PARAMETRIC DESCRIPTION OF 3-DIMENSIONAL SURFACE OF CRANIOFACIAL SHAPE

Deni Suwardhi*, Halim Setan*, Zulkepli Majid*, Anuar Ahmad*, Albert K.Chong**.
*Department of Geomatic Engineering, Faculty of Geoinformation Science & Engineering, UTM Skudai.
**School of Surveying, University of Otago. Dunedin, New Zealand.
Tel: 3 4797587
E-mail: deni_suwardhi@yahoo.com, halim@fksg.utm.my, zulkepli@fksg.utm.my

Abstract

This paper describes two methods to create a parametric model of a craniofacial structure. The first method uses spherical harmonics (SH) which describe the surface determined by mapping to a sphere. The second method is Hierarchical B-Spline which split the surface into several numbers of patches to obtain a given shape. The usual way of storing craniofacial data is voxel images that allow visual capturing of the objects, but it is lack of structural description since they are based on huge list of numbers. On the other hand new medical imaging modalities, such as Digital Close-Range Photogrammetry and Laser-scanner produce precise and smooth models of human tissues. Until now the model used in such applications are rough polygon meshes.

1.0 INTRODUCTION

Nowadays, multimedia data not only appear in image, sound or video forms, but could be in geometric surfaces forms. The latter has become more popular because of its availability of direct or indirect mesh sensor. Laser scanner is one example of direct sensor, whereas CT-Scanner, MRI-Scanner and Digital Close Range Photogrammetry are examples of indirect sensor. A wide variety of representation methods have been proposed for modeling these surfaces. Basically, they can be classified into three categories depending on the data source and the target application: mesh representation, parametric representation and implicit representation. For a simple visualization of smooth surfaces, the model widely used is the mesh approximation. When the data is acquired from sensors and the model should be suitable for a CAD use, parametric approximation could be well adapted using a standard model such as B-splines. If the approximation should be used for a more semantic description of an object, implicit models can be chosen.

Unfortunately, mesh surface representation data have large volume in discrete form. So it must be represented by other forms for efficient / effective manipulation and analysis. Based on the definition of shape in Iyer (2004), mesh surface is shape or geometry objects that have geometrical, morphological and topological aspects. Parametric shape representation is commonly used to solve the problem.

Parametric technique describes surface objects by mathematical relationships rather than just a set of vertices (as in polygonal models) or primitives (as in many 3D modelers). A parameterization of a surface can be viewed as one-to-one mapping from a suitable domain to the surface. The most popular parametric modelers is 3D shapes with a 2D sketch and then apply a sweep or extrusion or rotation to it (surface of revolution).

Many methods could reduce a shape into a simpler shape representation. Detailed reviews of various shape representation techniques are available in Loncaric (1998). There is difference between a shape representation and a shape description. Marr and Nishihara (1978) define a shape representation as: "A formal scheme for describing shape or some aspects of shape together with rules that specify how the scheme is applied to any particular shape. The result of using a representation to describe a given shape is a description of the shape in that representation. A description may specify a shape only roughly or in fine detail."

Loncaric (1998) distinguishes between a shape representation and a description in this way: "Shape representation methods result in a non-numeric representation of the original shape (e.g. a graph) so that the important characteristics of the shape are preserved. The word "important" in the above sentence typically has different meanings for different applications. Shape description refers to the method that results in a numeric descriptor of the shape and is a step subsequent to shape representation. A shape description method generates a shape descriptor vector (also called a feature vector) from a given shape. The goal of description is to uniquely characterize the shape using its shape descriptor "vector." In other words, shape description is an instantiation of a shape representation. This definition is used in this paper.

