CONTROL OF AN ACTIVE MAGNETIC BEARING SYSTEM USING SLIDING MODE TECHNIQUES

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ABSTRACT In this paper, stabilization of an Active Magnetic Bearing (AMB) system with varying rotor speed using Sliding Mode Control (SMC) techniques are considered. The gyroscopic effect and imbalance inherited in the system is proportional to rotor speed in which this nonlinearity effect causes high system instability as the rotor speed increases. Also, transformation of the AMB dynamic model into a new class of uncertain system shows that both the gyroscopic effect and the imbalance lie in the mismatched part of the system matrix. SMC design methods based on Linear Matrix Inequality (LMI) and H_2 techniques are proposed in which the sufficient condition that guarantees the stability of the reduced-order system is achieved. Then, a chattering-free control law is also established and discussed such that the system states are not only driven to reach the switching surface and to maintain the sliding mode, but also the control effort is reduced. The performance of the controllers applied to the AMB model is demonstrated through simulation works under various system conditions.

1. INTRODUCTION

An active magnetic bearing (AMB) system is a collection of electromagnets used to suspend an object and stabilization of the system is performed by feedback control. The system is composed of a floating mechanical rotor and electromagnetic coils that provide the controlled dynamic force. Due to this non-contact operation, AMB system has many promising advantages for high-speed and clean-environment applications. Moreover, adjustable stiffness and damping characteristics also make the system suitable for elimination of system vibration. Although the system is complex and considered an advance topic in term of its structural and control design, the advantages it offers outweigh the design complexity. A few of the AMB applications that receive huge attentions from many research groups around the globe are the flywheel energy and storage device (Mukhopadhyay et al., 2000), compressor (Komari et al. 1998), turbo molecular pump (Losch et al., 1999), Left Ventrical Assist Device (LVAD) and artificial heart (Lee et al., 2003).

The control issues for AMB system receive high interest from many research groups. AMB inherits many nonlinearity effects that cause the system to be very unstable and proper control is thus mandatory to stabilize the system. The main objective of this work is to present SMC techniques to overcome the most crucial nonlinearities in AMB which are the gyroscopic effect and the imbalance. The design of SMC controller involves two crucial steps which are commonly referred to as the reaching phase and the sliding phase (Spurgeon and Edwards, 1998). As apart of the design process, the AMB model is transformed into a so-called regular form to map the uncertainties present in the system which results both the gyroscopic and imbalance are mismatched, i.e. not in phase with the control input. This condition imposes design difficulties for the system to achieve asymptotic system stability. Two SMC techniques are proposed to overcome this challenge.

The outline of this paper is as follows: In Section 2, the model of the AMB system based on (Sivrioglu and Nonami, 1998) is illustrated and represented in its deterministic form which serves as a tool for the controller design. Section 3 covers the design of two types SMC control algorithm based on LMI and H_2 theory to overcome the AMB nonlinearity. The stability of the system under the designed controller is also shown. Then, in Section 4, the performances on the AMB system under the designed controller are illustrated through simulation works under various system conditions. Finally, the conclusion in Section 5 summarizes the contribution of the work.

2. MODELING OF ACTIVE MAGNETIC BEARING (AMB) SYSTEM

In order to synthesize the proposed sliding surface with the controller, a vertical shaft AMB system model for the application of turbo molecular pump system is derived based on the work done in (Sivrioglu and Nonami, 1998).

