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## Nonlinear Parametric Study of Photon in a Fibre Bragg Grating

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### Abstract

This paper examines the nonlinear effects of alpha and gamma changes on the motion of photon in a Fiber Bragg Grating. The study has been successfully performed out under Bragg resonance condition where the initial frequency of the light has the same value with the Bragg frequency. The Stokes parameter is chosen to provide important information on the total energy and energy difference between the forward and backward propagating modes. The motion of a particle moving is shown using the classic anharmonic potential where beta is set to zero for power conservation along the grating structures.

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### 1. Introduction

Nonlinear effects in Fiber Bragg Grating (FBG) are currently under intense investigation by many leading groups of researchers. Bragg gratings in optical fibers are excellent devices for studying nonlinear phenomena particularly based on the Kerr nonlinearity [1]. The stationary properties of one-dimensional Bragg gratings were first analyzed by Winful et.al [2]. Several group of researchers has done many researches and reported the existence of soliton in Fiber Bragg gratings [1-5]. Chen and Mills coined the term gap soliton in their numerical work covering the nonlinear optical super lattices [3]. Mills and Trullinger obtained an analytical solution for stationary gap solitary waves [4]. Sipe and Winful [5], Christoudolides and Joseph [6], Aceves and Wabnitz [7], de Sterke and Sipe [8] and recently K. Senthilnathan *et. al* has derive the formation of bright and gap soliton solution for nonlinear coupled mode equation, which governs the pulse propagation in FBG [9].

The motion of particle moving in FBG represents the pulse propagation in the grating structure of fiber optics to show the existence of optical soliton. In order to describe the photon motion, the function of potential energy is

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depicted. Photon can be trapped by some parameters of potential energy such as alpha and gamma. In this paper we the effect of alpha and gamma for obtaining the optimized points of the potential well is examine.

## 2. Theoretical Consideration

Wave propagation in optical fibre is analyzed by solving Maxwell's Equation with appropriate boundary conditions. In the presence of Kerr nonlinearity, using the coupled-mode theory, the nonlinear coupled mode equation is defined under the absence of material and waveguide dispersive effects. This is due to the dispersive factor arising from the periodic structure dominating near Bragg resonance conditions and it is valid only for wavelengths close to the Bragg wavelength. By substituting the stationary solution to the coupled mode equation and by assuming  $E_{\pm}(z, t) = e_{\pm}(z)e^{-i\hat{\delta}ct/\bar{n}}$ , we obtain

$$\begin{aligned} i\frac{de_f}{dz} + \hat{\delta}e_f + \kappa e_b + \left(\Gamma_s|e_f|^2 + 2\Gamma_x|e_b|^2\right)e_f &= 0 \\ i\frac{de_b}{dz} + \hat{\delta}e_b + \kappa e_f + \left(\Gamma_s|e_b|^2 + 2\Gamma_x|e_f|^2\right)e_b &= 0 \end{aligned} \quad (1)$$

Equation (1) represents the time-independent light transmission through the gratings structure where  $e_f$  and  $e_b$  are the forward and backward propagating modes [1]. In order to explain the formation of Bragg soliton, consider the Stokes parameter since it will provide useful information about the total energy and energy difference between the forward and backward propagating modes. In this study, we consider the following Stokes parameter [10].

$$\begin{aligned} A_0 &= |e_f|^2 + |e_b|^2, \\ A_1 &= e_f e_b^* + e_f^* e_b, \\ A_2 &= i(e_f e_b^* - e_f^* e_b), \\ A_3 &= |e_f|^2 - |e_b|^2 \end{aligned} \quad (2)$$

with the constraint  $A_0^2 = A_1^2 + A_2^2 + A_3^2$ . In FBG theory, the Nonlinear Coupled Mode (NLCM) equation requires that the total power  $P_0 = A_3 = |e_f|^2 - |e_b|^2$  inside the grating is constant along the grating structures [2]. The NLCM equations can be written in terms of Stokes parameter

$$\begin{aligned} \frac{dA_0}{dz} &= -2\kappa A_2, \quad \frac{dA_1}{dz} = 2\hat{\delta}A_2 + 3\Gamma A_0 A_2 \\ \frac{dA_2}{dz} &= -2\hat{\delta}A_1 - 2\kappa A_0 - 3\Gamma A_0 A_1, \quad \frac{dA_3}{dz} = 0 \end{aligned} \quad (3)$$