The following criteria have been commonly cited by Iyer (2004), for formulating or evaluating a shape representation:

a) Scope: The shape representation must be able to describe all classes of shape.
b) Uniqueness: There should be a one-to-one mapping between shapes and descriptions of shape within a representation.
c) Stability: For a particular shape representation, the shape descriptor must be stable for small changes in shape.
d) Sensitivity: The shape representation must be capable of capturing even subtle details of the shape.
e) Efficiency: It must be possible to efficiently compute and compare descriptors within a shape representation from input data.
f) Multi-scale Support: The representation must describe shape at multiple scales as a hierarchical structure.
g) Local Support: The representation must be information preserving and, if required, should be able to be computed locally.

This paper reports 3D object representation activity as a part of research on the development of a craniofacial spatial database and information system in Malaysia. Craniofacial is pertaining to the head and face of human, including soft-tissue and hard-tissue. For soft-tissue, mesh surface is obtained from laser scan whereas hard tissue-data is obtained from CT-Scan. The descriptor will be stored in database as a representation of 3D model of craniofacial object. In a similar way, mesh representation will be stored in database because the descriptor has functions for similarity searching, symmetry or asymmetry validation, or another spatial analysis as procedures in database server. Query result will be given in the mesh representation format.

Section 2 in this paper describes representation and surface fitting methods for spherical harmonic and hierarchical b-spline function. Section 3 presents experimental result. Section 4 presents the property of descriptor and section 5 is conclusion.

2.0 METHODS

2.1 Shape representation by spherical harmonic

The Spherical harmonic (SH) shapes description, originally described by Brechbühler *et. al.* (1991), is a hierarchical, global, multi-scale boundary description which is limited to represent objects of spherical topology. The basis functions of the parameterized surfaces are spherical harmonics. A key component of the processing scheme is the mapping of surfaces objects to parameterized surfaces prior to expansion into harmonics. Brechbühler *et. al.* (1995) presented procedures for the explicit parametric representation and global description of surfaces of simply connected 3-D objects. He parameterized the surface by defining continuous, one-to-one mapping from the surface of the original objects to the surface of a unit sphere.

2.1.1 Spherical harmonics descriptors

SH representation only consider surfaces that are topologically equivalent to a sphere, that is, surfaces that can be smoothly deformed into a spherical shape. A parametric representation of a surface in three dimensions can be expressed as follows:

$$\vec{X}(\theta,\phi)=\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} X(\theta,\phi) \\ Y(\theta,\phi) \\ Z(\theta,\phi) \end{pmatrix}, \tag{1}$$

where x, y, and z are Cartesian coordinates and $X(\theta,\phi)$, $Y(\theta,\phi)$, and $Z(\theta,\phi)$ are the corresponding coordinate functions of the surface variables θ and ϕ.

SH functions are defined by the following relation

$$Y_l^m(\theta,\phi)=\sqrt{\frac{2l+1(l-m)!}{4\pi(l+m)!}}P_l^m(\cos\theta)e^{im\phi}, \tag{2}$$

where $P_l^m(\cos\theta)$ are associated Legendre functions (with argument $\cos\theta$), and l and m are integers with $-l \le m \le l$. Note that when m is a superscript of a SH or associated Legendre function, as in $Y_l^m(\theta,\phi)$ or $P_l^m(\cos\theta)$, it identifies a unique function and is not an exponent.

$$P_l^m(w)=\frac{(-1)^m}{2^l l!}(1-w^2)^{\frac{m}{2}}\frac{d^{m+l}}{dw^{m+l}}(w^2-1)^l \tag{3}$$

SH are complex valued functions. Real valued functions are obtained by forming appropriate linear combinations of the complex functions. In this report, $Y_l^m(\theta,\phi)$ refer to real valued functions that are expressed as

$$Y_l^m(\theta,\phi)=N_{lm}P_l^m(\cos\theta)\cos(m\phi), \tag{4}$$

for $m \ge 0$ and

$$Y_l^m(\theta,\phi)=N_{lm}P_l^{|m|}(\cos\theta)\sin(|m|\phi), \tag{5}$$

for $m<0$, where N_{lm} is a normalization constant.

