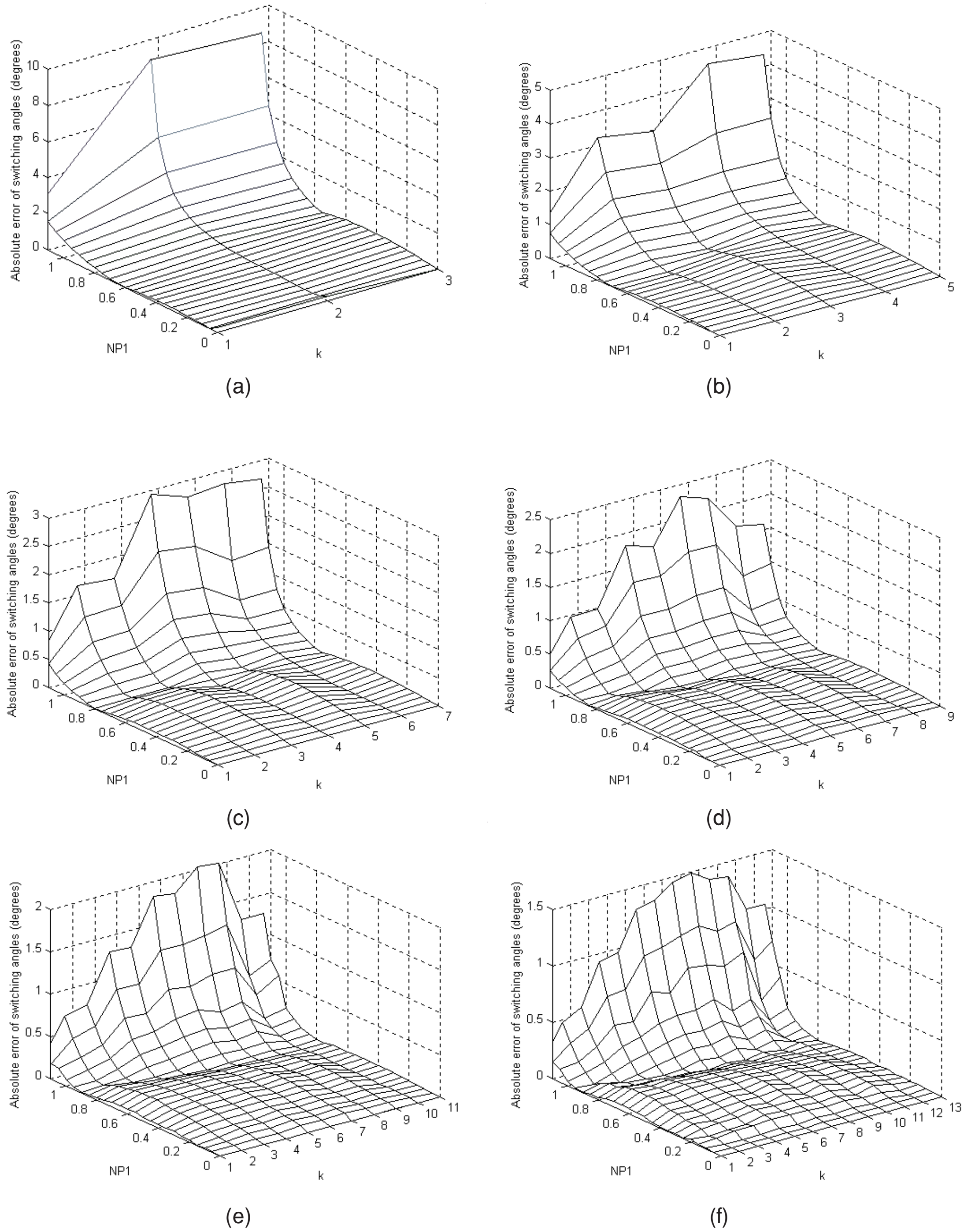


Figure 6 Variation of switching angle errors for (a) m=3, (b) m=5, (c) m=7, (d) m=9, (e) m=11, (f)m=13

cases, the angle error trend is very small for $NP1$ less than 0.8. In addition the errors reduce for increasing values of m. However for values of $NP1$ greater than 0.8, the errors increase drastically. The reason for this increase can be attributed to the departure of the trajectories from being straight lines for $NP1$ above 0.8. On this basis, for $NP1$ greater than 0.8, a correction factor need to be incorporated to the switching angles to reduce the error. The maximum angle errors for $0 < NP1 \le 0.8$ and $0.8 < NP1$ m 1.15 are tabulated in Table 1 and 2.