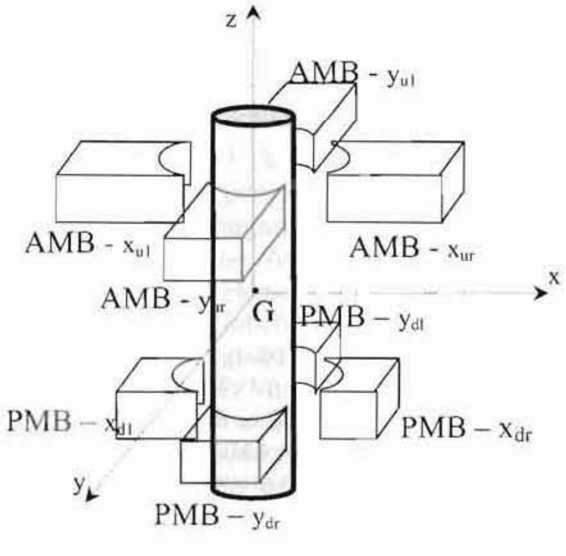

Fig. 1. Vertical Active Magnetic Bearing (AMB) System

The gyroscopic effect that causes the coupling between two axes of motions (pitch and yaw) will be also considered. Fig. 1 illustrates the five degree-of-freedom (DOF) vertical magnetic bearing in which the vertical axis (z-axis) is assumed to be decoupled from the system and hence controlled separately.

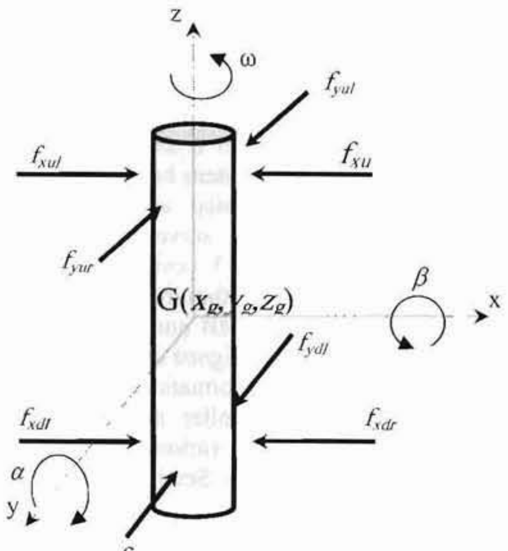

Fig. 2 The total forces acting on the AMB rotor.

The top part of the rotor of the system in Fig. 1 is controlled actively by the magnetic bearing, labeled as AMB, in which the coil currents are the inputs. The bottom part of the rotor however is levitated to the center of the system by using two sets of permanent magnets labeled as PMB. The rotation of rotor around the z-axis is supplied by external driving mechanism and considered a time-varying parameter. Fig. 2 illustrates the free-body diagram of the rotor which shows the total forces produced by the AMB and PMB of the system. Based on the principle of flight dynamics, the equations of motion of the rotor-magnetic bearing system is as follows:

$$\begin{aligned} m\ddot{x}_g &= f_{x_a} + f_{x_b} + m_{un} l\omega^2 \cos(\omega t) \\ J_r\ddot{\beta} &= -J_a\omega_z\dot{\alpha} + L_a f_{x_a} - L_b f_{x_b} \\ m\ddot{y}_g &= f_{y_a} + f_{y_b} + m_{un} l\omega^2 \sin(\omega t) \\ J_r\ddot{\alpha} &= J_a\omega_z\dot{\beta} - L_a f_{y_a} + L_b f_{y_b} \end{aligned} \quad (1)$$

The terms $m_{un}l\omega^2\cos(\omega t)$ and $m_{un}l\omega^2\sin(\omega t)$ are the imbalances due the difference between rotor geometric center and mass center. These imbalances cause the whirling motion and the magnitude is proportional to the speed of rotation, ω. The gyroscopic effect is represented by the term $-J_a\omega_z\dot{\alpha}$ and $J_a\omega_z\dot{\beta}$, where it can be noticed that this will cause the coupling between the axes of motions proportional to the speed.