SHs are ideal for representing functions defined on a unit sphere since they form a complete orthonormal basis set. A function $F(\theta,\phi)$, dependent on spherical coordinates θ and ϕ, can be expressed as an expansion of spherical harmonic functions $Y_l^m(\theta,\phi)$ as follows:

$$F(\theta,\phi)=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}c_{lm}Y_l^m(\theta,\phi), \tag{6}$$

where c_{lm} are expansion coefficients determined by evaluating the surface integral that represents the inner product of the two functions

$$c_{lm}=\int_0^{\pi}\int_0^{2\pi}F(\theta,\phi)Y_l^m(\theta,\phi)\sin\theta d\theta d\phi \tag{7}$$

Note that this integral covers the entire surface. Approximations are computed to a finite value of l which is called the order of the expansion.

2.1.2 *Surface fitting to SH*

Basically, the problem of approximating the surface of 3D objects consists in finding a model that represents a set of data points or vertices :

$$(x_i, y_i, z_i) = \in R^3, \forall i = 0,..., nvert$$

The parametrization defines the function $F(\theta,\phi)$ only for the parameter coordinates of the vertices. Let *nvert* denote the number of vertices and *i* be the index of a vertex, $0 \leq i < nvert$. So, $F(\theta_i, \phi_i) = F_i$ is defined. An interpolating function between these sample points is needed for the evaluation of the integral (7). An adaptation of bilinear interpolation could be used for this purpose.

The straightforward discretization of the integral,

$$c_{lm} \approx \frac{4\pi}{n_{vert}} \sum_{i=0}^{n_{vert}-1} F_i Y_l^m(\theta_i, \phi_i) \qquad (8)$$

does not give the precise coefficients of a series representing the object, especially craniofacial object. The reason is that although the function $Y_l^m(\theta,\phi)$ are orthonormal, their values evaluated at some set of parameter pairs (θ_i, ϕ_i) will generally not form an orthonormal set of vectors. Some indexing scheme $j(l,m)$ adopted to assign a unique index *j* to every pair l, m, like e.g. $j(l,m) = l^2 + l + m$. When the degree of the spherical harmonics is limited, i.e., $0 \leq 1 < n_j$, *j* is also limited by $j < n_j = n_l^2$. All needed values of our basis functions is arranged in an *nvert x* n_j matrix B where $b_{i,j(l,m)} = Y_l^m(\theta_i, \phi_i)$. In the usual case where n_j is significantly smaller than *nvert*, the columns of B are approximately orthogonal. The object space coordinate of all vertices is arranged in an *nvert x 3* matrix $X = (F_0, F_1, ..., F_{nvert-1})^T$ and all coefficients in the *nj x 3* matrix $C = (c_0^0, c_1^{-1}, c_1^0, c_1^1, ...)^T$. The equations 8 for all *l* and *m* take the compact form $c_{lm} \approx \frac{4\pi}{n_{vert}} B^T X$. But a spherical harmonic series should really pass near the real positions of our vertices, ie. $X = BC + E$ so that the error value in matrix E could be small. These so called normal equation are solved with least square sums over the column of E by

$$C = (B^T B)^{-1} B^T X \qquad (9)$$

This is not too different from (8) because the symmetric $n_j x n_j$ matrix $\frac{4\pi}{n_{vert}} B^T B$ is close to the identity matrix.

This approximation procedure has been applied to molecular surfaces by Duncan and Olson (1995). In their methods, the function $F(\theta,\phi)$ defines the *radius function*, that is, the distance from an origin inside the surface to the surface point with angular coordinate (θ,ϕ). This radius function is represented by a spherical harmonic expansion. An important restriction is that for an accurate approximation, the radius function must be a single value. That is, each ray from the origin must have one and only one intersection with the surface. Surfaces with this property are called star like.

2.2 Shape representation by Hierarchical B-splines

Hierarchical B-splines are extension of B-splines. Refer to Forsey & Bartels (1988) for a complete description of hierarchical B-splines. They introduced the concept of locally refining a B-spline surface. In 1995, Forsey & Bartels completed with writing a paper about surface fitting to hierarchical B-splines from point cloud data.

