Application Of Control Theory In Inventory And Order Based Production Control System (IOBPCS)

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ABSTRACT

The use of efficient production and inventory control systems is of great importance for industry. This paper gives a review of the research by Towill [1] concerning control theory application in an inventory and order based production control systems (IOBPCS). This paper studies the ability of such Inventory System to recover from shock demands and to protect the manufacturing process from random sales variations. Some simulation results are plotted. One good compromise design achieved has the time-to-adjust inventory, demand averaging time and production delay time all of comparable magnitude.

INTRODUCTION

It is generally recognized that an efficient production control system can only be designed and operated if the dynamic behaviour of the constituents parts is properly understood. System dynamics means that the dynamic properties of a system are analysed. The purpose of a production -inventory control systems is to transform usually incomplete information about market and available production resources into coordinated plans for production and replenishment of raw materials. There are usually both deterministic and stochastic variations in demand and available capacity [4], [5]. Furthermore production and delivery times are often quite long. An interesting question is whether methods and concepts from control theory can be successfully applied in connection with production and inventory control. This is aimed at increasing production by designing scheduling and control systems using control theory simulation to reduce fluctuations in both inventory and production rate.

There are some basic concepts that obviously have an impact on related areas like production and inventory control. One such concept which is important is feedback control ie. a control policy which is a function of the present state [2]. Such a policy has the advantage that it is in a sense adaptive with respect to modelling errors caused. A block diagrams usually constitutes a very efficient way of illustrating how a control system works.

SIMULATION MODELLING

The production inventory control system specified for the study in this paper is the IOBPCS shown in Figure 2.1. Table 1, shows the IOBPCS simulation model for a step change in sale rate, and random sale demand. Towill [2] has found the exponential delay particularly suitable for industrial dynamics simulation, and so it is the discrete version which will be used in the model. Discrete systems operate on numbers which are sampled at regular interval, say ΔT apart. The exponential delay may be readily approximated by drawing the analogy with exponential smoothing. Appendix A explains this relationship in detail. For example AVCON equation may be written as shown in Table 1. The relationship between αAVCON and TAC is shown in Table 2. Table 3 shows the terms use in the typified Industrial Dynamic simulation [3].

The production control law, which determines ORATE, the order rate placed on the factory is composed of two parts. One is due to the present inventory deficit (EINV), and the other due to AVCON. The AVCON and EINV is the effect from feedforward and feedback control law respectively. TAC and TAI are imposed to control ORATE effectively. There is a production delay (TPROD) between the planning of orders, and their completion period (COMRATE). The actual or present inventory level

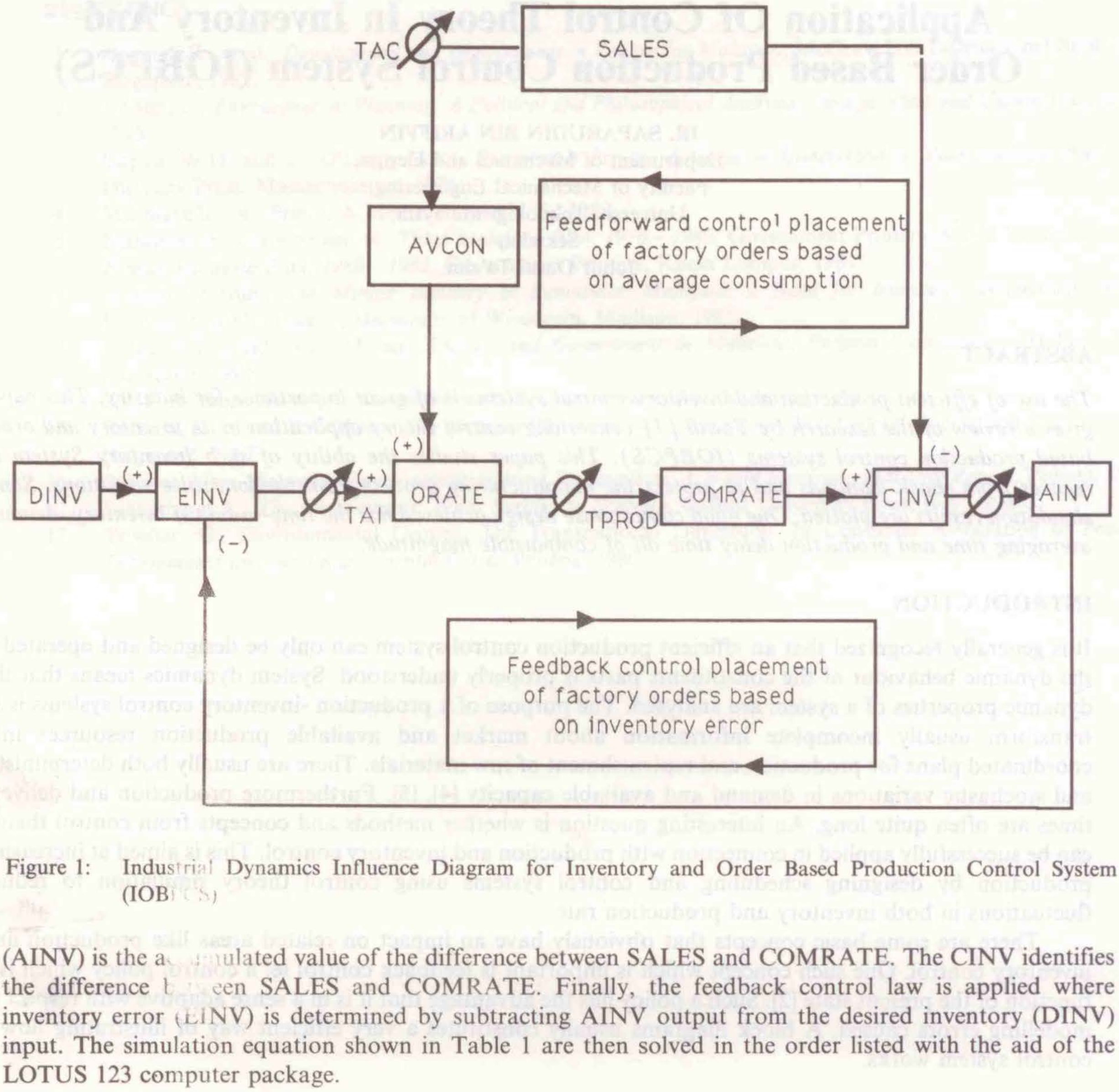

Figure 1: Industrial Dynamics Influence Diagram for Inventory and Order Based Production Control System (IOBPCS)

(AINV) is the accumulated value of the difference between SALES and COMRATE. The CINV identifies the difference between SALES and COMRATE. Finally, the feedback control law is applied where inventory error (EINV) is determined by subtracting AINV output from the desired inventory (DINV) input. The simulation equation shown in Table 1 are then solved in the order listed with the aid of the LOTUS 123 computer package.