In Equation (3), we drop the distinction between the Self-Phase Modulation and cross effect modulation effects and hence it becomes  $3\Gamma = 2\Gamma_x + \Gamma_s$  and it can be clearly show that the total power,  $P_0 (=A_3)$  inside the grating is found to be constant or conserved along the grating structure [2]. In the construction of the anharmonic type oscillator it is necessary to use the conserved quantity. It is in the form  $\hat{\delta}A_0 + \frac{3}{4}\Gamma A_0^2 + \kappa A_1 = C$ , where  $C =$

constant of integration and  $\hat{\delta} =$  detuning parameter. Using Equation (3), we obtain

$$\frac{d^2 A_0}{dz^2} - \alpha A_0 + \beta A_0^2 + \gamma A_0^3 = 4\hat{\delta}C \quad (4)$$

where  $\alpha = 2[2\hat{\delta}^2 - 2\kappa^2 - 3\Gamma C]$ ,  $\beta = 9\Gamma\hat{\delta}$  and  $\gamma = \frac{9}{4}\Gamma^2$ . Equation (4) contains all the physical parameter of the NLCM equation [1].

In order to describe the motion of a particle moving with the classic anharmonic potential, we have the solution as follows,

$$V(A_0) = -\alpha \frac{A_0^2}{2} + \beta \frac{A_0^3}{3} + \gamma \frac{A_0^4}{4} \quad (5)$$

It represents the potential energy distribution in the Fiber Bragg Grating structures [4].

### 3. Bragg Resonance Conditions

From Equations (3) and (4), we can easily define the function of refractive index and frequency dependence,

$$\hat{\delta} = \frac{n_0}{c(\omega_0 - \omega_B)} \quad (6)$$

Equation (6) shows the detuning parameter which provides a measurement of detuning from the Bragg resonance condition [1]. The Bragg frequency is given by

$$\omega_B = \frac{\pi c}{n_0 \Lambda} \quad (7)$$

Assuming the NLCM equations under the Bragg Resonance Condition, where  $\omega_B = \omega_0$  and hence the detuning parameter  $\hat{\delta} = 0$ . Under this condition, the anharmonic oscillator equation reduces to the well known unforced and undamped duffing oscillator equation [1].

### 4. Results and Discussion

According to Equation (5),  $\beta$  is not considered due to power conservation along the propagating of this FBG structure. The qualitative aspects of the potential well will change if we vary the nonlinearity parameter of the wave equation. The description can be shown as follows:

Figure 1 depicts the double-well potential under Bragg resonance condition where  $\beta = 0$ ,  $\gamma = 0.23$  and  $\alpha$  is varies from 0.1 to 1.0. Photon with power of less than total power,  $P_0$  will only travel inside the well unless the energy exceeds the potential level. This would allow the photon to move outside the well.

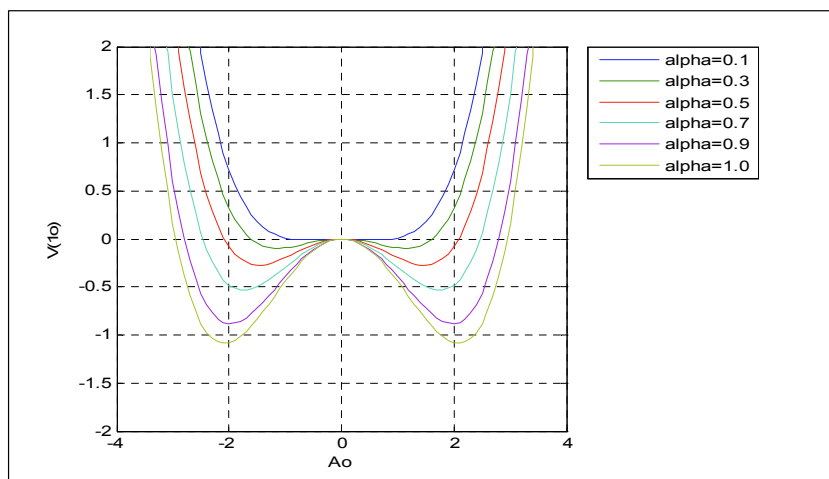


Figure 1. The motion of photon in double well potential when  $\alpha = 0.1$  to 1.0.

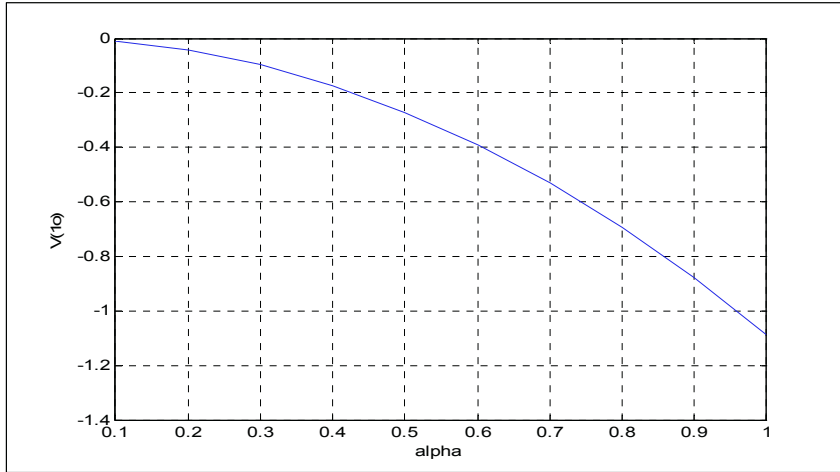


Figure 2. The optimized point of the double well potential when  $\alpha = 0.1$  to  $1.0$ .