2.2.1 Hierarchical B-Splines descriptors

The basis functions used to define B-spline curves and surfaces are the B-spline functions. They are piecewise polynomial functions defined as follows. Given a non-decreasing sequences of r real numbers $U = \{u_i, u_i, \ldots, u_i\}$, the *i*th B-spline function of pth degree (i.e. order $p + 1$), $N_{i,p}(u)$, is recursively defined as

$$N_{i,0}(u) = \begin{cases} 1 & u_i \le u < u_{i+1} \\ 0 & otherwise \end{cases} \qquad (10)$$

$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u) \qquad (11)$$

while the 0/0 quotient is defined to be zero.

The u_i values are referred to as *knots* and U is the *knot vector*. Note that consecutive knots can have equal values: if u_i through u_{i+k-1} are all equal, u_i is said to have multiplicity k. The unique values in the knot vector are referred to as breakpoints. The semi-open interval $[u_i, u_{i+1}]$ is called *i*th knot span.

B-spline surface defined by control vertices $V_{i,j}$ and basis functions $N_{i,k}(u)$, $M_{j,l}(w)$ of some polynomial order k and l, respectively

$$Q(u,w) = \sum_{i=1}^{n+1} \sum_{j=1}^{m+1} V_{i,j} N_{i,k}(u) M_{j,l}(w) \qquad (12)$$

The idea of hierarchical B-splines is to increase the number of patches to obtain a given shape. The number of patches on a hierarchical B-splines parametric surface is increased by the insertion of additional knots. This process is known as refinement or subdivision. Refinement allows manipulation of smaller details of the surface. This is of particular interest when representing objects such as faces, where greater levels of detail are desired in areas such as the eye, nose, mouth and ear.

Given the surface defined by Equation 12, it is possible to refine the basis functions, and thus the surface, such that they are linear combinations of smaller basis functions

$$N_{i,k}(u) = \sum_r \alpha_{i,k}(r) B_{r,k}(u) \qquad (13)$$

$$M_{j,l}(w) = \sum_s \alpha_{j,l}(s) B_{s,l}(w) \qquad (14)$$

where k and l represent the polynomial order of the basis functions. A larger number of control points is also generated, and the new refined surface definition is

$$Q(u,v) = \sum_r \sum_s W_{r,s} B_{r,k}(u) B_{s,l}(v) \qquad (15)$$

$W_{r,s}$ is related to $V_{i,j}$ by

$$W_{r,s} = \sum_i \sum_j \alpha_{i,k}(r) \alpha_{j,l}(s) V_{i,j} \qquad (16)$$

Since the conversion from one matrix of control points to the other is done via the conversion of the basis functions, refinement is non-local, because of the large-scale replacement of **V**’s by **W**’s over an entire row or column. More **W**’s are generated than are generally needed.

Forsey & Bartels (1988) use *overlays* to minimize the number of **W** vertices that must be stored, and *offset referencing* to simplify the editing process.

2.2.2 Surface fitting to hierarchical B-Spline

The previous sections describe the characteristics and generation of B-spline surfaces from a known defining polygon net. The inverse problem is also of interest; i.e., given a known set of data on a surface, determine the defining polygon net for a B-spline surface that best interpolate that data.

Refer to equation 12 and note that the $Q(u,w)$'s are the known surface data points. The polygon net points and basis functions can be determined for a known order and a known number of defining polygon net vertices in each parametric direction provided that the parametric values *u,w* are known at the surface data points. Hence, for each known surface data point equation 12 provides a linear equation in the unknown defining polygon net vertices $V_{i,j}$. Writing equation 12 out for a single surface data point yields

$$D_{1,1}(u_1,w_1) = \qquad (17)$$

$$N_{1,k}(u_1)[M_{1.l}(w_1)V_{1,1} + M_{1.l}(w_1)V_{1,1} + ... + M_{1.l}(w_1)V_{1,1}] +$$

