THE IMPLEMENTATION OF VARIANCE COMPONENT ESTIMATION (VCE) USING MINQE APPROACH INTO AN ADJUSTMENT OF HEIGHT NETWORKS

Zainal Abidin Md Som

Department of Geomatics Engineering, Faculty of Geoinformation Science and Engineering
University of Technology Malaysia, 81310 UTM Skudai , Malaysia
Email : zainal@fksg.utm.my

Abstract

The adjustment of height network involves a simple procedure which can be easily performed by a linear parametric adjustment. The required input data normally consists of levelling line particulars such as height differences and lines distances. The observational variance is normally introduce by the latter which then determines the weight matrix for the adjustment. Such simple variance estimation has to be re-evaluated particularly in cases where the levelling data is gathered from heterogeneous sources such as GPS or from a combination of several levelling instruments. The approach of VCE using Minimum Norm Quadratic Estimation (MINQE) has been implemented in the adjustment of height network in order to examine the formulation of the stochastic model. The example of such implementation is shown using two different levelling networks – a precise levelling and a GPS levelling. The results of this study show that in some cases, the conventional way of determining weight matrix in height network has to be reconsidered.

Key Words: *Variance component estimation, MINQE, least squares adjustment, height network*

1.0 INTRODUCTION

The adjustment of height network involves a simple procedure which can be easily performed by a linear parametric adjustment. The required input data normally consists of levelling line particulars such as height differences and lines distances. By linear parametric adjustment, the solution sought is;

$$\hat{x} = -(A^T PA)^{-1} A^T Pl \qquad (1)$$

where A is the design matrix (m x n), P is the weight matrix (n x n) and l is the (n x 1) vector. The A-matrix consists only the values of 1, -1 and 0 depending on how the levelling points were arranged. As for the vector l, it contains the different between the computed and the observed values of height differences.
It is obvious that there is not much problem in solving the A-matrix as well as l-vector as both of them were generated by the input data of levelling line particulars. What left to be deal is the weight matrix P.

The observational variance is normally introduced by adopting the length of each leveling line S_i, (i.e., $\sigma_i^2 = 1/S_i^2$), which then determines the covariance matrix of the observations as $\Sigma = 1/\sigma_i^2$. Finally, the weight matrix for the adjustment is computed as, $P = \sigma_0^2 \Sigma^{-1}$.

This paper is aimed at introducing the possibility of implementing the method of variance component estimations (VCE) as a means to compute the variances in leveling network. One of the most frequently VCE techniques used in surveying is the method of Minimum Norm Quadratic Estimation (MINQE). The theoretical background of MINQE was introduced by Rao in a series of papers (Rao

1970, 1971 and 1979). Examples of MINQE applications in surveying implementation can be found, among others, in Persson (1980), Chen et. al (1990), Kuang (1993). A more recent work on MINQE and variance component estimation can be found in Satirapod et. al (2002) and Bacigal et. al (2004).

2.0 THE MINQE METHOD OF VARIANCE COMPONENT ESTIMATION

Consider the following functional model;

$$l + v = Ax \tag{2}$$

where l is the vector of observations, x is the vector of unknown parameters, A is the design matrix as respectively described previously and v is the vector of residuals. Next, we consider that the covariance matrix of the observations, C_l is decomposed into;

$$C_l = \sum_{1}^{m} \theta_i T_i \tag{3}$$

where $\theta_1, \theta_2, ..., \theta_m$ are called variance-covariance components which are to be estimated, and $T_1, T_2, ..., T_m$ are the corresponding coefficient matrices.

A quadratic function $l^T Ml$ is said to be a minimum norm quadratic unbiased estimator of the linear function, $\sum_{i=1}^{m} P_i \theta_i$, with unbiasedness and invariance in x, if the quadratic matrix M is determined from the following optimization problem (Rao, 1970);

$$Tr\{MC_l MC_l\} = \text{minimum} \tag{4a}$$

subject to MA = 0, and (4b)

$$MA = 0, Tr\{MT_i\} = p_i, \text{ all } i \tag{4c}$$

where $Tr\{\}$ is the trace operator of a matrix. Since C_l in equation (3) is unknown, the approximate values of $\theta_i (i = 1,2,...,m)$ have to be provided. Let $\theta_i^{(0)}$ be a *priori* value of θ_i. Then the approximate covariance matrix of the observations is calculated as

$$C_l^{(0)} = \sum_{1}^{m} \theta_i^{(0)} T_i$$

Solving equations (4a) to (4c) gives the estimated variance-covariance components as (Rao, 1971);

$$\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2, ..., \hat{\theta}_m)^T = [S(\theta^{(0)})]^{-1} q(\theta^{(0)}) \tag{5}$$