Table 1 Maximum errors in switching angles for $0 < NP1 \le 0.8$

m	Maximum error for odd switching angles (degree)	Maximum error for even switching angles (degree)
3	0.6795	0.8967
5	0.3242	0.4535
7	0.2759	0.3469
9	0.2136	0.2232
11	0.1784	0.1582
13	0.1533	0.1154

Table 2 Maximum errors in switching angles for $0.8 < NP1 \le 1.15$

m	Maximum error for odd switching angles (degree)	Maximum error for even switching angles (degree)
3	8.3785	8.6192
5	4.2015	4.3793
7	2.6184	2.8355
9	2.2420	2.1003
11	1.9446	1.9688
13	1.4446	1.4038

4.0 ERROR CORRECTION

To account for the relatively large error for the case of $0.8 < NP1 \le 1.15$, an error correction factor is incorporated. For $NP1 > 0.8$, and k is odd,

$$\alpha_{k(corrected)} = \alpha_k - \Delta D_k \tag{13}$$

with

$$\Delta D_k = \frac{(NP1-0.8)^2}{0.09} \times \left[-\frac{52}{m}\left[\frac{k}{m+5} - 0.5\right]^2 + \frac{13}{m} \right], \text{ k odd} \tag{14}$$

While for even switching angles at $NP1 > 0.8$, the error correction factor is incorporated as before in the previous equation shown:

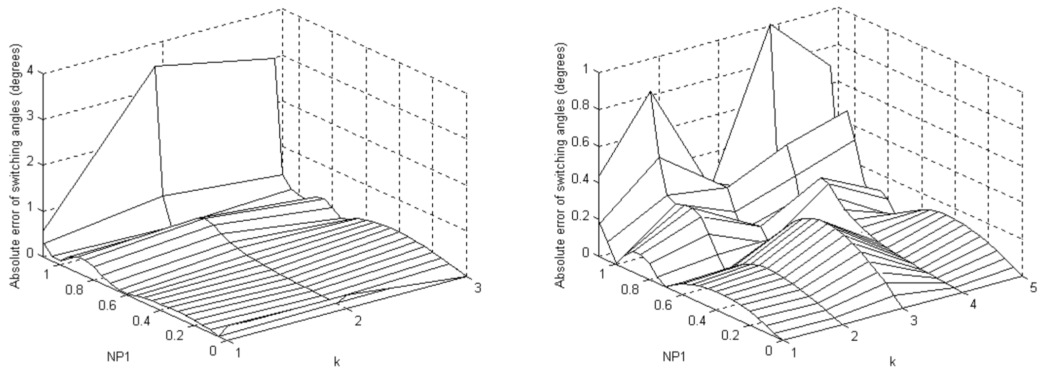
$$\alpha_{k(corrected)} = \alpha_k - \Delta D_k \tag{15}$$

with

$$\Delta D_k = \frac{(NP1-0.8)^2}{0.09} \times \left[-\frac{52}{m}\left[\frac{k}{m+3} - 0.5\right]^2 + \frac{13}{m} \right], \text{ k even} \tag{16}$$

5.0 ACCURACY OF GENERALIZED ALGORITHM INCORPORATING CORRECTION FACTOR

Figures 9(a) through (f) show the absolute error between the exact switching angles and those calculated with the approximated quadratic equations, incorporating the correction factors. Comparing Figures 6(a) through (f), it could be seen that the maximum errors have been reduced by a factor of 3-6 times. Table 3 shows the maximum errors at $0.8 < NP1 \leq 1.15$ with and without error correction factor.