Table 1: The IOBPCS Simulation Calculations

Average Sales Rate Estimation Equation
$AVCON\,(k+1) = AVCON\,(k) + \alpha\, AVCON\,[SALES\,(k) - AVCON(k)]$

Factory Orders Rate Equation
$ORATE\,(kt1) = AVCON(kt1) + (EINV\,(k\,)/TAI]$

Factory Output Rate Equation
$COMRATE\,(k+1) = COMRATE(k) + \alpha\, PROD\,[ORATE(k+1) + COMRATE(k)]$

Inv [illegible] Equation
EI[illegible] $DINV - AINV\,(k+1)$

Table 2: The Exponential Smoothing Equation (see Appendix A for detail)

$$\alpha\,\mathrm{AVCON} = \left[\frac{1}{1+\dfrac{\mathrm{TAC}}{\Delta \mathrm{T}}}\right]; \quad \alpha\,\mathrm{PROD} = \left[\frac{1}{1+\dfrac{\mathrm{TPROD}}{\Delta \mathrm{T}}}\right]$$

Table 3: Terms used in IOBPGS Simulation

DINV	=	Desired inventory level
AINV	=	Actual (Present) inventory level
EINV	=	Inventory error
SALES	=	Instantaneous Sales Rate
CINV	=	Change in inventory level
ORATE	=	Order placed rate
GOMRATE	=	Completion production Rate
AVGON	=	Average sales (deliveries) rate
TAG	=	Averaged time to find AVGON
αAVGON	=	Multiplier used in simulation with the effect of TAG
TPROD	=	Production delay time
αPROD	=	Multiplier used in simulation with the effect of TPROD
TAI	=	Time to reduce inventory deficit to zero

SIMULATION RESULTS

A Step Increase In Sales

The dynamic response parameters to be tuned by a system designer are TAG and TAI. Assume that the average sales rate is 100 widgets/week and the factory has target inventory level of 600 widgets. Some sample results for the behaviour of EINV to sudden change in SALES +10 widgets/week are plotted in Figure 2, 3 and 4. The production delay (TPROD) has been assumed to be 4 weeks. TAC has been varied from 0 to 16 weeks, with TAI fixed at 4 weeks **(Figure 2)**. Then TAG has been fixed at 4 weeks and TAC varied from 1 to 16 weeks. **(Figure 3)** Finally, TPROD has been varied from 2 to 8 weeks with TAI and TAC fixed at 4 and 8 weeks respectively.

Inspection of Figure 2, 3, 4 shows that the setting of TAI = 4 weeks and TAG = 8 weeks with TPROD fixed at 4 weeks appears to be a good design, since unnecessary fluctuation of EINV has been avoided; At the same time the actual inventory level (AINV) would not drop too low and will not be excessively long as shown in Figure 5. The time to recover to the desired Inventory (DINV) is also not excessively long as shown in Figure 6.

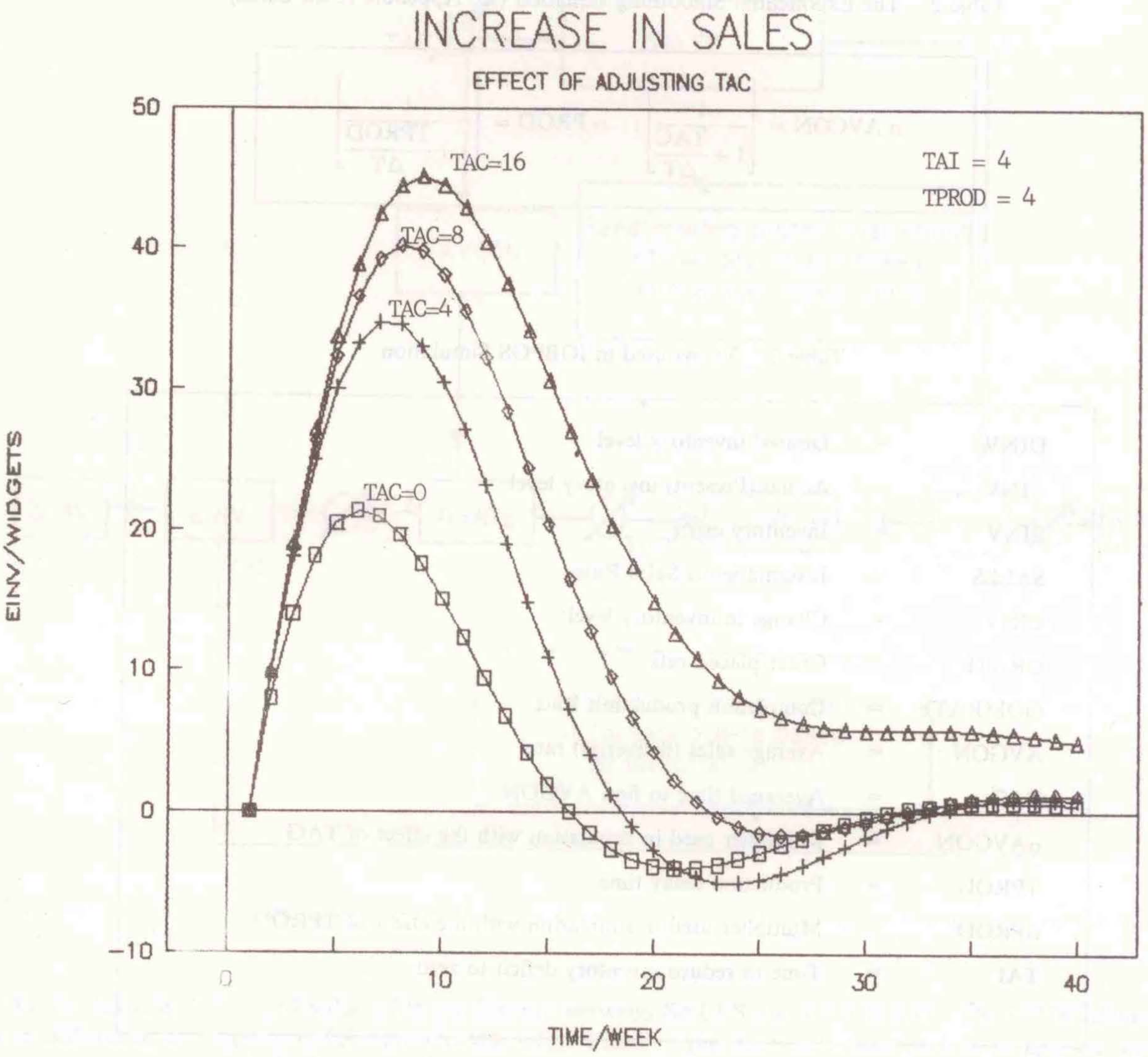

Figure 2: Dynamic Response of IOBPCS to a step increase in Sale (Effect of varying TAC)