where the (i, j) th element of matrix $S(\theta^{(0)})$ is

$$s_{ij} = Tr\{R(\theta^{(0)}) T_i R(\theta^{(0)}) T_j\}, \tag{6a}$$

and the *i*th component of vector $q(\theta^{(0)})$ is

$$q_i = l^T R(\theta^{(0)}) T_i R(\theta^{(0)}) l, \tag{6b}$$

and

$$R(\theta^{(0)}) = (C_l^{(0)})^{-1} [I - A[A^T (C_l^{(0)})^{-1} A]^{-1} A^T (C_l^{(0)})^{-1}], \tag{6c}$$

where I is identity matrix, $\theta^{(0)} = (\theta_1^{(0)}, \theta_2^{(0)}, ..., \theta_m^{(0)})^T$, and $R(\theta^{(0)})$ stands for matrix R which is evaluated with $\theta^{(0)}$. The same symbol definition as $R(\theta^{(0)})$ is used for the others. If the survey network under study is a free network, matrix $[A^T (C_l^{(0)})^{-1} A]$ is singular and a generalized inverse must be used.

It is clear from equation (5) that if some a priori values of the variance-covariance components (i.e., values $\theta_i^{(0)} (i = 1,2,...,m)$ are adopted to initialize the solution procedure, an iterative process, referred to as the iterative MINQE, should be followed (Rao, 1979). Let the estimators from equation (4), denoted by $\theta^{(k)} (k = 1)$, be chosen as the approximate values of θ. Then the MINQUE is recomputed, and the estimators from the next iteration are:

$$\hat{\theta}^{(k+1)} = [S(\hat{\theta}^{(k)})]^{-1} q(\hat{\theta}^{(k)}) . \qquad (7)$$

The iterative process stops when the solution (as in equation 7) ceases to change.

The covariance matrix of the estimated variance-covariance components is computed by (Chen et. al, 1990) as;

$$C_{\hat{\theta}} = 2[S(\hat{\theta})]^{-1} \qquad (8)$$

As a guide, for the case of the parametric adjustment model, the procedures for the computations of the variance-covariance components are summarized as shown in Figure 1 follows, where δ is a convergence tolerance.

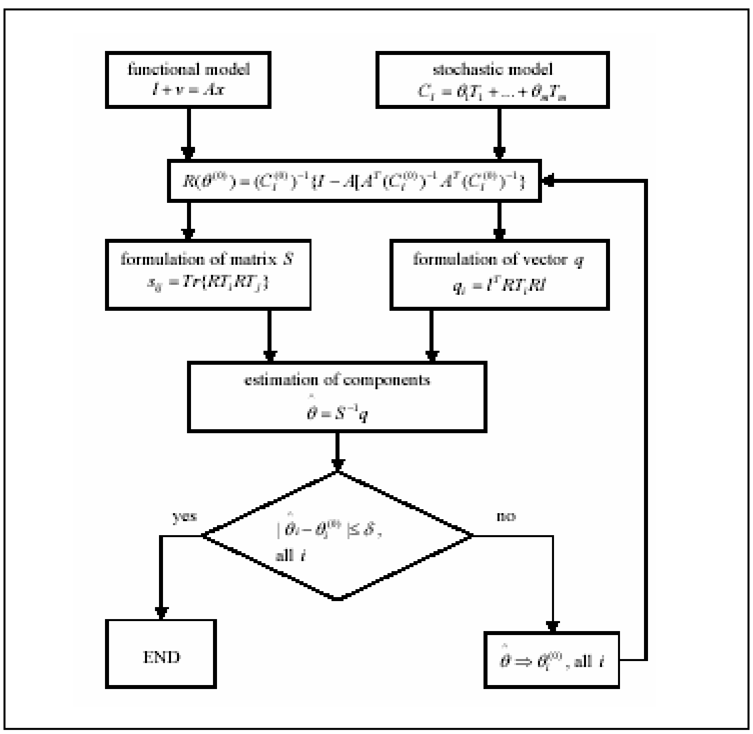

Figure 1: The computation procedure of variance-covariance components using MINQUE

3.0 EXAMPLES OF IMPLEMENTING MINQE IN LEVELLING ADJUSTMENT

In this study, the method of MINQE was implemented in assessing the variance of levelling lines for two different types of levelling network – a GPS levelling and a precise levelling networks. These two types of levelling network were chosen as examples because they constitutes as the most practice in engineering and mapping control purposes.

3.1 GPS HEIGHT NETWORK

For the GPS levelling, the data set was taken from a network in Maracaibo area, Venezuela (Figure 2). The network was established in order to monitor the subsidence occurrence in that area due to oil extraction. The Maracaibo network is chosen for two reasons – its readily available data courtesy of UNB Research Group (Chrzanowski, 1995) and the availability of its MINQE results of VCE (Chen et. al, 1991). The importance of the latter is to enable them to serve as a calibration in implementing the MINQE routine developed for this study.

