GENERALIZED FUZZY TOPOGRAPHIC TOPOLOGICAL MAPPING

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Abstract

Fuzzy Topographic Topological Mapping (FTTM) is a model for solving neuromagnetic inverse problem. FTTM consists of four topological spaces which are connected by three homeomorphisms. FTTM 1 and FTTM 2 were designed to present 3-D view of an unbounded single current and bounded multicurrent sources, respectively. It has been showed that FTTM 1 and FTTM 2 are homeomorphic and this homeomorphism will generate another 14 FTTM. There is a conjecture stated that if there exist *n* numbers of FTTM, then they will generate another $n^4 - n$ new FTTM. In this thesis, the conjecture is proven by using geometrical features of FTTM. In the process, several definitions such as sequence of FTTM, sequence of polygon, sequence of cube with combination of two, three and four FTTM are developed. Some geometrical and algebraic properties of sequences of FTTM are identified and proven. A new conjecture is also proposed in this thesis which states that the number of generating Fk_n if there are *k* components and *n* models of F*k* is $n^k - n$. Surprisingly, the nonzero sequence of cube with combination of two, three and four FTTM appeared in Pascal's Triangle.

Abstrak

Pemetaan Topologi Topografi Kabur (FTTM) adalah satu model untuk menyelesaikan masalah songsangan neuromagnetik. FTTM terdiri daripada empat ruang topologi yang mana dihubungkan dengan tiga homeomorfisma. FTTM 1 dan FTTM 2 direka bentuk bagi menunjukkan pandangan tiga dimensi sumber arus tunggal tidak terbatas dan sumber arus berbilang terbatas setiap satunya. Ia telah ditunjukkan bahawa FTTM 1 dan FTTM 2 adalah homeomorfik dan homeomorfisma ini akan membentuk 14 FTTM yang lain. Terdapat satu konjektur yang menyatakan bahawa jika wujud sebanyak *n* FTTM, maka bilangan FTTM yang baru yang dijanakan adalah $n^4 - n$. Dalam tesis ini, konjektur itu telah dibuktikan dengan menggunakan ciri-ciri geometri FTTM. Dalam proses itu, beberapa takrifan seperti jujukan FTTM, jujukan poligon dan jujukan kubus dengan gabungan dua, tiga dan empat FTTM telah dibangunkan. Ciri-ciri geometri dan aljabar jujukan FTTM telah dikenalpasti serta dibuktikan. Satu konjektur baru juga telah dicadangkan di dalam tesis ini yang menyatakan bahawa bilangan Fk_n yang akan dijana jika terdapat sebanyak *k* komponen dan *n* model F*k* ialah $n^k - n$. Menariknya, jujukan kubus yang tidak kosong dengan gabungan dua, tiga dan empat FTTM wujud dalam Segitiga Pascal.

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CHAPTER 1

INTRODUCTION

1.1 Background of Research

Fuzzy Topographic Topological Mapping (FTTM) was developed by Fuzzy Research Group (FRG) at UTM in 1999. FTTM was built to solve neuromagnetic inverse problem i.e. to determine the location of epileptic foci in epilepsy disorder patient (Tahir *et al.*, 2000). The model which consists of topological and fuzzy structures is composed into three mathematical algorithms (Fauziah, Z., 2002). FTTM have four components which are magnetic contour plane (*MC*), base magnetic plane (*BM*), fuzzy magnetic field (*FM*) and topographic magnetic field (*TM*) as shown in Figure 1.1. FTTM Version 1 was developed to present a 3-D view of an unbounded single current source (Fauziah, Z., 2002 & Liau L. Y., 2001) while FTTM Version 2 was developed to present 3-D view of a bounded multi current source (Wan Eny Zarina *et al.*, 2002).

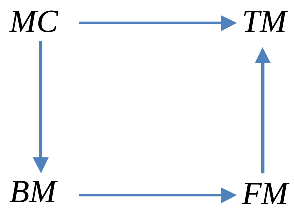

Figure 1.1 Model of FTTM

1.2 Statement of Problem

FTTM Version 1 consists of three algorithms, which link between four components. The four components are MC_1, BM_1, FM_1 and TM_1. Besides that, FTTM Version 2 also consists of three algorithms, which link between four components. The four components are MC_2, BM_2, FM_2 and TM_2. FTTM Version 1 as well as FTTM Version 2 is specially designed to have equivalent topological structures between its components (Liau L. Y., 2006). In other words, there are homeomorphisms between each component of FTTM Version 1 and FTTM Version 2 (see Figure 1.2).

