UNSTEADY MICROPOLAR BOUNDARY LAYER FLOW AND CONVECTIVE HEAT TRANSFER

ANATI BINTI ALI

UNIVERSITI TEKNOLOGI MALAYSIA

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To my husband Kamarudin, my daughters Amira Al Wardah and Athirah Al Husna, and family

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ABSTRACT

Most industrial fluids such as polymers, liquid crystals and colloids contain suspensions of rigid particles that undergo rotation. However, the classical Navier-Stokes theory normally associated with Newtonian fluids is inadequate to describe such fluids as it does not take into account the effects of these microstructures. In this research, the unsteady boundary layer forced and mixed convection of micropolar fluids are considered where the unsteadiness is due to an impulsive motion of the free stream. Both small and large time solutions as well as the occurrence of flow separation, followed by flow reversal are taken into account. The two-dimensional flow along the entire surface of a cylinder and a sphere is solved numerically using the Keller's box scheme in a three dimensional grid where the discretization is made either on a net cube, or a zig-zag grid in the case of flow reversal. The numerical results show that as the micropolar material parameter increases, the thickness of both velocity and microrotation boundary layers, as well as the peak value of the skin friction coefficient along the body surface, also increase. Meanwhile, the value of the Nusselt number, in the case of micropolar fluids, is lower near the forward stagnation point and higher near the rear stagnation point compared to Newtonian fluids. It is also found that the separation time is brought forward in both cases of weak and strong concentration of microelements in the assisting mixed convective flows. However, in the opposing case, the separation time is delayed for a flow past a cylinder, while for a flow past a sphere, only the weak concentration of microelements can give similar results.

ABSTRAK

Kebanyakan bendalir industri seperti polimer, hablur cecair dan koloid mengandungi ampaian zarah tegar yang mengalami putaran. Walaubagaimanapun, teori klasik Navier-Stokes yang kebiasaannya dikaitkan dengan bendalir Newtonian adalah tidak sesuai untuk menerangkan bendalir tadi memandangkan ia tidak mengambil kira kesan mikroputaran. Dalam penyelidikan ini, kesan olakan paksaan dan olakan campuran bagi bendalir mikropolar dipertimbangkan di mana aspek ketidakmantapan aliran adalah disebabkan oleh gerakan dedenyut arus bebas. Kedua-dua penyelesaian bagi nilai masa yang kecil dan nilai masa yang besar serta kejadian pemisahan aliran yang diikuti dengan aliran menentang diberi perhatian. Aliran di sepanjang permukaan silinder dan sfera ini diselesaikan secara berangka menggunakan skim kotak Keller dalam tiga matra di mana pendiskretan dilakukan pada grid berbentuk kiub atau berbentuk zig-zag bagi aliran menentang. Penyelesaian berangka yang diperolehi menunjukkan ketebalan lapisan sempadan bagi kedua-dua halaju dan mikroputaran serta nilai puncak bagi tegasan ricih di sepanjang permukaan objek adalah meningkat apabila parameter bahan mikropolar meningkat. Sementara itu, nilai nombor Nusselt bagi bendalir mikropolar adalah lebih rendah di sekitar titik genangan hadapan dan lebih tinggi di sekitar titik genangan belakang berbanding bendalir Newtonian. Kajian juga mendapati bahawa bagi olakan campuran membantu dalam bendalir mikropolar, pemisahan lapisan sempadan berlaku lebih awal bagi kedua-dua konsentrasi elemen mikro yang rendah dan tinggi. Bagaimanapun, bagi kes olakan campuran menentang, masa pemisahan dapat ditangguhkan bagi aliran melewati silinder manakala bagi aliran melewati sfera, hanya konsentrasi elemen mikro yang rendah dapat memberikan keputusan serupa.