$$\vdots$$

$$N_{1,k}(u_1)[M_{1.l}(w_1)V_{1,1} + M_{1.l}(w_1)V_{1,1} + ... + M_{1.l}(w_1)V_{1,1}] +$$

where for an *r x s* topologically rectangular set of data and / Writing an equation of this form for each data point yields a system of simultaneous equations. In matrix form the result is

$$[D] = [C][B] \qquad (18)$$

where $C_{i,j} = N_{i,k}M_{j,l}$. For *r x s* topologically rectangular surface point data, [D] is an *r * s x 3* matrix containing the three-dimensional coordinates of the surface point data, [C] is an *r * s x n * m* matrix of the products of the B-spline basis functions, and [B] is an *n * m x 3* matrix the three-dimensional coordinates of the required polygon net points.
If [C] is square, the defining polygon net is obtained directly by matrix inversion, i.e.,

$$[B] = [C]^{-1}[D] \qquad (19)$$

In this case the resulting surface passes through each data point.

If [C] is not square the problem is overspecified and a solution can only be obtained in some mean sense. In particular the solution given by

$$[B] = [[C]^T[C]]^{-1}[C]^T[D] \qquad (20)$$

In Forsey and Bartels (1988), a compact means of representing spline surfaces derived from successive refinement was described. Three essential features of this representation were :

a) That only the modified portions of a hierarchical surface need to be stored in a data structure;
b) That each level of refinement, or overlay, was represented as an offset from reference position derived from a level of lower refinement
c) That editing operates on points which were selected directly from the composite surface itself, rather than through control vertices.

The last feature is not relevant to surface fitting, which is concerned with the static approximation of given data. The other two features, however, lend themselves to a compact and efficient means of fitting surfaces to data.

The approach taken utilizes least square to fit a template spline surface to the data, for example, a spline approximation to a plane, sphere, cylinder, or torus. The template is chosen to model the main topology of the data appropriately. The fitting operates on each coordinate is shown by equation 20. The residual between surface and the data,

$$r = \sum_{i=1}^{n+1} \sum_{j=1}^{m+1} V_{i,j} N_{i,k}(u_r) M_{j,l}(w_s) - P_{r,s} \tag{21}$$

is computed for all r. Out-of-tolerance points are those points *(u_r, w_s)* of the domain for which the data lie further than a selected tolerance from the surface. If no points are out of tolerance, the data have been fitted sufficiently well by the surface. The response to the existence of any out-of-tolerance points will be to refine the surface and repeat the fitting process only for regions that are out-of-tolerance.

3.0 EXPERIMENTAL RESULTS

The approximation method has been tested on three surfaces:

a) A soft-tissue surface is generated from laser scanner
b) A soft-tissue surface is generated from CT-scanner

3.1 Approximation of a soft-tissue from laser scanner

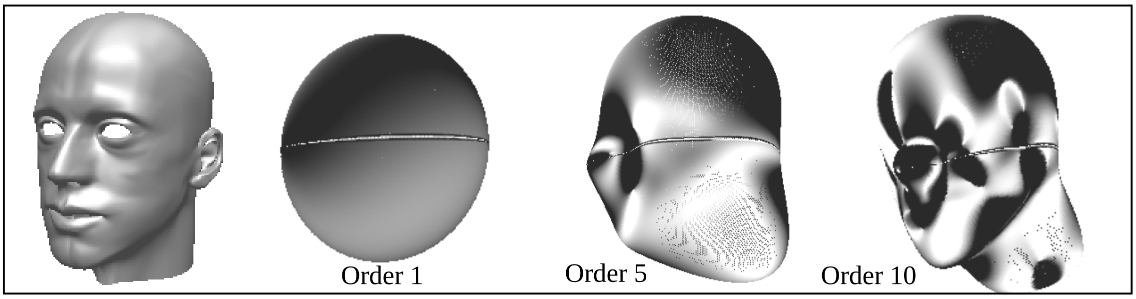

Figure 1. **Mesh surface from Laser scan and its SH reconstruction**

Figure 1 shows mesh surface model of head taken from laser-scanner (number of vertices: 36047, Number of triangle face : 718080).