Using the fact that FTTM 1 and FTTM 2 are homeomorphic componentwise, there are at least another 14 elements of FTTM that can be identified easily.

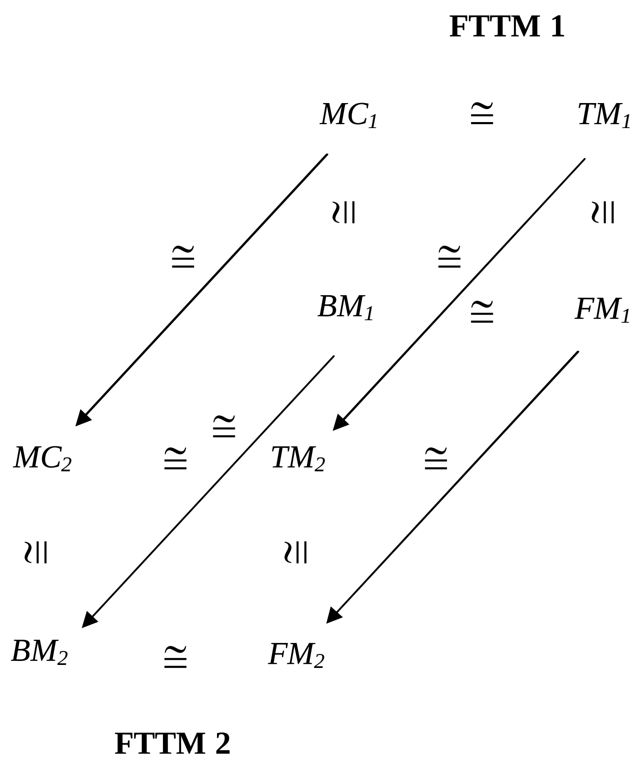

Figure 1.2 Homeomorphisms between FTTM 1 and FTTM 2

Li Yun (2006) first noticed that if there were two elements of FTTM that are homeomorphic to each other componentwise, it would generate more homeomorphisms. The numbers of generating new elements of FTTM are

$$\left[\binom{2}{1}\times\binom{2}{1}\times\binom{2}{1}\times\binom{2}{1}\right]-2=14 \text{ elements.}$$

Consequently, Liau Li Yun (2006) has proposed a conjecture; if there exist n elements of FTTM, then the numbers of new elements are $n^4 - n$.

1.3 Objectives of Research

The aims of this research are as follows;

a) to study geometrical features of FTTM.

b) to prove the conjecture (the number of generating FTTM).

c) to find new features on generating FTTM.

1.4 Scope of Research

This research will focus on the goal to prove the conjecture, namely the number of generating FTTM. In order to achieve this goal, the basic concepts of FTTM and how Li Yun produced this conjecture will be studied. Basic concepts of FTTM will be discussed.

The process of proving the conjecture includes understanding the geometrical features of FTTM. Thus, number theory and discrete mathematics are the two areas of mathematics that can provide some of the tools to accomplish the task. Methods of proving, sequences including Fibonacci number and Pascal’s Triangle will also be covered in this work.

1.5 Importance of Research

As mentioned earlier in this chapter, FTTM is a technique to determine the location of epileptic foci in epilepsy disorder patient. By proving the conjecture, many FTTM can be generated. In other words, other versions of FTTM besides FTTM Version 1 and FTTM Version 2 can be developed in solving the neuromagnetic inverse problem to determine the location of epileptic foci in epilepsy disorder patient by proving the conjecture.

1.6 Framework of Research

This thesis consists of six chapters. The first chapter discusses the background of the research, statement of problem, objectives of research, scope of research, importance of the research and framework of research.

Chapter 2 consists the literature review of this work. It presents the concept of FTTM and its generalization. This chapter also discusses extension of Fibonacci numbers and Pascal’s Triangle.

Chapter 3 consists the methodology of this work. Some methods of proving will be presented and the reason why these methods failed will be explained. Finally, the suitable method to prove the conjecture will be revealed.