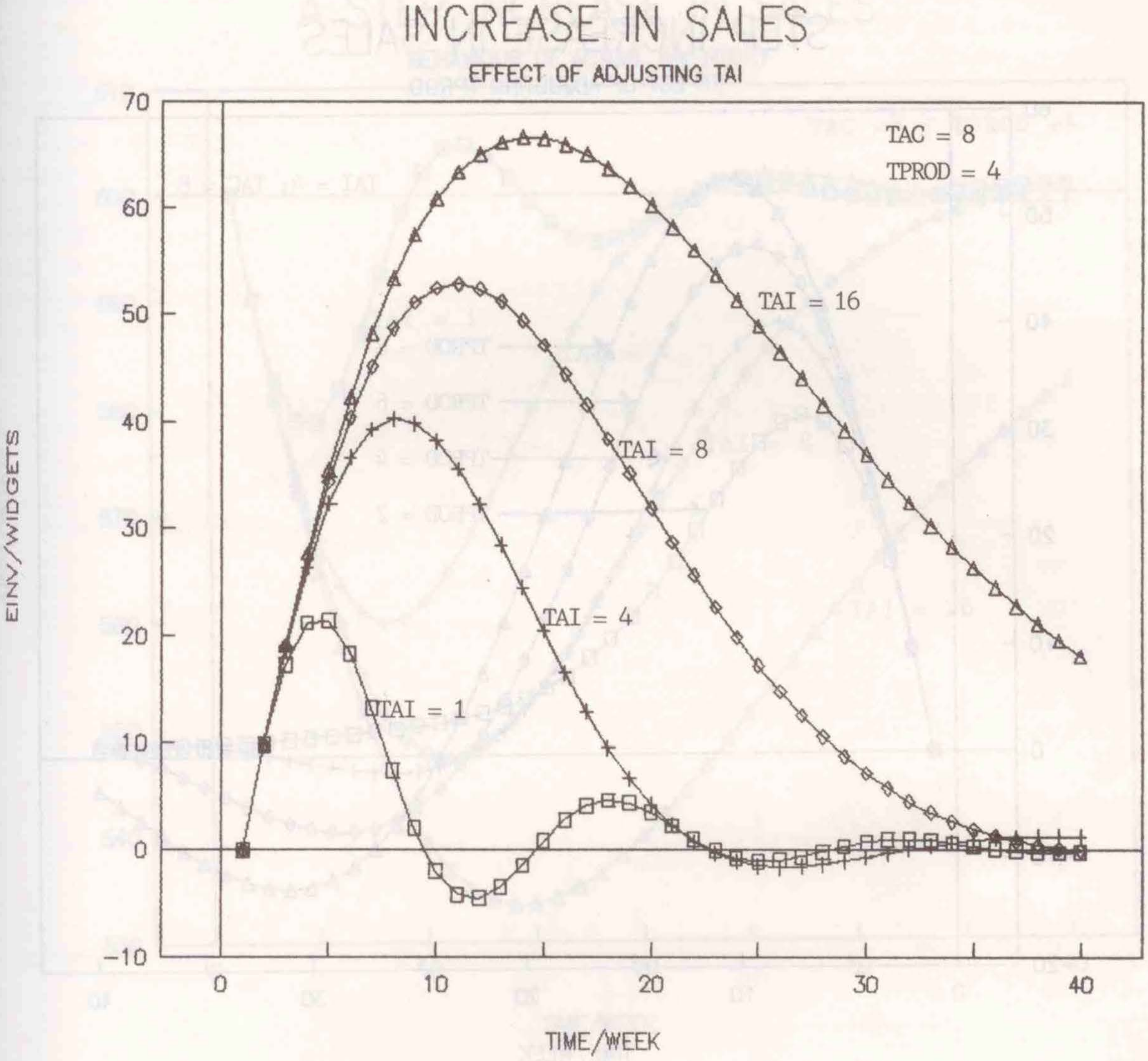

Figure 3: Dynamic Response of IOBPCS to a step increase in sale (Effect of verying TAI)