3.2 Approximation of a soft-tissue from CT-scanner

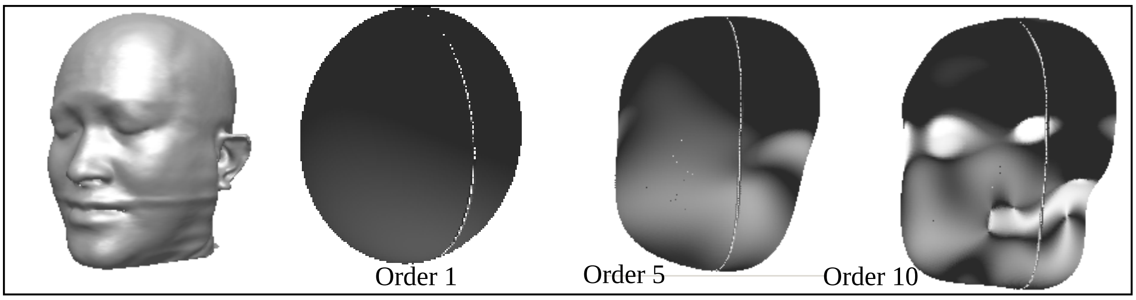

Figure 2. Soft-tissue Mesh surface from CT-scan and its SH reconstruction

Figure 2 shows mesh surface model of head taken from laser-scanner (number of vertices: 24324, Number of triangle face : 48638).

4.0 PARAMETRIC DESCRIPTION APPLICATIONS

As mentioned in the Section 1.0, this paper reports 3D object representation activity as a part of research on the development of a craniofacial spatial database and information system in Malaysia. The descriptor will be stored in database as a representation of 3D model of craniofacial object. In the similar way, mesh representation will be stored in database because the descriptor has functions for similarity searching, symmetry or asymmetry validation, or another spatial analysis as procedures or functions in database server. Query result still will be given in the mesh representation format.

The following explains subsequently similarity search concept, shape average calculation, and crest line extraction as applications of parametric descriptor.

4.1 Similarity search application

In our prototype content-based 3D model craniofacial database system (Figure 4), an entry in the database stores a 3D model along with descriptor for the model. Then, steps of a query/ retrieval to the database will be explained as follows:

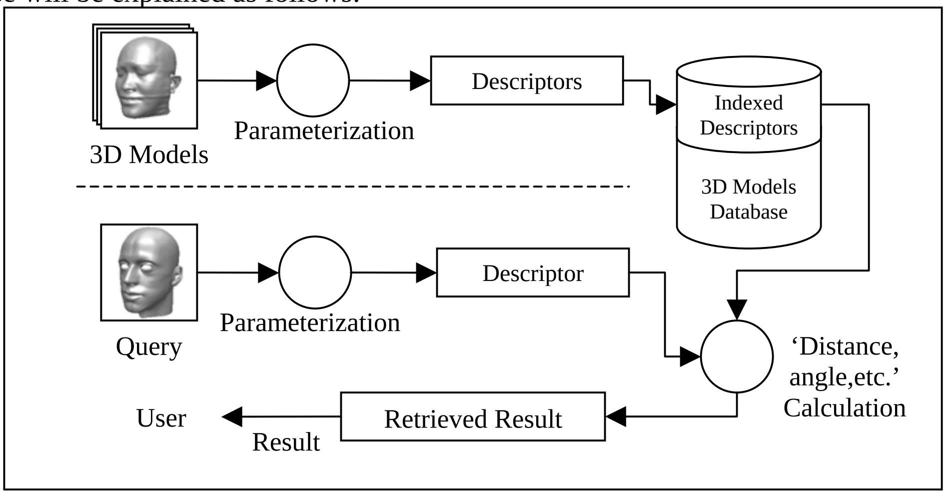

Figure 4. Overview of the shape similarity search system.

a) *Query formation*: users will present the system with an example 3D shape and asks for k-nearest shapes.
b) *Shape representation*: a descriptor of the example will be needed by the database system. So, parameterization of the example is calculated.
c) *Dissimilarity computation*: application in database server will compared a few different methods to compute distance, or dissimilarity, between a pair of descriptors.
d) *Query Result*: database server will give the k-nearest 3D shapes that distances lie in a selected tolerance.
e) *Indexing and retrieval*: In the implementation, to speed up the query process, we will develop the spatial indexing methods. The descriptors will be treated as indexing objects.