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a	-	radius of the cylinder
c	-	thermal conductivity
C_f	-	skin friction coefficient
C_p	-	specific heat at constant pressure
erf	-	error function
F	-	body force
f or F	-	reduced stream function
g	-	acceleration due to gravity
Gr	-	Grashof number
h or H	-	reduced microrotation
j	-	microinertia density
K	-	micropolar material parameter
n	-	ratio of the microrotation vector component to the fluid skin friction at the wall
N	-	non-dimensional component of the microrotation vector normal to xy-plane
Nu	-	Nusselt number
$\overline{p}$	-	dimensional pressure
p	-	non-dimensional pressure
O	-	order of magnitude
$\overline{p}_d$	-	dynamic pressure
$\overline{p}_h$	-	hydrostatic pressure
Pr	-	Prandtl number

q_w	-	convection heat transfer coefficient
$\bar{r}(\bar{x})$	-	dimensional radial distance from symmetrical axis to surface of the sphere
$r(x)$	-	non-dimensional radial distance from symmetrical axis to surface of the sphere
Re	-	Reynolds number
s or S	-	reduced temperature
$\bar{t}$	-	dimensional time
t	-	non-dimensional time
t_s or T_s	-	separation time
$\bar{T}$	-	dimensional fluid temperature
T	-	non-dimensional fluid temperature
T_∞	-	external temperature
T_w	-	surface temperature
$\bar{u}, \bar{v}$	-	dimensional velocity components along $\bar{x}$ and $\bar{y}$ axes
u, v	-	non-dimensional velocity components along x and y axes
$\bar{u}_e(\bar{x})$	-	dimensional external velocity
U_∞	-	reference velocity
$\bar{x}, \bar{y}$	-	dimensional Cartesian coordinates measured along the surface of the cylinder (or sphere) and normal to it, respectively
x, y	-	non-dimensional Cartesian coordinates measured along the surface of the cylinder and normal to it, respectively

Greek symbols

α	-	mixed convection parameter

β	-	thermal expansion coefficient
γ	-	spin gradient viscosity
η	-	similarity variable
κ	-	vortex viscosity
λ	-	the derivative of the external velocity with respect to x at the forward and rear stagnation points
μ	-	dynamic viscosity
υ	-	kinematic viscosity
ρ	-	density
τ_w	-	wall shear stress
ψ	-	stream function

Superscripts

$'$	-	differentiation with respect to η or Y

Subscripts

s	-	steady-state flow
w	-	wall condition
∞	-	far field condition
a	-	temporary reference point

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CHAPTER 1

INTRODUCTION

1.1 General Introduction

As fluid flow past an object, a thin layer of fluid near the surface is created. This layer is called the boundary layer, which was presented by Ludwig Prandtl in the early 1900's. Within this layer, the velocity changes from zero at the surface to the free stream value away from the surface. The study on boundary layer flow is significant for many problems in aerodynamics, including wing stall, the skin friction drag on an object, and the heat transfer that occurs in high speed flight. The unsteady case of boundary layer flow has become a very important branch of fluid mechanics research including in an area of convective heat and mass transfer. The presence of time variable in the unsteady problem as an extra independent variable increases the complexity of its solution procedure. Literature reviews on unsteady boundary layers can be found in the review paper by Riley (1975) and in the book by Telionis (1981).

For the fluid flow behavior which cannot be explained in the basis of Newtonian fluids, the theory of micropolar fluids may play its role. The theory of micropolar fluids, first proposed by Eringen (1966, 1972) described fluids which consist of rigid, randomly oriented particles suspended in a viscous medium. Industrial colloidal fluids, polymeric suspensions and liquid crystals are some examples of micropolar fluids. Besides, the presence of dust in the air and blood flow in arteries and capillaries may also be modeled using micropolar fluid dynamics. In addition to the classical velocity vector field, the governing equations for the flow of a micropolar fluid involve a microrotation vector

and a gyration parameter. The researches on unsteady boundary layer flow of micropolar fluid in the vicinity of stagnation points have received much attention. However, solutions for boundary layer flow in micropolar fluid for the entire body have been obtained only for steady cases.

The problems of unsteady two-dimensional boundary layer flow and heat transfer of a viscous and incompressible micropolar fluid past a circular cylinder and a sphere are considered in this study. The unsteadiness is due to the impulsive motion of the free stream velocity and the sudden change in the surface temperature. These unsteady problems involved the occurrence of boundary layer separation and flow reversal. The three dimensional Keller's box scheme which combine the standard differencing and modified differencing are used for the solution of these problems. The standard differencing is used when there is no flow reversal in the previous spatial coordinate. Otherwise, the modified differencing is applied. The present study is able to present the numerical solutions not only in the vicinity of stagnation points but also for the entire surface of cylinder and sphere. In addition, the numerical solutions obtained are also valid from small time solutions to large time solutions.