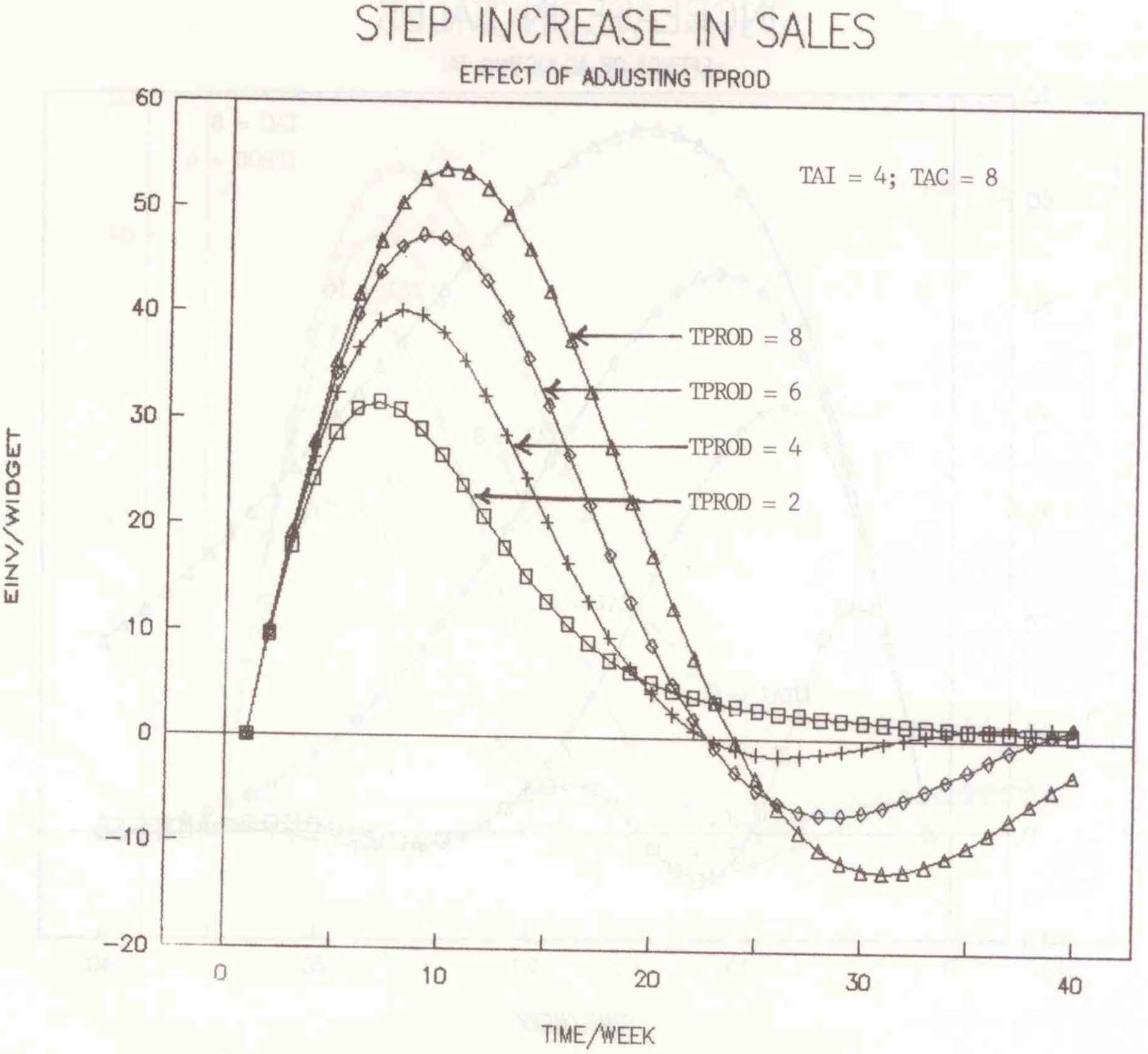

Figure 4: Dynamic Response of IOBPCS to step increase in Sale (effect of adjusting Production delay)

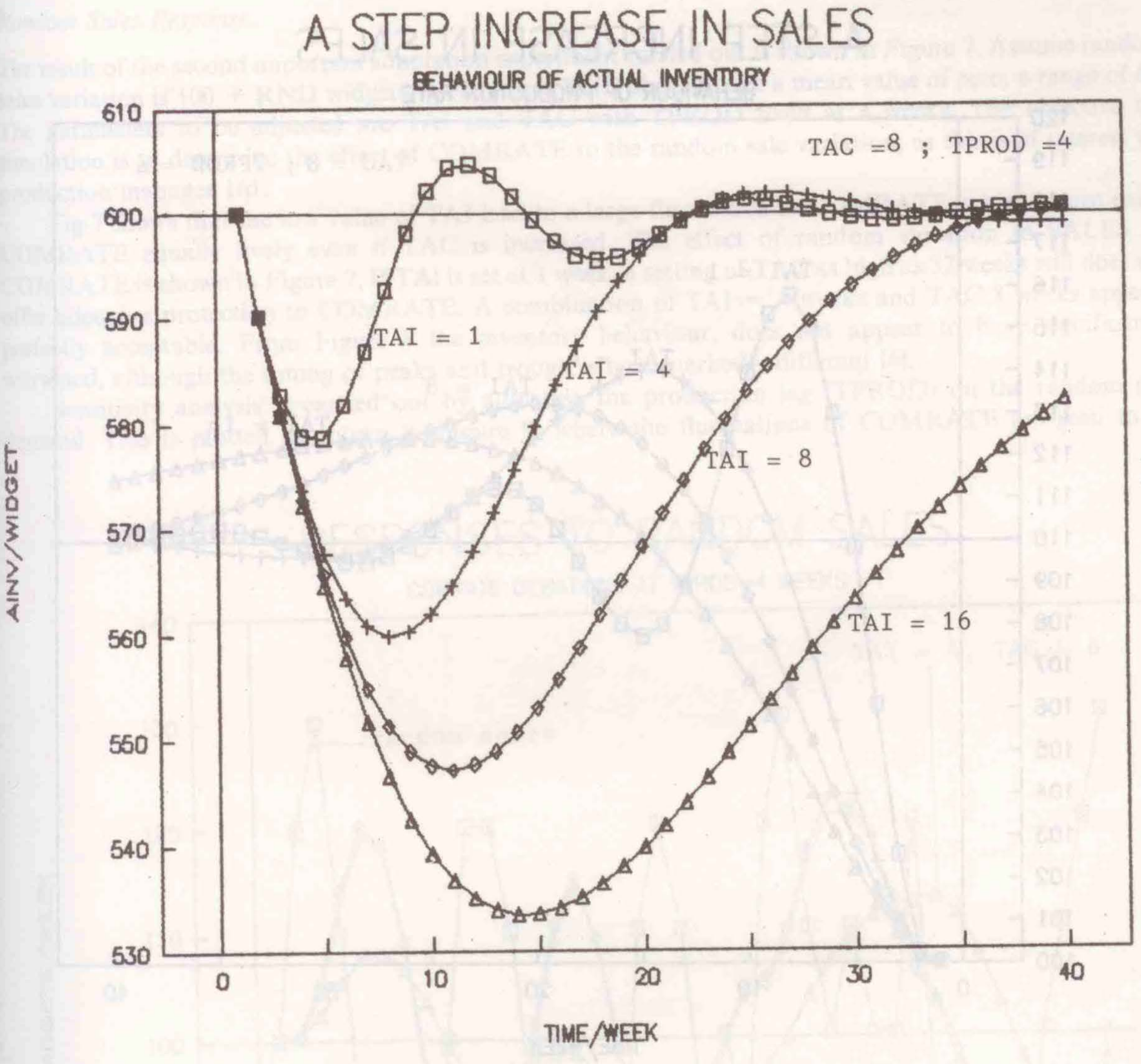

Figure 5: Dynamic responses of actual inventory (AINV) to the desired inventory (DINV) of 600 widgets.