4.2 Shape average application

The set of calculated spherical harmonic descriptor can be used to perform any statistical operation (Briet *et.al* ,1999). In their application they analyzed several skulls, calculated the average set of descriptors c_{lm} which they used to reconstruct an average skull.

The following scenario, which is representative of the queries that medical researches would like to ask, illustrates a sample session with such a system in which each step generates a database query (based on Brief *et.al.* 1999):

- The medical researcher may start by specifying and querying a set of voxel-defined surface, $g^j(x,y,z)$, with age, sex, and race criteria, for example 17-18 old, male and Malay race.
- After dataset, $g^j(x,y,z)$, is collected then each data j in dataset will be transformed into the functions of surface $r^j(\vartheta,\varphi)$ by segmentation and calculation of the euclidean distance.
- Calculation of the spectra of all shapes $r^j(\vartheta,\varphi)$.

$$c_{i,j} = \iint Y_i^*(\vartheta,\varphi) r^j(\vartheta,\varphi) d\vartheta d\varphi \qquad (22)$$

- Calculation of the norm spectrum by simply calculating the mean values

$$c_{i,norm} = \frac{1}{n}\sum_{j=1}^{n} c_{i,j} \qquad (23)$$

- Transformation of this spectrum back to the norm shape

$$r_{norm}(\vartheta,\varphi) = \sum_i c_{i,norm} Y_i(\vartheta,\varphi) \qquad (24)$$

$r_{norm}(\vartheta,\varphi)$ is used as "normal" reference for a comparison of the shape of a patient with defects. The comparison may be one of spatial query type in the system.

4.3 Crest lines extraction

The database will store craniofacial data and their anatomical components. For example, skull will differed to mandible, frontal, temporal, etc (see Figure 5). Hence, we need one method to separate the skull to its anatomical components.

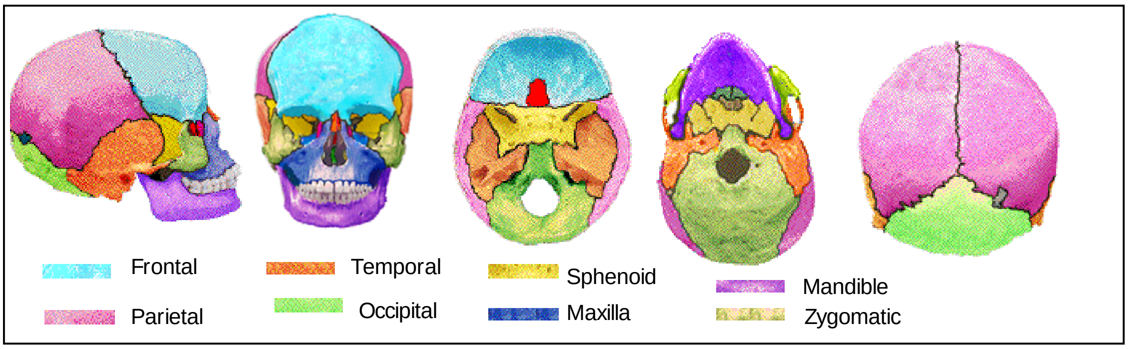

Figure 5. Anatomical of human skull components.

To achieve this, we will combine three techniques: the extraction of 3D feature lines or crest lines, their matching using 3D deformable line models, and the extension of the deformation to the whole shape space using warping techniques.