The control forces produced by the AMB are given by the following equations:

$$\begin{aligned} f_{xu} &= 2K_{du}x_g + 2L_u K_{du}\beta + 2K_{iu} i_{xu} \\ f_{yu} &= 2K_{du}y_g - 2L_u K_{du}\alpha + 2K_{iu} i_{yu} \end{aligned} \quad (2)$$

where $f_{xu} = f_{xur} - f_{xul}$ and $f_{yu} = f_{yur} - f_{yul}$ are the net forces produced by the AMB on each x- and y-axis respectively (the same net force for bottom PMB as well). This is possible by having the AMB coil wound to produce differential current mode. For the PMB, the net forces produced are given by the following equations:

$$\begin{aligned} f_{xb} &= -2C_b\dot{x}_g + 2C_bL_b\dot{\beta} - 2K_b x_g + 2K_bL_b\beta \\ f_{yb} &= -2C_b\dot{y}_g - 2C_bL_b\dot{\alpha} - 2K_b y_g - 2K_bL_b\alpha \end{aligned} \quad (3)$$

The above equations (1), (2) and (3) can be integrated to produce the AMB model in the following form:

$$\dot{X}(t) = A(\omega,t)X(t) + BU(t) + F(\omega,t) \quad (4)$$

where $X = [x_g, \beta, y_g, \alpha, \dot{x}_g, \dot{\beta}, \dot{y}_g, \dot{\alpha}]^T$ are the states of the system, $A \in^{8x8}$ is the system matrix, $B \in^{8x2}$ is the input matrix, $U(t) = [i_{xu}\ i_{yu}]^T$ the input currents and $F(\omega,t)$ is the disturbance vector due to the imbalances. In order to synthesize the controller, the AMB model is treated as an uncertain system in which deterministic approach to classify the system is used based on (Osman and Roberts, 1995). By using this approach, the AMB model can be decomposed into its nominal and uncertain part as shown below

$$\dot{X}(t) = (A + \Delta A(\omega, t))X(t) + BU(t) + F(\omega, t) \qquad (5)$$

where $\Delta A(\omega,t)$ represents the uncertainty of the system matrix and $F(\omega,t)$ is the disturbance matrix associated with speed dependent of imbalance. A and B are the nominal constant matrices of the system. The decomposition into this deterministic form is possible due to the fact that the bounds are known. The elements of the $\Delta A(\omega,t)$ and $F(\omega,t)$ can be calculated based on these available bounds. The rotational speed is given as follows:

$$0 \ rad/\sec \le \omega \le 3142 \ rad/\sec . \qquad (6)$$

Then, by using these bounds and the values of the system parameters as shown in Table 1, each element of the system and disturbance matrices can be calculated and specified in the following form:

$$\underline{a}_{ij} \le a_{ij}(\omega,t) \le \overline{a}_{ij} \qquad (7a)$$

$$\underline{d}_j \le d_j(\omega,t) \le \overline{d}_j \qquad (7b)$$

Table 1 Parameters for AMB Model.

Symbol	Parameter	Value	Unit
m	Mass of Rotor	1.595	kg
J_a	Moment of Inertia about rotational axis	1.61×10^{-3}	kg.m^2
J_r	Moment of Inertia about radial axis	3.83×10^{-3}	kg.m^2
L_u	Distance of upper AMB to G	0.0128	m
L_b	Distance of lower PMB to G	0.0843	m
K_{iu}	Linearized force/current factor	200	N/A
K_{du}	Linearized force/displ. factor	2.8×10^5	N/m
K_b	Stiffness coefficient of PMB	1.0×10^5	N/m
C_b	Damping coefficient of PMB	48	kg/s
m_{un}	Static imbalance	0.6×10^{-3}	m
l	Distance of unbalance mass from G	0.02	m
ω	Rotor rotational speed	0 – 1047 (0 – 10000)	rad/sec (rpm)

for $i = 1, \ldots, 8$, and $j = 1, \ldots, 8$, where $a_{ij}(\omega,t)$ and $d_j(\omega,t)$ are the element of $\Delta A(\omega,t)$ and $F(\omega,t)$ matrices respectively. The upper and lower bars indicate the maximum and minimum values of the elements. The element of matrix A and $\Delta A(\omega,t)$ can be calculated based on these bounds by using the following procedure:

$$a_A(i,j) = \frac{\overline{a}_{ij} + \underline{a}_{ij}}{2}, \quad a_{\Delta A}(i,j) = \overline{a}_{ij} - a_A(i,j) \qquad (8)$$

for i-th row, j-th column elements of A and $\Delta A(\omega,t)$. For the disturbance matrix, $F(\omega,t)$, only the maximum values of the elements are needed since these values represent the highest disturbance values caused by the imbalance which should be eliminated from the system.