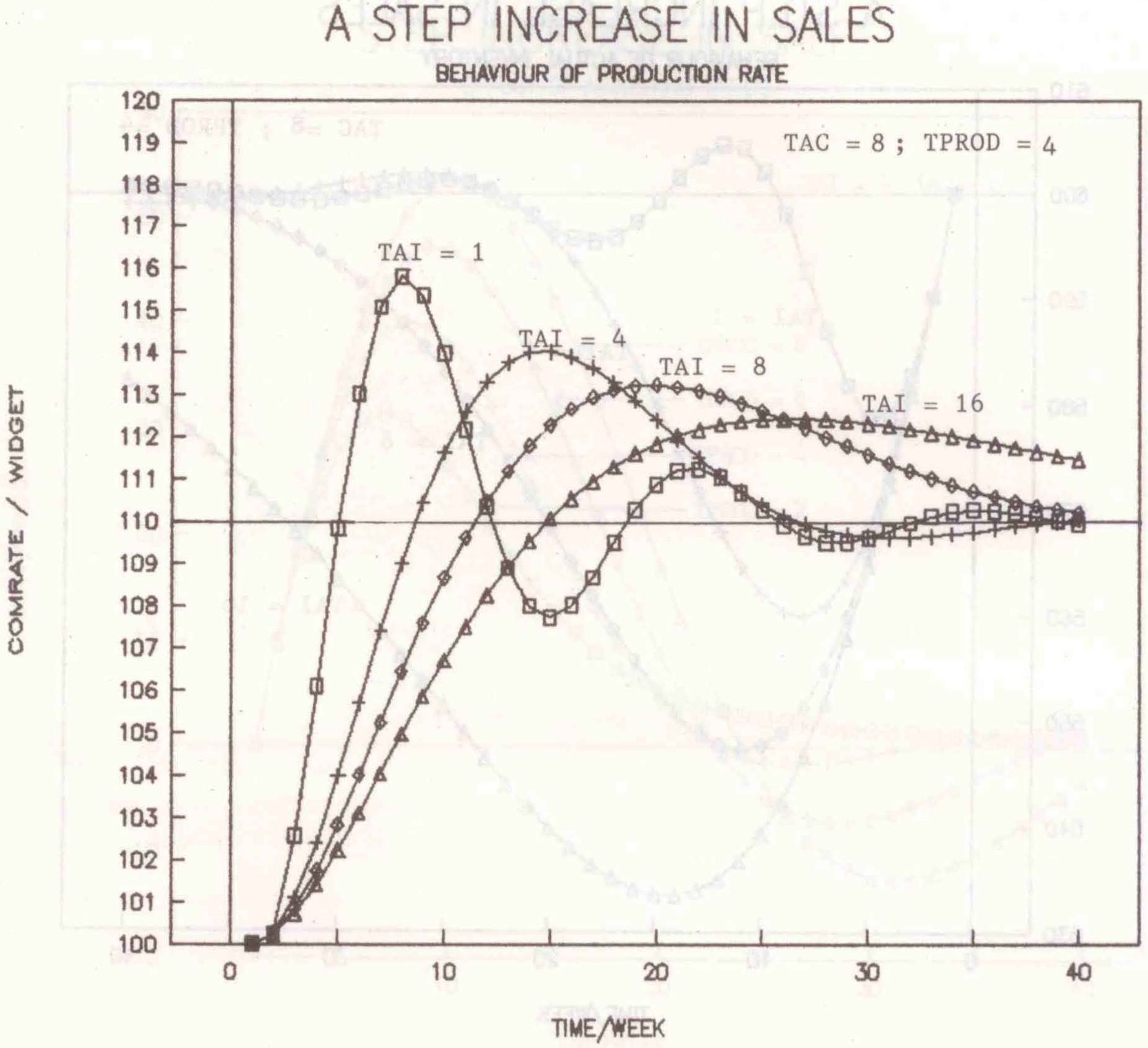

Figure 6: Dynamic responses to a sudden SALES of + 10 widgets/week (effect of varying TAI)

Random Sales Response

The result of the second important simulation experiment carried out is shown in Figure 7. Assume random sales variation is 100 + RND widgets/week, where RND should have a mean value of zero, a range of 60. The parameters to be adjusted are TAI and TAC with TPROD fixed at 4 weeks. The objective the simulation is to determine the effect of COMRATE to the random sale variation, as this is of interest the production manager 161.

Fig. 7 shows that the low value of TAI lead to a large fluctuation in COMRATE, which inturn cause COMRATE equally lively even if TAC is increased. The effect of random variation in SALES on COMRATE is shown in Figure 7. If TAI is set at 1 week, a setting of TAC as high as 32 weeks still does not offer adequate protection to COMRATE. A combination of TAI = 4 weeks and TAC 8 weeks appears perfectly acceptable. From Figure 8 the inventory behaviour, does not appear to have significantly worsened, although the timing of peaks and trough is now markedly different [6].

Sensitivity analysis is carried out by adjusting the production lag (TPROD) on the random sale demand. This is plotted as shown in Figure 9, where the fluctuations of COMRATE are seen to be