(a) (b)

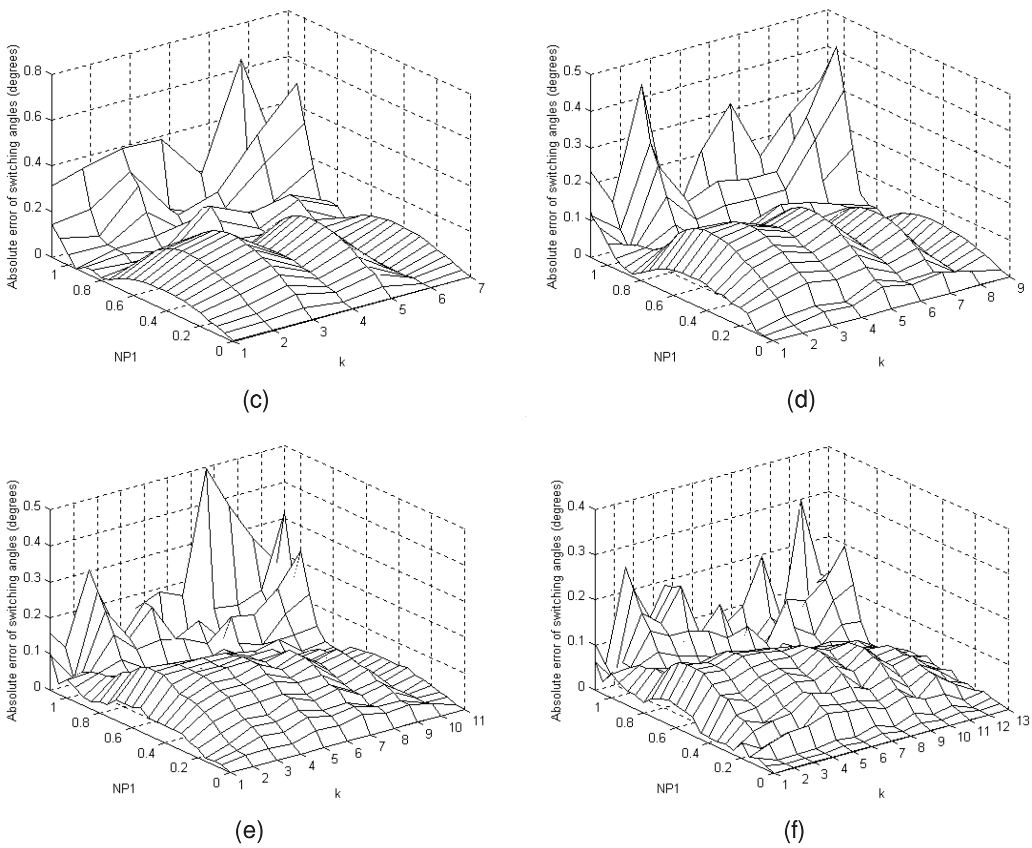


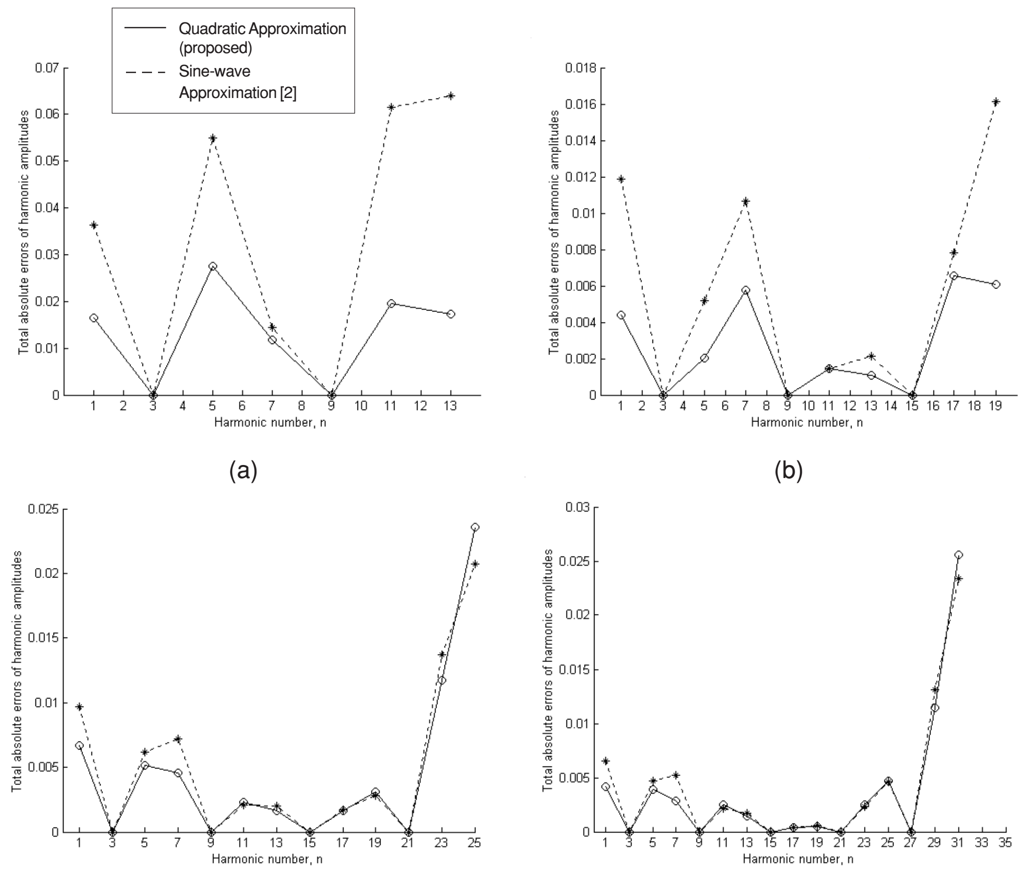
Figure 9 Variation of error, mincorporating error correction for *NP*1>0.8 (for m=3,5,7,9,11,13)

Table 3 Maximum error in switching angles with and without error correction for 0.8 < *NP*1 ≤ 1.15

m	Without correction		With correction	
	Max. error for odd switching angles (degree)	Max. error for even switching angles (degree)	Max. error for odd switching angles (degree)	Max. error for even switching angles (degree)
3	8.3785	8.6192	2.8490	3.3764
5	4.2015	4.3793	0.6626	0.9819
7	2.6184	2.8355	0.3697	0.6173
9	2.2420	2.1003	0.4186	0.2294
11	1.9446	1.9688	0.3606	0.4798
13	1.4446	1.4038	0.2411	0.2844

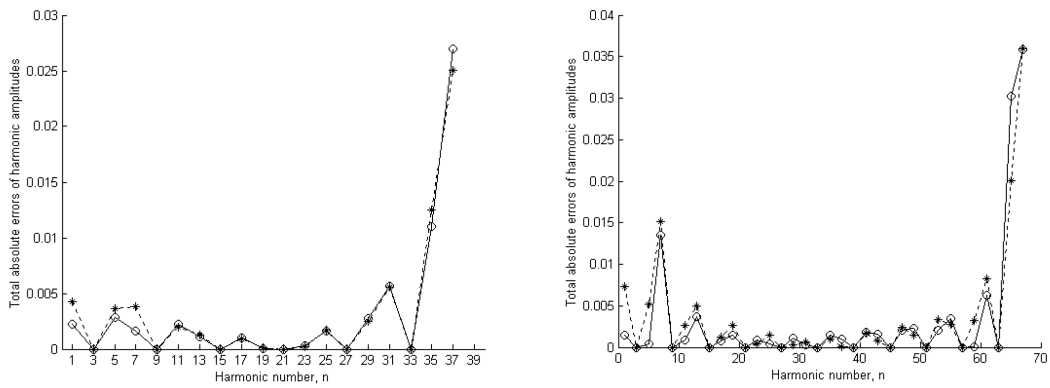
6.0 HARMONICS PERFORMANCE

Finally the harmonics performance of the proposed algorithm is evaluated. Figures 10(a) through (f) show the comparison of the total absolute errors of the harmonic amplitudes between the proposed approach using quadratic approximation and the sine-wave approximation method suggested by [2]. The harmonic error is defined as the ratio of the harmonic to the fundamental component. From these figures, it could be seen that for small m, the amplitudes of the harmonics error (that are supposedly eliminated) is smaller than the ones generated by the sine-wave approximation approach. For larger m, the harmonics using both methods are nearly the same. In the overall, the harmonic errors are very small; the worst-case harmonic is less than 1.5% of the fundamental component. For higher value of m, the harmonics will improve. Therefore it can be concluded that the proposed algorithm is accurate enough to substitute for the original harmonic elimination equations. Furthermore, it is expected that the proposed method requires less computing time because it involves only multiplication procedure.