The problem statements are given in Section 1.2, followed by objectives and scope of this study. In Section 1.4, the research methodology is described. Further, Section 1.5 presents the significance of the study, followed by the outline of the thesis in Section 1.6. Finally, the governing equations are presented in Section 1.7.

1.2 Problem Statements

How do the micropolar fluids models compare with the Newtonian fluid models in the problem of unsteady boundary layer flow and heat transfer past a cylinder and a sphere? What are the effects of micropolar material parameter to the flow characteristics such as skin friction and heat transfer coefficient? How do the velocity, microrotation and temperature profiles affected due to the presence of micropolar fluid? Can

micropolar fluid flow give any advantages to the flow separation compared to Newtonian fluid? How does the case of a sphere differ with the case of a cylinder?

1.3 Objectives and Scope

To calculate the flow characteristics such as in Section 1.2 above by solving the following problems:

1. Unsteady boundary layer flow past an impulsively started circular cylinder in a micropolar fluid.

2. Unsteady forced convection boundary layer flow from a circular cylinder in a micropolar fluid.

3. Unsteady mixed convection boundary layer flow from a circular cylinder in a micropolar fluid.

4. Unsteady mixed convection boundary layer flow past a sphere in a micropolar fluid.

The models are assumed to be incompressible and two-dimensional. Here, the calculations are done for some values of micropolar material parameter K, $K = 0, 1, 2$ and for some values of Prandtl number, $\Pr = 0.7, 1$ and 7 following the work by Cebeci (1978, 1979), Ingham and Merkin (1981), Lok *et al*. (2003a, b) and Nazar *et al*. (2003a, b).

1.4 Research Methodology

The solutions of the four problems stated in the previous section are sought by applying the following research methodology:

1.4.1 Mathematical Formulation

The governing equations for problems outlined in the objectives are first given in dimensional form. The non-dimensionalization of equations is done by introducing appropriate non-dimensional variables. Through this step, the governing equations can be transformed into the non-dimensional form of equations. Next, these non-dimensional forms of equations are simplified using the boundary layer approximation.

The non-dimensional forms of boundary layer equations are then simplified further by introducing the stream function. Using this step, the number of equations is reduced since the continuity equation is automatically satisfied by the definition of stream function. Besides, the number of independent variables is also reduced.

1.4.2 Numerical Computation

A Keller's box scheme in a three dimensional grid as being applied by Cebeci (1978, 1979) is used to solve the partial differential equations. This method consists of discretization using a finite difference method, Newton's method for linearization and block-elimination method. At the forward and rear stagnation points, the discretizations are done on a two dimensional grid as being applied by Lok *et al*. (2003a, b, 2006) and Nazar *et al*. (2002a, b, c, 2003a, b). Other than these points, the discretization should be made either on the net cube or on the zig-zag grid. This algorithm is developed using Matlab 7.0.

1.5 Significance of Study

Devices cooled by mixed convection such as electrical heaters and transformers, solar central receivers exposed to wind currents, electronic devices cooled by fans, nuclear reactors cooled during emergency shutdown, heat exchangers placed in a low-velocity-environment are some examples of applications of transient flows in technologies and industries. Moreover, the effect of buoyancy forces on the heated boundary layer flow can give a significance impact on the unsteady separation process.

The Newtonian fluids principle has its limitation. It is unable to describe the behavior of fluid containing particles as micropolar fluids do. In fact, micropolar fluid is a good model for studying many complicated fluid motion. Hoyt and Fabula (1984) and Vogel and Patterson (1964) conducted experiments with fluids containing minute amounts of polymeric additives. It was found that there is a reduction in skin friction near a rigid body (Hassanien *et al*., 1999). Gray and Hilliard (1966) in his invention, introduce relatively small amounts of a non-Newtonian fluid, a long-chain polymer such as polyethylene oxide, into the water adjacent the bow of the ship. This alters the shear characteristics of the fluid in boundary layer of the ship which decreases the overall frictional drag of the vessel. This leads to the increasing ship speed and it decreases the required power to maintain a given vessel speed. It is such advantages in the fields of aeronautics and submarine navigation.