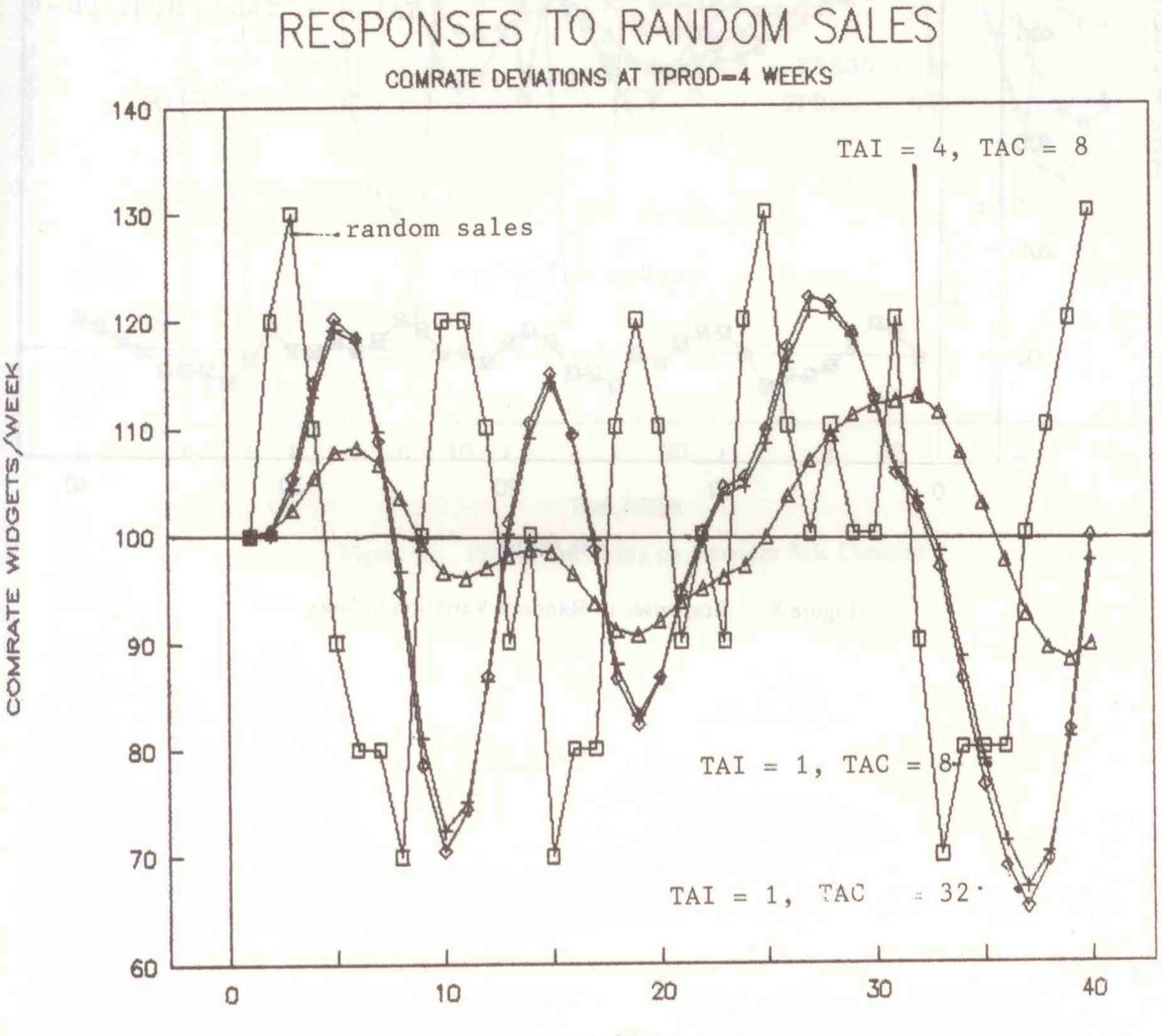

Figure 7: Responses to random variation in Sales

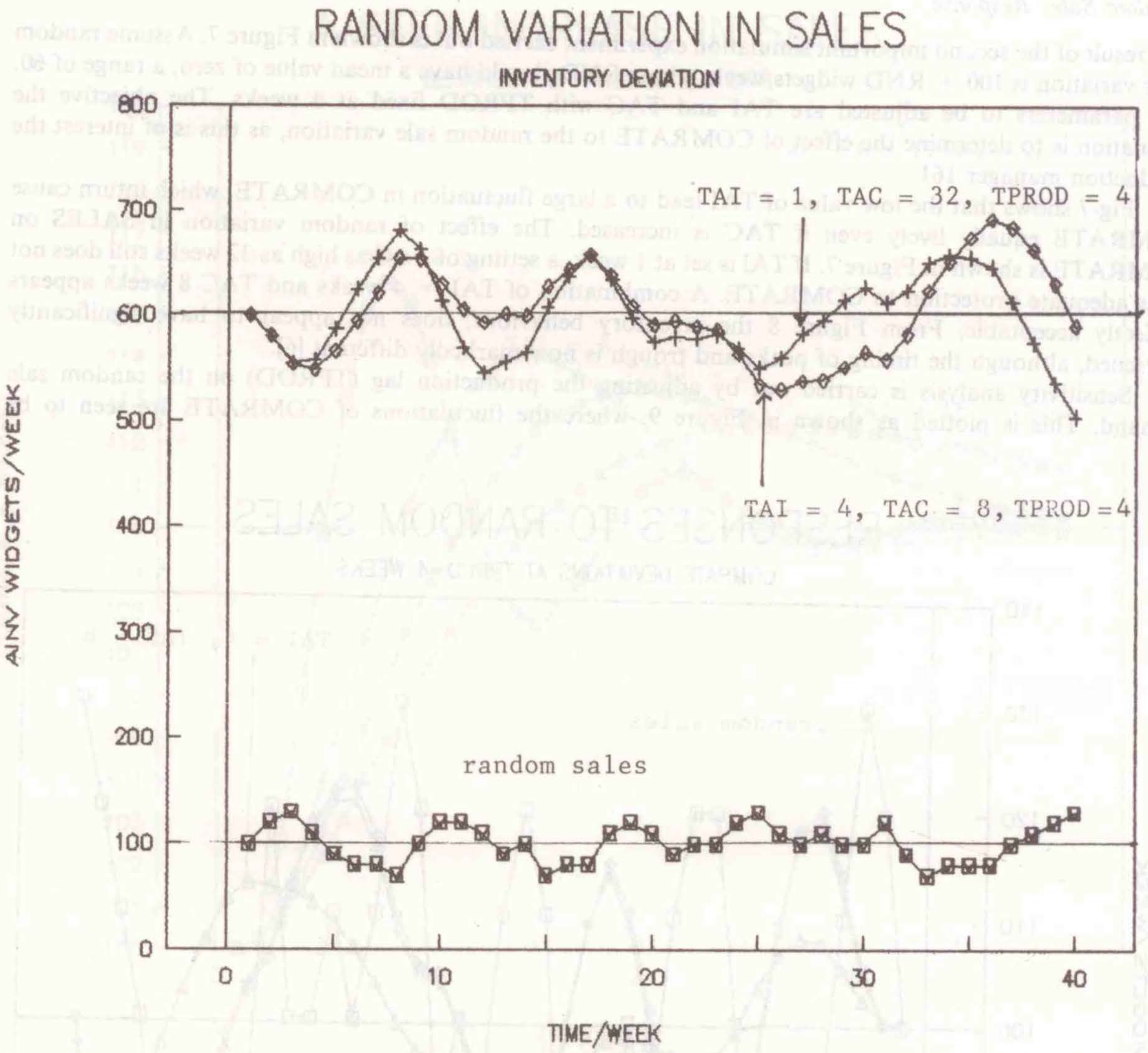

Figure 8: Responses to Random Variation in Sales

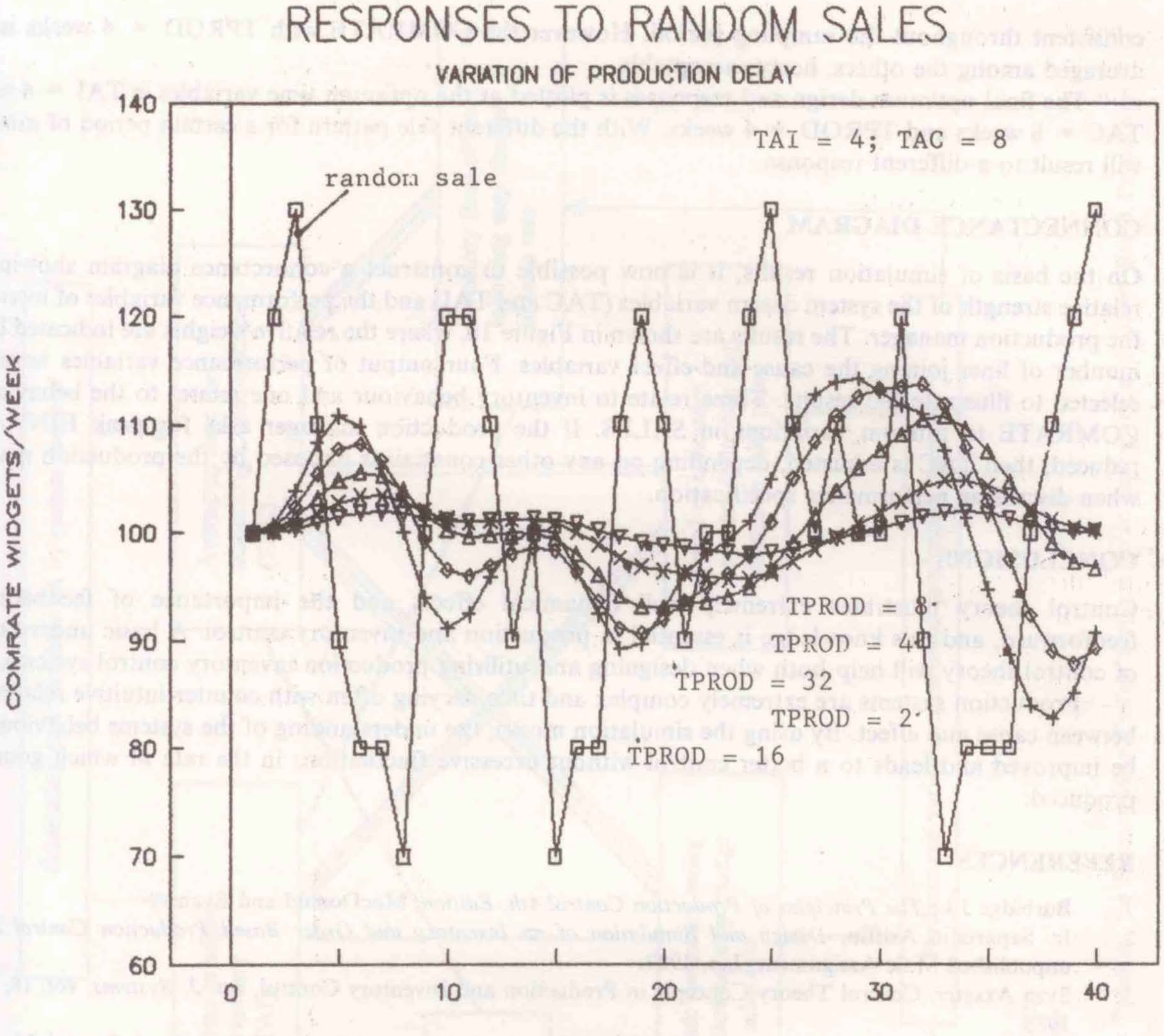

Figure 9: Production Delay on Random Sale Demand

consistent throughout the sampling period. However the COMRATE with TPROD = 4 weeks is seen averaged among the others, hence, acceptable.

The final optimum design and responses is plotted at the optimum time variables ie TAI = 4 weeks, TAC = 8 weeks and TPROD = 4 weeks. With the different sale pattern for a certain period of sampling will result to a different response.

CONNECTANCE DIAGRAM

On the basis of simulation results, it is now possible to construct a connectance diagram showing the relative strength of the system design variables (TAC and TAI) and the performance variables of interest to the production manager. The results are shown in Figure 10, where the relative weights are indicated by the number of lines joining the cause-and-effect variables. Four output or performance variables have been selected to illustrate the results. Three relate to inventory behaviour and one relates to the behaviour of COMRATE to random variations in SALES. If the production manager asks for peak EINV to be reduced, then TAC is adjusted, depending on any other constraints imposed by the production manager when discussing performance specification.

CONCLUSIONS

Control theory illustrates extremely well dynamical effects and the importance of feedback and feedforward, and this knowledge is essential in production and inventory control. A basic understanding of control theory will help both when designing and utilising production inventory control systems.

Production systems are extremely complex and time varying often with counter-intuitive relationship between cause and effect. By using the simulation model, the understanding of the systems behaviour may be improved and leads to a better control without excessive fluctuations in the rate at which goods are produced.