(a) (b)

(c) (d)



(e) (f)

Figure 10 Total absolute errors for harmonics. (a)m=5 and *NP*1=1 (b)m=7 and *NP*1=0.8 (c)m=9 and *NP*1=1 05 (d) for m=11 and *NP*1=0.5 (e)m=13 and *NP*1=0.5 (f)m=23 *NP*1=1.1

7.0 CONCLUSION

The report has successfully proposes an algorithm to calculate the switching angles using harmonic elimination PWM scheme for odd number of switching angles per quarter cycle. The algorithm results in quadratic equations which require only the multi plication process and therefore, can be implemented efficiently on a microprocessor. Any changes to the number of harmonics to be eliminated and also the fundamental amplitude of the pole switching waveform can be made online using this optimized PWM scheme.

Thus both the harmonics incidents and the fundamental components can be independently controlled. For AC traction application, the flexibility to change the fundamental amplitude is particularly important when the DC link voltage of the inverter varies due to the absence of a preconditioning chopper. Such a variation can be taken into account by a microprocessor, by online recalculation of the new switching angles such that the fundamental component of the inverter output voltage is maintained constant.

Finally it is important to note that although this paper has concentrated on the use of on-line HEPWM for VSI in railway traction application, the algorithm derived can be also be suitably used in industrial VSI drives.

REFERENCES

[1] Patel H. S. and R. G. Hoft. 1973. Generalized Techniques of Harmonic Elimination and Voltage Control in Thyristor Inverters: Part I-Harmonic Elimination. *IEEE Transaction on Industrial Applications*. Vol. IA-9, No.3.

[2] Taufiq, J. A., B. Mellitt and C. J. Goodman. 1986. Novel Algorithm for Generating Near Optimal PWM

Waveforms for AC Traction Drives. *IEE Proceedings*. Vol 133, Pt. B, No. 2.
[3] Bowes S. R. and P. R. Clark. 1992. Simple Microprocessor Implementation of New Regular-Sampled Harmonic Elimination PWM Techniques. *IEEE Transaction on Industry Applications*. 28(1).
[4] Enjeti P. N., P. D. Ziogas and J. F. Lindsay. 1990. Programmed PWM Techniques to Eliminate Harmonics: A Critical Evaluation. *IEEE Transaction on Industry Applications*. 26(1).

APPENDIX A (Derivation of switching angle equation for odd k)

From Figure 4, a suitable equation for the curves using the quadratic fit would be,

$$\Delta_k = -a(k-b)^2 + c \qquad \text{(A1)}$$

From the same figure,

$$c = 0.4025 \qquad \text{(A2)}$$

$$b = \frac{m}{2} + 0.5 = \frac{m+1}{2} \qquad \text{(A3)}$$

$$\Delta_k = 0.35 \text{ at } k = 0.5 \qquad \text{(A4)}$$

Substitute (A1), (A2) and (A3) in (A1),

$$\begin{aligned} 0.35 &= -a\left(0.5 - \frac{m+1}{2}\right)^2 + 0.4025 \\ a &= \frac{0.21}{m^2} \end{aligned} \qquad \text{(A5)}$$

Substitute (A2), (A3) and (A5) in (A1),

$$\Delta_k = -\frac{0.21}{m^2}\left(k - \frac{m+1}{2}\right)^2 + 0.4025 \text{, for odd k} \qquad \text{(A6)}$$

With a straight line approximation, Δ_k will reduce linearly to zero at $NP1$ equal to zero. Thus, the generalized algorithm for the odd switching angles, for any value of m and $NP1$, is given by the equation below,

$$\alpha_k = \frac{60^\circ(k+1)}{m+1} - \left[\frac{2\times 60^\circ}{m+1} \times \frac{\Delta_k \times NP1}{0.8}\right] \text{, for k odd} \qquad \text{(A7)}$$