Besides applications in aeronautics and submarine, the study of micropolar fluid flows with heat transfer has important engineering applications. For instance the applications in power generators, refrigeration coils, transmission lines, electric transformers and heating elements. It serves as the basis of understanding some of the important phenomena occurring in heat exchanger devices (Elbarbary and Elgazery, 2005). Eringen (2001) demonstrated the adequacy of applying micropolar fluid theory to describe liquid crystal behavior. According to him, other possible substances that can be modeled by micropolar fluids are anisotropic fluids, magnetic fluids, clouds with dust, muddy fluids and biological fluids. Moreover, the micropolar fluid theory may have

applications in the understanding flow of colloidal fluids, fluids with additives, suspension solutions, blood flows, fluids with bar like elements etc. In addition, it also could be applied in a number of processes that occur in industry. Such applications include the extrusion of polymer liquids, solidification of liquid crystals, cooling of metallic plate in a bath, ferro liquids, etc. (Nazar *et al.,* 2003b). The study of micropolar fluid flow is necessary due to the increasing importance of the flow behavior which cannot be characterized by Newtonian relationships.

1.6 Thesis Outline

This thesis consists of seven chapters. Chapter 1 begins with the background of the research which outlines the general introduction, problem statements, objectives and scope, research methodology and significance of this study. The governing equations of this research are shown in the next section followed by the literature review. The literature review is divided into four sections specifically for each problem considered in this study.

Details of the method applied in this research are discussed in Chapter 2. The problem of unsteady mixed convection boundary layer flow past a cylinder in a micropolar fluid is taken as an example to show how this method is applied. Chapter 3 discusses the problem of unsteady boundary layer flow past an impulsively started circular cylinder in a micropolar fluid. Both cases of small time and large time along a cylinder surface are considered.

In Chapter 4, the research in Chapter 3 is extended to include the effect of temperature differences between the cylinder surface and its surrounding fluid. An extra equation and boundary condition for temperature should also be taken into account. We further extend this problem to a mixed convection case in Chapter 5. The effect of buoyancy force that appears in the momentum equation is investigated for both opposing and assisting cases.

Chapter 6 discusses the problem of unsteady mixed convection boundary layer flow past a sphere in a micropolar fluid. As for the case of a cylinder, we investigate the effect of buoyancy force for both opposing and assisting cases. All of the cases considered in Chapter 3-6 are solved numerically using the three dimensional Keller's box scheme outlined in Chapter 2. The obtained numerical results which include the velocity, microrotation and temperature profiles as well as the skin friction and the heat transfer coefficient are presented in each of these chapters. Results for the separation point and separation time are also included. Finally, the summary of this research are given in Chapter 7. In this chapter, we also include the suggestions for future research.

1.7 Governing Equations

The governing equations for the unsteady micropolar fluid flow and heat transfer consist of the continuity, momentum, thermal energy and microrotation equations. The continuity equation expresses the principle of mass conservation whereas the momentum equation is derived using the Newton's second law of motion. The thermal energy equation is based on the first law of thermodynamics while the microrotation equation is formulated from the fundamental principle of conservation of angular momentum for polar fluids with the non-symmetric stress tensor and couple stress tensor. In vector form, these equations are expressed as (Guram and Smith, 1980; Lukaszewich, 1999):

$$\overline{\nabla}\cdot\overline{\mathbf{u}} = 0, \tag{1.1}$$

$$\rho\frac{D\overline{\mathbf{u}}}{D\bar{t}} = -\overline{\nabla}\overline{p} + (\mu+\kappa)\overline{\nabla}^2\overline{\mathbf{u}} + \kappa\left(\overline{\nabla}\times\overline{\mathbf{N}}\right) + \mathbf{F}, \tag{1.2}$$

$$\rho C_p\frac{D\overline{T}}{D\bar{t}} = c\,\overline{\nabla}^2\overline{T}, \tag{1.3}$$

$$\rho j\frac{D\overline{\mathbf{N}}}{D\bar{t}} = \gamma\,\overline{\nabla}^2\overline{\mathbf{N}} + \kappa\left(-2\overline{\mathbf{N}} + \overline{\nabla}\times\overline{\mathbf{u}}\right). \tag{1.4}$$