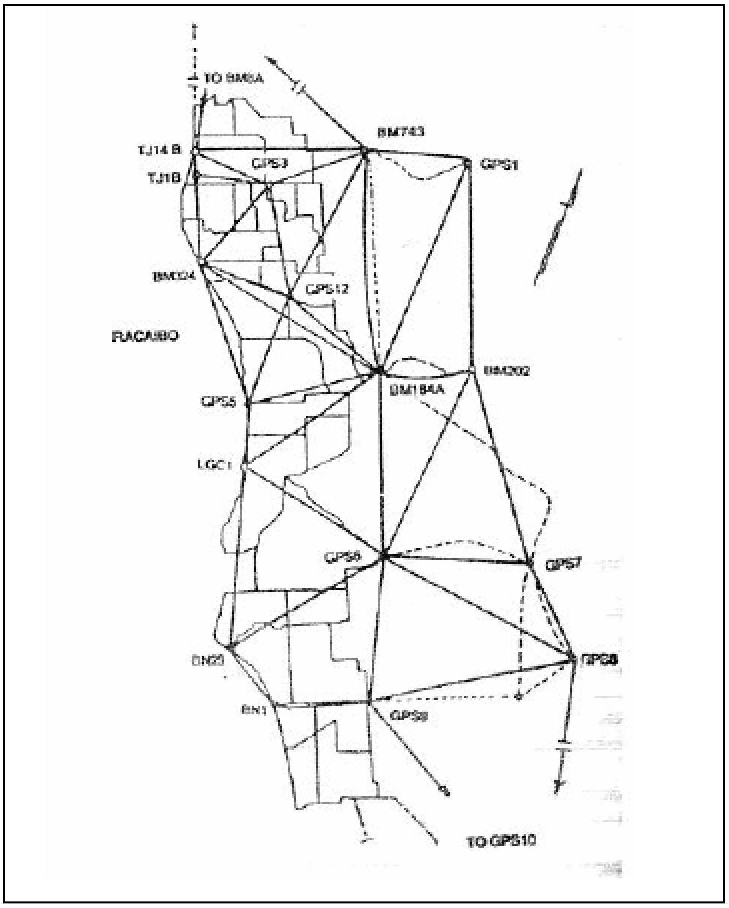

Figure 2: Maracaibo GPS Leveling Network, Venezuela (Chen et. al, 1991)

The accuracy of GPS-derived baseline components are generally expressed by

$$\sigma^2 = a^2 + b^2S^2 \qquad (9)$$

where a and b are the variance components to be estimated and s is the baseline length. The value of a and b depend on the type of receiver, observation environment, satellite geometry, observation time, processing data technique, and so on. They should be estimated from the observation rather taken directly from the values claimed by manufacturer as normally practice.

Chen et. al (1990) also pointed out that in some cases the error model (eq. 9) is not always suited. One reason could be due to uncertainties in tropospheric modelling as experienced in the Maracaibo Network. Therefore, two alternative error models, $\sigma^2 = a^2$ and $\sigma^2 = b^2S^2$ has been suggested by Chen et. al (1990). In this study, the technique of MINQE was employed to determine the estimated values for a and b which results are tabulated in Table 1.

The results indicate that model $\sigma^2 = a^2$ should be chosen to compute the variance for the Maracaibo Network data used in this study. The error component a was estimated as 29mm which is very similar with the results as reported in Chen et. al (1990).

Table 1: Results of VCE for Maracaibo GPS Network using MINQE

	Component a	Component b	Outcome
Model 1	35 mm (± 28 mm)	1.4 ppm (± 2.2 ppm)	Model Rejected
Model 2	29 mm (± 16 mm)	-	Model Accepted
Model 3	-	3.0 ppm (± 1.7 ppm)	Model Rejected

3.2 PRECISE LEVELLING NETWORK

As for precise levelling network, the data used is taken from the Precise Levelling Network covering the Johor Region. The levelling network was surveyed by Jabatan Ukur & Pemetaan Malaysia (Jupem). This network is chosen because it was previously adjusted using an available established software (Hasbullah Meri, 1998). That mean the adjustment results obtained in this study can be compared particularly for the case of traditional way of weighting scheme.