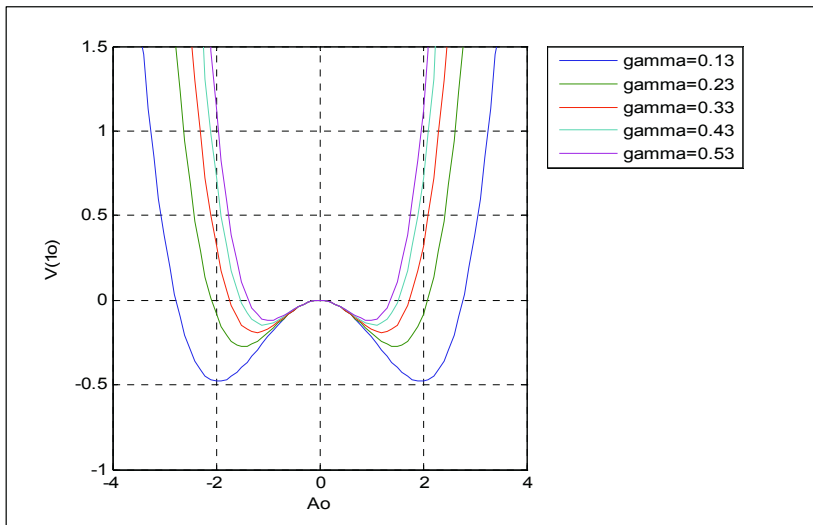


Figure 3. Under Bragg resonance condition the system possesses double well potential when  $\gamma = 0.13$  to  $0.53$ .

Figure 2 explains the optimized point for varies of alpha,  $\alpha$ . The graph clearly shows that the optimized points decreased exponentially when  $\alpha$  are increased. However, it is no longer valid when  $\alpha \gg 1$ , since it turns into linear a relationship. Figure 3 shows the motion of photon in double well potential under varies of gamma under Bragg resonance condition from 0.13 to 0.53. It noticed that the increment of gamma ( $0.53 < \gamma < 1$ ) will reduce the double well potential to a single well potential. Figure 4 describes the optimized point for varies of gamma,  $\gamma$ . The parametric of gamma produces for potential energy function increases exponentially. However, when  $\gamma \gg 1$ , linearity is observed. This shows that it is invalid if  $\gamma \rightarrow \infty$ .

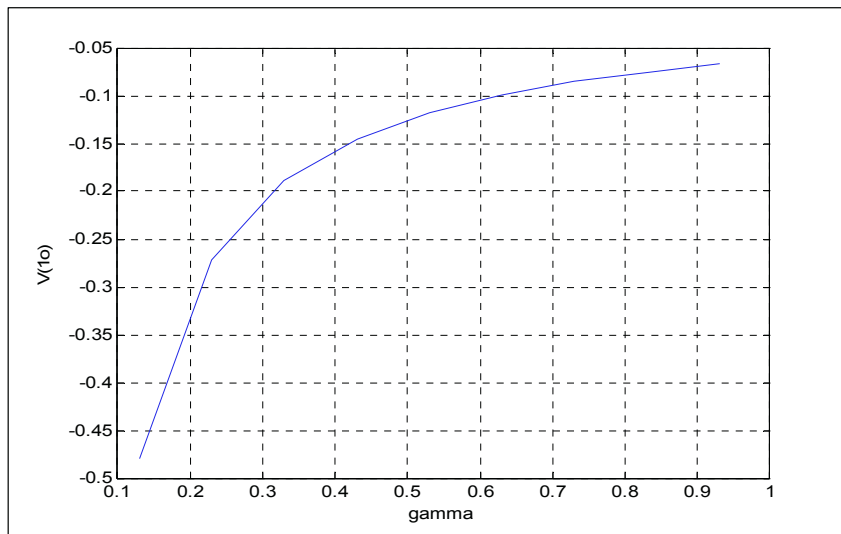


Figure 4. The optimized point of the double well potential when  $\gamma = 0.1$  to  $1.0$ .

## 5. Conclusions

By using the Stokes parameter in nonlinear coupled mode theory under Bragg resonance condition, we successfully developed that the changes of nonlinearity parameter will depend on the motion of photon in the potential well. This will influence in the existence of Bragg soliton in Fiber Bragg Grating.

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