In the above equations, $\overline{\mathbf{u}}$ is the velocity vector, $\overline{\mathbf{N}}$ is the microrotation vector normal to the $\bar{x}, \bar{y}$-plane, $\overline{T}$ is the temperature of the fluid, $\overline{p}$ is the pressure, $\bar{t}$ is the time, ρ is the density of the fluid, μ is the dynamic viscosity, κ is the vortex viscosity, j is the microinertia density, γ is the spin-gradient viscosity, **F** is the body force, C_p is the specific heat at constant pressure and c is the thermal conductivity. The symbol $\overline{\nabla}^2$ is the Laplacian operator where the gradient is defined as $\overline{\nabla} = \frac{\partial}{\partial \bar{x}}\mathbf{i} + \frac{\partial}{\partial \bar{y}}\mathbf{j}$. Here **i** and **j** are the unit vectors, $\bar{x}$ is the coordinate measured along surface of the cylinder (or sphere) and $\bar{y}$ is the coordinate measured in the normal direction to the wall.

Equations (1.1) – (1.4) are applied for the problem of unsteady mixed convection boundary layer flow in a micropolar fluid. For the problem of isothermal unsteady boundary layer flow in a micropolar fluid, Equation (1.3) may be neglected while the buoyancy term **F** in Equation (1.2) is taken as **F** = 0. For the problem of unsteady forced convection flow in a micropolar fluid, Equations (1.1)-(1.4) are considered while the buoyancy term **F** in Equation (1.2) is taken as **F** = 0.

1.8 Literature Review

The literature review for this study is presented in this chapter. The literature reviews in the next four sections are given based to the four problems outlined in the objectives of this study. In Section 1.8.1, we present the literature review for the problem of unsteady boundary layer flow past an impulsively started circular cylinder while in Section 1.8.2, we present the literature review of the forced convection boundary layer flow past a cylinder. The literature review for the problem of unsteady mixed convection boundary layer flow past a cylinder is presented in Section 1.8.3 followed by the literature review on the unsteady mixed convection boundary layer flow past a sphere in Section 1.8.4.

1.8.1 Unsteady Boundary Layer Flow Past an Impulsively Started Circular Cylinder

The classical problem of unsteady boundary layer flow of a viscous and incompressible fluid past a circular cylinder has been considered by many authors, such as, Collins and Dennis (1973a, b), Bar-Lev and Yang (1975) and Cebeci (1979). Cebeci (1979) used the Keller's box in three dimensional grid with zig-zag differencing in dealing with the flow reversal. The generalized differential quadrature (GDQ) and generalized integral quadrature (GIQ) approach have been applied by Shu *et al.* (1996).

In micropolar fluids, the research on steady boundary layer flow past a cylinder has received considerable attention. Nath (1976) considered the steady problem around a cylinder and a sphere in micropolar fluids. The solutions were obtained using an implicit finite difference. He found that in micropolar fluid, the separation occurs at earlier streamwise location as compared to Newtonian fluids. In contrast with the microrotation profiles and microrotation gradient, the skin friction and velocity profiles are almost insensitive to microrotation parameter. Hassanien *et al.* (1996) considered a steady boundary layer flow at an axisymmetric stagnation point on infinite circular cylinder. They developed a numerical procedure based on Chebychev polynomials and found that micropolar fluids display a reduction in drag compared to those for Newtonian fluid. The wall shear and couple stresses increases with the increasing values of the micropolar material parameter, K.

The unsteady boundary layer flow of a micropolar fluid near the forward stagnation point of a plane surface which is impulsively started from rest was studied by Lok *et al.* (2003a). Their numerical results for the transient solution were obtained by implementing the Keller's box scheme in two-dimensional grid. They found that the skin friction coefficient increases as the values of the material parameter K increase. The velocity and microrotation profiles attain the steady flow case as time progresses. In a next paper, Lok *et al.* (2003b) extend the above problem to the case of rear stagnation point. Near the rear stagnation point, the separation occurs at the same value of time t

for any value of K when the case of weak concentration of microelements, $n = 1/2$ is taken into account.

Based on Cebeci (1979) and Lok *et al*. (2003a, b), the first problem of this research is formulated. The unsteady boundary layer flow past an impulsively started circular cylinder in micropolar fluid is studied. Here, the unsteady boundary layer flow of a micropolar fluid is considered not only at the forward and rear stagnation points, but also for other streamwise points along the cylinder surface.