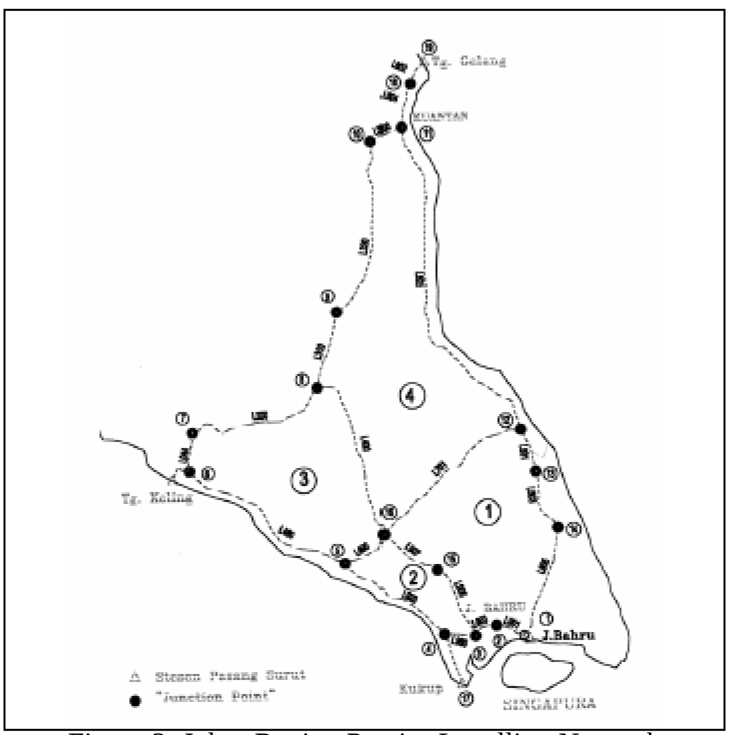

Figure 3: Johor Region Precise Levelling Network

In this study, the adjustment of the Johor region precise levelling was initially implemented by adopting the conventional way of weighting scheme (i.e., $\sigma_i^2 = 1/S_i^2$), where S is the length of leveling line. The adjustment was done using a self-developed software written in Fortran. The estimated height obtained for all the benchmarks were very similar with the results reported by Hasbullah Meri (1998).

Further, the work was concentrated in examining a number of alternative error models to be used in determining the variances for the leveling data. A total of four alternative error models as suggested by Chen & Chrzanowski (1985) as follows were tested;

i) $\sigma^2 = a\ell + b\ell^2 + c|\Delta h| + d\Delta h^2$

ii) $\sigma^2 = a\ell + b\ell^2 + d\Delta h^2$

iii) $\sigma^2 = a\ell + b\ell^2$

iv) $\sigma^2 = a\ell + d\Delta h^2$

The MINQE method was employed to determine the variance components of (i.e., estimates value of a, b c and d) and the results obtained is as tabulated in Table 2.

	a (mm)	b (mm)	c (mm)	d (mm)	Outcome
Model 1	0.00003 (Passed)	0.0000… (Passed)	291.33 (Passed)	493.61 (Passed)	Model Accepted
Model 2	0.000002 (Passed)	0.0000… (Failed)	-	10.2 (Passed)	Model Rejected
Model 3	0.000003 (Passed)	0.0000… (Passed)	-	-	Model Accepted
Model 4	0.000001 (Passed)	-	-	72.6 (Passed)	Model Accepted

Table 2: Results of VCE for Johor Precise Levelling data using MINQE

4.0 RESULTS AND ANALYSIS

The above results reveal that three models (i.e., Model 1, 3 and 4) have passed the test and they should be considered as the correct error model for the purpose of computing the variance for the data given. Model 2 seems not suitable hence rejected because one of the variance components (i.e., parameter b) failed the local test.

Subsequently, a series of least squares adjustment were run through on the Johor levelling data. The purpose of the adjustments is to examine the impact of the weight matrix created by three different error models (i.e., Model 1, 3 and 4) respectively. The main criteria used in the comparison is the outcome of the global test on the aposteriori variance factor. The outcome and comparison of the three separate adjustment is shown in Table 3.

Table 3: Comparison of LSE adjustment results

	Global test	Outcome
Model 1	0.111877 < 1.0 < 2.575711	Passed - Model Accepted
Model 3	0.000205 < 1.0 < 0.004726 (?)	Failed - Model Rejected
Model 4	0.000205 < 1.0 < 0.004726 (?)	Failed - Model Rejected

The results shown in Table 3 clearly indicates that Model 1 (i.e., $\sigma^2 = a\ell + b\ell^2 + c|\Delta h| + d\Delta h^2$) is the most suitable model which should be adopted in variance computation for the Johor precise levelling data. The other two models, Model 3 and 4, eventually failed the chi-squares test hence their candidature as error model should be discarded.

5.0 CONCLUSIONS

The implementation of variance component estimation using MINQE approach in the adjustment of height networks has been successfully presented. The works reveal the suitability of MINQE in assessing the right choice of error model to be used in computing the variance for leveling works.

The results obtained pointing out that in some cases, the conventional error model in GPS height has to be reconsidered. In the case of precise levelling, the weight matrix no longer depends solely on the distance of the leveling line. They might be cases where the height difference has to be considered in the weight matrix computation.

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