In their paper, Stylianou and Farin (2000) mention that crest lines are local shape features of a surface. They are as the set of all point satisfying

$$D_{t_1} k_1(u,v) = 0 \qquad (25)$$

where k_1 is the largest principal curvature and t_1 is its direction in the *(u,v)*-domain.

They are shape features with the main characteristic of using local information to yield a global description of the surface. Crest lines are anatomically meaningful, like on the skull, the crest lines represent the salient lines (the orbits, the nose, the mandible, or the temples).

5.0 CONCLUSION

Based on literature reviews, experimental results and applications that use the parametric descriptors, we compared the spherical harmonic and hierarchical B-spline method for representation of 3D craniofacial objects. Seven criteria from Iyer (2004) were referred to in our comparison. Table 1 gives the comparison.

Table 1: Representation method comparison

Criteria	Spherical Harmonic	Hierarchical B-spline
Scope	Limited to star like shape	Not limited
Uniqueness	Yes	Yes
Stability	Only for global change	Global or local change can detect
Sensitivity	Only for global change	Global or local change can detect
Efficiency	For high order is not simple computing. But, spectral values of the descriptor easy to compare to other descriptor objects	Simple computing. Not easy to compare one descriptor to another.
Multi-scale Support	Yes	Yes
Local Support	No	Yes

As part of the further work, we would like to implement the applications in section 4.0. We also would like to improve an algorithm for calculating the descriptors.

Acknowledgements

This research is part of a prioritized research IRPA vot. 74537 sponsored by Ministry of Science, Technology & Innovation Environment (MOSTI) Malaysia.

REFERENCES

1. C. Brechbühler, G. Gerig and O. Kübler, (1995), *Parametrization of closed surfaces for 3-D shape description*, Computer Vision and Image Understanding, pp. 154—170.
2. C. Brechbühler, G. Gerig and O. Kübler, (1991), *Towards representation of 3D shape: Global surface parametrization*, Visual Form, Proc. of the IWVF (Internatinal Workshop on Visual Form), Capri.
3. Iyer, Natraj, (2004), *A Multi-scale Hierarch ical 3D shaper representation for similar shape retrieval.* , proceedings of the TMCE, Laussanne, Switerland, edited by Horvarth and Xirouchakis.
4. D.R. Forsey and R. H. Bartels, (1988). *Hierarchical b-spline refinement.,* Computer Graphics, 22(3):pp. 205-212.
5. D.R. Forsey and R.H. Bartels., (1995), *Surface Fitting with Hierarchical Splines.*, ACM Transactions on Graphics, Vol. 14, No.2, April 1995, Pages 134-161.
6. Brief, J., Hassfeld, S., Redlich, T., Muhling, J., Walz, M., Krempein, R., Munchenberg, J., Worn, H., Rembold, U., Grabowski, H., Raczkowsky, J., Burgert, O., Salb, T., & Dilllmann, R. (1999). *3D Norm data – the first step towards semiautomatic virtual craniofacial surgery*. In Proceedings of the 13th International Congress and Exhibition on Computer Assisted Radiology and Surgery (CARS'99) (pp.559-563). Paris: Elsevier Press.
7. Bruce S. Duncan and Arthur J. Olson (1995), *Approximation and Characterization of Molecular Surfaces*, http://www.scripps.edu/pub/olson-web/papers/sph/sph.html,

8. Stylianou G. and Farin,G. (2000), *Crest lines extraction from 3D triangulated meshes*, Electronic Papers Procedding NSF/DoE Lake Tahoe Workshop on Hierarchical Approximation and Geometrical Methods for Scientific Visualization,October 15-17, 2000.
9. Marr, D., and Nishihara, H.K., (1978), *Representation and Recognition of the Spatial Organization of Threedimensional Shapes*, Proceedings of the Royal Society of London: Part B, Vol. 200, No. 1140, pp. 269-294.
10. Loncaric S., (1998), *A Survey of Shape Analysis Techniques*, Pattern Recognition, Vol. 31, No. 8, pp. 983-1001.