1.8.2 Unsteady Forced Convection Boundary Layer Flow Past a Cylinder

Extending his work on boundary layer flow problem, Cebeci (1978) investigate the heat transfer aspects of an impulsively started circular cylinder. The Nusselt number near the forward stagnation point tends rapidly to its steady-state value as the value of time becomes large. Kim *et al*. (1987) studied the numerical calculations of the velocity and temperature fields for unsteady flows over a single circular cylinder and a bundle of cylinders. They considered periodic disturbances, natural or externally imposed. Periodic and averaged heat transfer was examined. Eswara and Nath (1992) considered the unsteady forced convection flow over a longitudinal cylinder, which is moving in the same or in the opposite direction to the free stream. The governing partial differential equations have been solved using an implicit finite-difference scheme in combination with a quasilinearization technique.

All of the above mentioned researches are done in a Newtonian fluid medium. Besides study of forced convection on Newtonian fluid, research on this area also considered for micropolar fluids. Gorla *et al*. (1983) studied the steady micropolar boundary layer flow and heat transfer over a flat plate. They considered boundary conditions of isothermal wall, constant surface heat flux and insulated wall with viscous dissipation effects. A study of unsteady boundary layer flow, heat and mass transfer of a laminar incompressible micropolar fluid was conducted by Kumari and Nath (1984).

They considered the flow at the stagnation point of a two-dimensional and an axisymmetric body when the free stream velocity and the wall temperature vary arbitrarily with time.

Gorla and Ameri (1985) investigated the steady laminar boundary layer flow and heat transfer of a micropolar fluid in axial flow along a continuous, moving cylinder. Both uniform surface temperature and uniform surface heat flux boundary conditions were considered by these authors. The steady-state boundary layer flow and heat transfer of a micropolar fluid in the vicinity of an axisymmetric stagnation point on a cylinder was presented by Hassanien and Salama (1997). They found that micropolar fluids display drag reduction and the friction factor is lower when compared to Newtonian fluids. Further, the steady micropolar fluid flow and heat transfer in an axisymmetric stagnation point on a horizontal cylinder with suction was presented by Elbarbary and Elgazery (2005). They assumed that the fluid density and the thermal conductivity vary linearly with temperature, while the fluid viscosity is assumed to vary as a reciprocal of a linear function of temperature.

Mahfouz (2007) presented the solutions of unsteady forced convection flow from a circular cylinder in micropolar fluid by considering the full governing equations in order to predict the vortex shedding process. The problem of unsteady boundary layer flow and heat transfer has potential relevance to many practical applications. Hence, the second problem of this research will consider the problem of unsteady forced convection boundary layer flow past a cylinder in micropolar fluid. As in our first problem in the previous section, the unsteady micropolar fluid case for all points along a cylinder is considered but here we include the effect of forced convection.

1.8.3 Unsteady Mixed Convection Boundary Layer Flow past a Cylinder

The problem of steady mixed convection boundary layer flow past a circular cylinder in a Newtonian fluid has been considered by Merkin (1977). He found that

heating the cylinder delays separation and if the cylinder is warm enough, there will be no separation at all. On the other hand, cooling the cylinder brings the separation point nearer to the lower stagnation point and for a sufficiently cold cylinder; there will not be a boundary layer on the cylinder. Ingham and Merkin (1981) extended this problem to the unsteady case. In this study, the horizontal cylinder is placed in a stream flowing vertically upwards. The numerical solutions of the unsteady boundary layer equations were presented for any station along the cylinder.

The steady problem of mixed convection flow past a cylinder in micropolar fluid has been studied by several authors. Mohammedien (1996) presented the steady mixed convection in an axisymmetric stagnation flow of micropolar fluid on a vertical cylinder. The mixed convection boundary layer flow of a micropolar fluid over an isothermal horizontal cylinder has been studied by Nazar *et al*. (2003c). Both cases of a heated and cooled cylinder were considered. The Keller box method was applied to solve the problem for both cases of heated and cooled cylinder. The calculation was made up to the point of separation of the boundary layer.