APPENDIX B (Derivation of switching angle equation for even k)

From Figure 5, a suitable equation for the curves using the quadratic fit would be of the form,

$$\Delta_k = -a(k-b)^2 + c \tag{B1}$$

From the figure, it could be seen that $\Delta_k = 0.325$ at $k = m - 1$ for m = 13 and above but assume m = 11 and below to have the same point too. With $\Delta_k = 0.325$ at $k = m - 1$,

$$0.325 = -a(m - 1 - b)^2 + c \tag{B2}$$

While $k = \dfrac{m-1}{2}$ for every curve, is found to be very near the value of 0.183. With Δ_k = 0.183 at $k = \dfrac{m-1}{2}$,

$$0.183 = -a\left(\frac{m-1}{2} - b\right)^2 + c \tag{B3}$$

Since $\Delta_k = 0$ at k = 0, then

$$0 = -ab^2 + c \tag{B4}$$

Subtract (B4) from (B2),

$$0.325 = -a(m-1-b)^2 + ab^2$$
$$0.325 = -a\left[(m-1-b)^2 - b^2\right]$$

From $a^2 - b^2 = (a+b)(a-b)$,

$$0.325 = -a(m-1)(m-1-2b) \tag{B5}$$

Subtract (B4) from (B3),

$$0.183 = -a\left(\frac{m-1}{2} - b\right)^2 + ab^2$$
$$0.183 = -a\left[\left(\frac{m-1}{2} - b\right)^2 - b^2\right]$$

From $a^2 - b^2 = (a+b)(a-b)$,

$$0.183 = -a\left(\frac{m-1}{2}\right)\left(\frac{m-1}{2} - 2b\right)$$

$$0.366 = -a(m-1)\left(\frac{m-1}{2} - 2b\right) \tag{B6}$$

Divide (B5) with (B6),

$$0.888 = \frac{(m-1-2b)}{\frac{m-1}{2} - 2b}$$

$$b = 2.482(m-1) \tag{B7}$$

Substitute (B7) in (B5),

$$0.325 = -a(m-1)\big(m-1-2(2.482)(m-1)\big)$$

$$a = \frac{0.082}{(m-1)^2} \tag{B8}$$

Substitute (B7) and (B8) in (B4),

$$0 = -\left[\frac{0.082}{(m-1)^2}\big(2.482(m-1)\big)^2\right] + c$$

$$c = 0.505 \tag{B9}$$

Substitute (B7), (B8) and (B9) in (B1) yields:

$$\Delta_k = -\frac{0.082}{(m-1)^2}\left[k - 2.482(m-1)\right]^2 + 0.505 \tag{B10}$$

From the curves in the Figure 5, it could be seen that only for m=13 and above, Δ_k = 0.325 at k=m–1 while for m=11 and below, at k=m–1, Δ_k gradually decreases from 0.325 in a curve as shown in the figure. So Equation (B10) is only accurate for m=13 and above and an error correction method needs to be carried out so that the errors for m=11 and below could be reduced to an acceptable range. From the errors generated through programming from m=3 to m=13 for Δ_k (error= $\Delta_{k\ \text{quadratic}} - \Delta_{k\ \text{actual}}$), the errors increase from k=2 to k=12. Therefore, while the values for $1/m^2$ may somewhat be like a majority of the errors, it is more suitable to use another expression to represent the equations. k/m^3 is found to be a more suitable replacement with the variable k in the equation minimizing the errors at a smaller k, as is the condition with the errors generated.

As a conclusion, the correct the errors of Equation (B10), k/m^3 is subtracted from it as shown below,

$$\Delta_k = -\frac{0.082}{(m-1)^2}\left[k - 2.482\,(m-1)\right]^2 + 0.505 - \frac{k}{m^3} \quad \text{for even k} \tag{B11}$$

The generalized algorithm for the even switching angles for any value of m and *NP*1, is then given by:

$$\alpha_k = \frac{60^\circ \times k}{m+1} + \left[\frac{2 \times 60^\circ}{m+1} \times \frac{\Delta_k \times NP1}{0.8}\right], \quad \text{for even k} \tag{B12}$$