Chapter 4 exposes the geometrical features of FTTM. Some definitions will be presented. This chapter consists the actual proof of the conjecture.

Chapter 5 consists the extension results. It reveals the relation between generating FTTM and Pascal's Triangle. This chapter also contains new theorems, corollaries and conjecture.

Chapter 6 consists the summary and some recommendations for future works.

The framework of this research can be summarized in Figure 1.3.

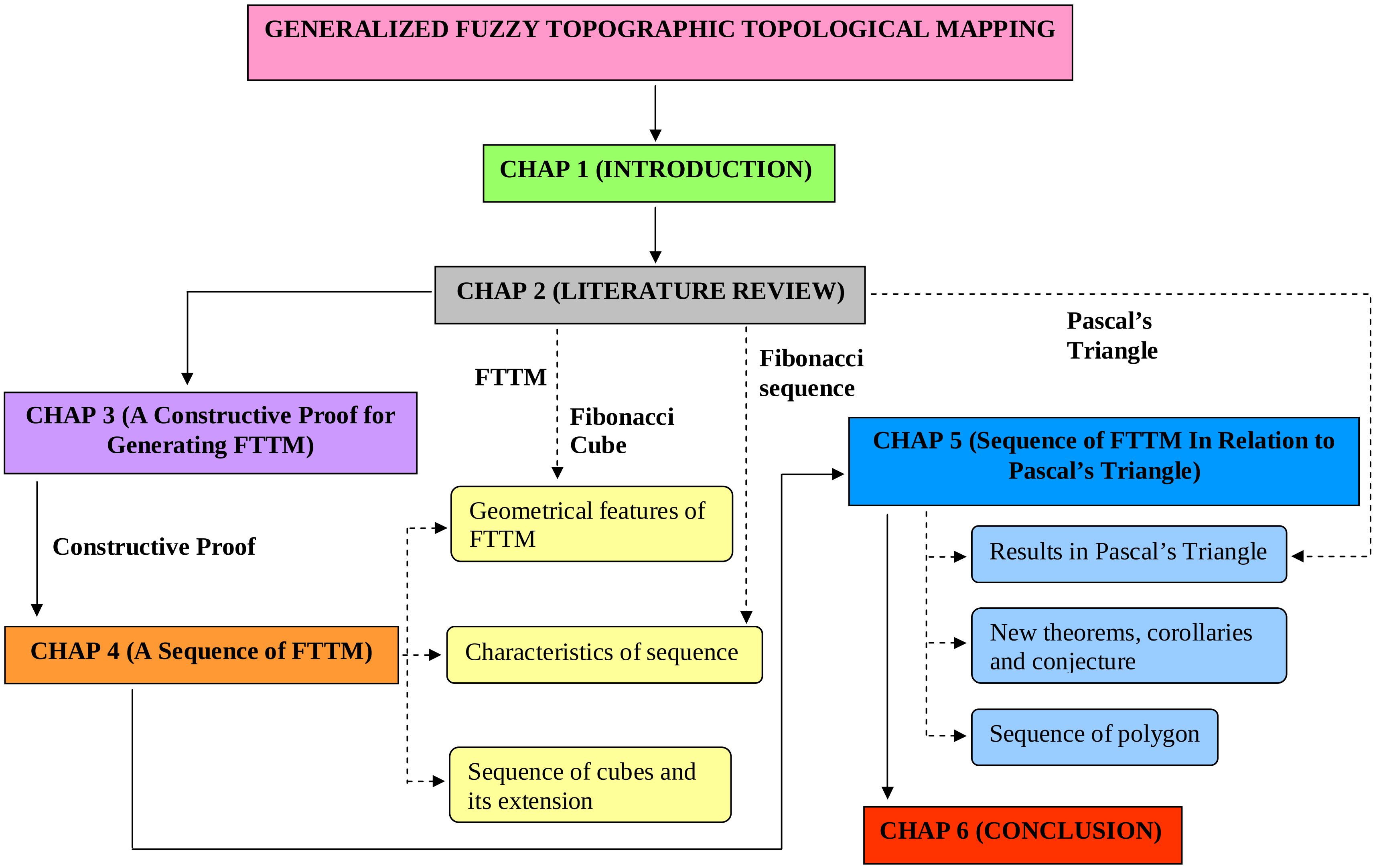

Figure 1.3 Framework of research

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