The unsteady convection boundary layer flow also has been applied successfully to micropolar fluid models. Kumari and Nath (1989) studied the unsteady laminar mixed convection boundary layer flow of a thermomicropolar fluid over a long thin vertical cylinder when the free stream velocity varies arbitrarily with time. The wall temperature is assumed to be time dependent while the temperature in the free stream is considered as a constant. The boundary layer solutions of unsteady combined convection of a micropolar fluid among a vertical plate have been presented by Gorla (1995). It was assumed that the free stream is arbitrary varied with time. Takhar *et al*. (1998) considered the problem of mixed convective unsteady three dimensional flow of a micropolar fluid near the forward stagnation point of a blunt nosed body. Here, the free stream temperature is taken as constant while the dissipation effects near the stagnation point are assumed to be negligible. The unsteady two-dimensional flow of micropolar fluid past a semi-infinite porous plate in a porous medium was studied by Kim (2001). They considered the free stream velocity which follows an exponentially increasing or

decreasing small perturbation law. Wang and Chen (2001) studied the transient force and free convection of micropolar fluid flow over a vertical wavy surface. More recently, Ibrahim and Hamad (2006) investigated the unsteady mixed convection boundary layer on a horizontal cylinder in micropolar fluid. Their focus is on the flow near the stagnation point of a non-isothermal circular cylinder. Kumar *et al*. (2006) considered the problem of mixed convection on a moving vertical cylinder with suction in a moving micropolar fluid medium. In micropolar fluid, the problem of unsteady mixed convection was discussed by Lok *et al.* (2006) who considered the problem in the vicinity of the stagnation point on a vertical surface.

The effect of mixed convection on an unsteady boundary layer flow of micropolar fluid will be considered in the third problem of this research. The flow and heat properties for different time and for all points along the circular cylinder will be discussed. Comparing our problem with Ingham and Merkin (1981), another extra equation should be considered due to the presence of micropolar fluid which is the angular momentum equation.

1.8.4 Unsteady Mixed Convection Boundary Layer Flow Past a Sphere

Numerical solutions for transient flow past a sphere in a viscous fluid has been presented by Dennis and Walker (1972). This unsteady problem is solved using an iterative scheme based on the Crank-Nicolson implicit finite difference approximation. The pressure at the rear stagnation point is found to be initially large and negative. As the value of time increases, this pressure rises to a maximum at separation and then slowly decreases to a steady value. Recently, Al-Ghamdi (2004) investigated numerically the impulsively started fluid flow about a solid sphere subjected to a uniform gas stream. A wide range of Reynolds numbers are chosen in this study. The increase of the Reynolds number leads to the decrease to the boundary layer thickness. Besides research on flow past a sphere, the study on the convective effect on a sphere also has reached much attention among many authors. Juncu (2007) presented a

computational study of the unsteady forced convection heat/mass transfer from spheres in a uniform viscous flow. Three cases were considered: equal spheres with identical physical properties, equal spheres with different physical properties and spheres of different sizes with identical/different physical properties.

The steady laminar free convection on an isothermal sphere in a micropolar fluid has been considered by Nazar *et al*. (2002a). They concluded that the heat transfer coefficient values are lower, while the skin friction parameter values are higher for micropolar fluids than those for Newtonian fluid when the Prandtl number is fixed. Simultaneous to this study, Nazar *et al*. (2002b) considered the case of constant surface heat flux of the steady free convection sphere in micropolar fluids. Recently, Cheng (2008) extended the work of Nazar *et al*. (2002a) to examine the steady natural convection from a sphere in micropolar fluids with constant wall temperature and concentration. They observed that the natural convection heat and mass transfer from a sphere in Newtonian fluids are higher than those in micropolar fluids.

Nazar *et al*. (2002c) studied the steady case of the mixed convection boundary layer flow about a solid sphere in Newtonian fluid. Both the assisting and opposing cases were considered in this study. The extension of this paper to the problem of micropolar fluid can be found in Nazar *et al*. (2003b). The effects of the material and mixed convection parameters on the local skin friction and local heat transfer coefficients were illustrated. One of their findings is that the buoyancy forces retard the fluid which makes the position of the boundary layer separation is brought nearer to the lower stagnation point of the sphere.

The present problem is formulated due to the lack attention given to the problem of unsteady flow past a sphere in micropolar fluid. The unsteady mixed convection boundary layer flow past a sphere will be investigated as the fourth problem of this research.