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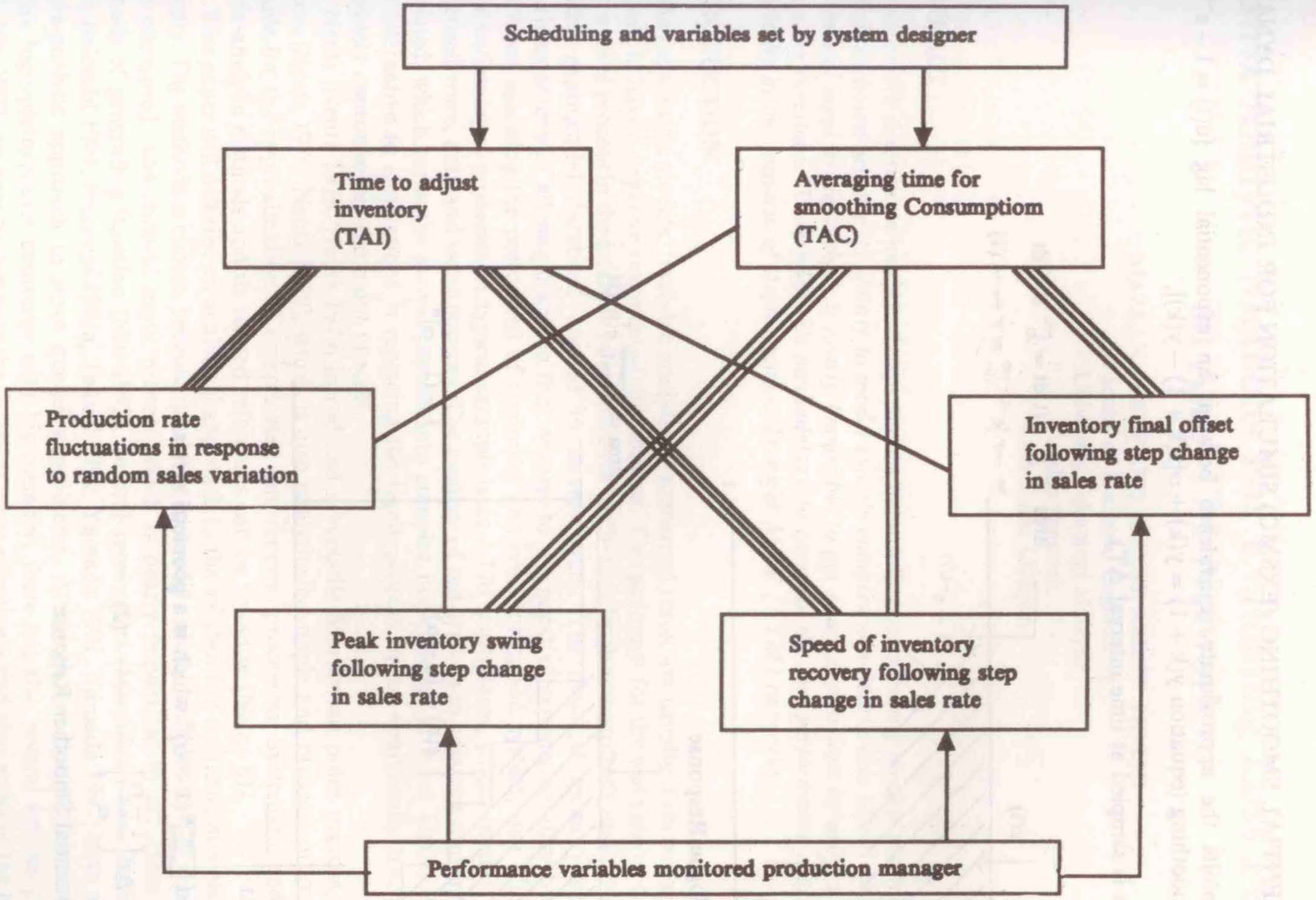


Key ≡ large influence
– little influence

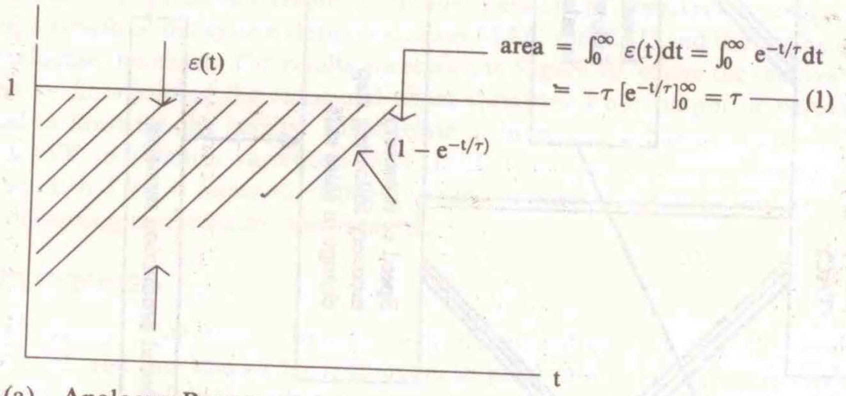
Figure 10: Connectance Diagram for Summarising Design Options in an Inventory and Order Based Production Control System

APPENDIX

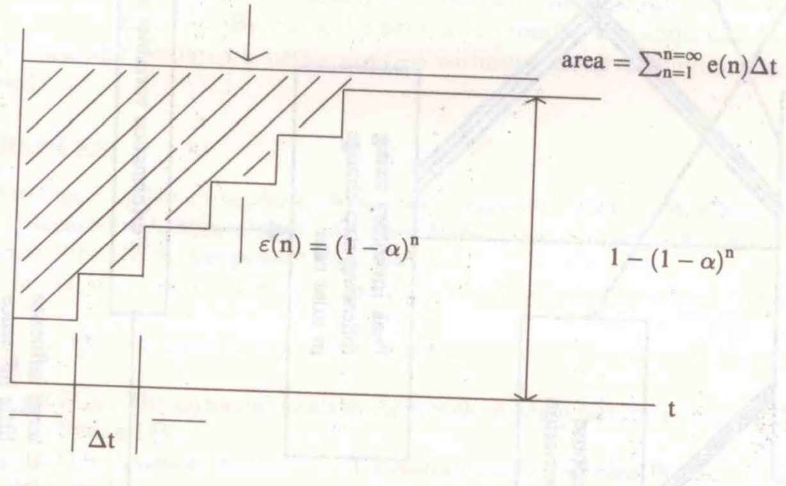
THE EXPONENTIAL SMOOTHING (EXSMO) SIMULATION FOR INDUSTRIAL DYNAMICS.

(EXSMO' exploits the approximate equivalence between an exponential lag $[u(t) = 1 - e^{-t/\tau}]$ and exponential smoothing (equation $y(k+1) = y(k) + \alpha[x(k+1) - y(k)]$,

where the data is sampled at time interval ΔT).



(a) Analogue Response



Area $= \Delta t \sum_{n=1}^{n=\infty}(1-\alpha)^n$ which is a geometri series

$$:.\ \text{Area} = \Delta t\left[\frac{1-\alpha}{\alpha}\right] \text{———— (2)}$$

(b) Exponential Smoother Reponse

Figure A/1

Equating the area under the error curves for both models step response, we have the fundamental relationship;

$$:.\ \tau = \Delta t\left[\frac{1-\alpha}{\alpha}\right]$$

$$\text{or } r\alpha = \left[\frac{1}{1+\frac{\tau}{\Delta t}}\right]$$

Where τ time constant
Δt sampling peroid.

In which the quivalence bercomes more and more accurates as $(\tau/\Delta t)$ iincrease (as does the work needed to perform the simulation).