3. SLIDING MODE CONTROL DESIGN

3.1 Surface design without imbalance

Consider a class of uncertain system given by:

$$\dot{x}(t) = (A + \Delta A(\omega,t))x(t) + Bu(t) + Ew(\omega,t) \qquad (9)$$

where $x(t) \in \Re^n$ is the system states, $u(t) \in \Re^m$ is the control input, E is the disturbance matrix and ω is any time-varying scalar function. A and B is the system and input matrices, respectively, and B is of full rank. $\Delta A(\omega,t)$ represent the system uncertainty. It can be noticed that the system is the new representation of system (5).To complete the description of the uncertain dynamical system, the following assumptions are introduced and assumed to be valid.

A1) For existence purposes, $\Delta A(\cdot,\cdot)$ and $w(\cdot,\cdot)$ are continuous on their arguments.

A2) There exist known positive scalars γ_1 and γ_2 and a function $h(x,p,t)$ such that

$$\|\Delta A(\omega,t)\| \le \gamma_1 .$$

$$\|w(\omega,t)\| \le \gamma_2 .$$

A3) The pair (A,B) is controllable.

Define a linear switching surface as

$$\sigma = Sx = 0 \qquad (10)$$

where $S \in \Re^{m \times n}$ is of full row rank m. By referring to (Spurgeon and Edwards, 1998), the switching surface parameter matrix S should be selected such that

E1) The matrix SB is nonsingular

E2) Given that $E = 0$, the reduced $(n\text{-}m)$ order system of the system (9) restricted on the switching surface $Sx=0$ is globally exponentially stable for all allowable uncertainties satisfying assumption *A2*).

To continue with the design, let $\Phi \in \Re^{n\times(n-m)}$ be any full column rank matrix such that $B^T\Phi = 0$ and $\Phi^T\Phi = I_{n-m}$. Then it can be established that

$$\begin{bmatrix} \Phi^T \\ (B^TB)^{-1}B^T \end{bmatrix} \begin{bmatrix} \Phi & B \end{bmatrix} = \begin{bmatrix} I_{n-m} & 0 \\ 0 & I_m \end{bmatrix} \tag{11a}$$

or equivalently

$$\Phi^T\Phi + B(B^TB)^{-1}B^T = I_n. \tag{11b}$$

From this, it is obvious that the nonsingular transformation matrix can be defined as follows:

$$T = \begin{bmatrix} \Phi^T \\ (B^TB)^{-1}B^T \end{bmatrix} \tag{12}$$

Substituting (12) into system (9) gives

$$\begin{aligned} \dot{x}(t) &= (A + \Phi\Phi^T\Delta A(x,p,t))x(t) + \Phi\Phi^T Ew(\omega,t) \\ &+ Bu(t) + (B^TB)^{-1}B^T \times \\ &\{\Delta A(x,p,t)x + Ew(\omega,t)\} \end{aligned} \tag{13}$$

This new system representation has shown that the system uncertainty and disturbance are mismatched as denoted by the last term in (13). As stated by *E2*), when $E = 0$, then the mismatched disturbance is not present which physically means that the imbalance effect is not present in the AMB system. This implies that the gyroscopic is the only nonlinearity presents and with this condition remains true, based on (Husain et al., 2008b), the following theorem is introduced such that the sliding surface can be designed to achieve asymptotic system stability.

Theorem 1.(Husain et al., 2008b) *Consider the system (9) and suppose assumption (A1)-(A3) hold. Then there exist a matrix S satisfying (E1) and (E2), if there is a matrix $P > 0$ and positive scalar ε such that*

$$\begin{aligned} &\Phi^TA^T\Phi P + P\Phi^TA\Phi + \delta^{-1}P^2 \\ &-(\delta\gamma_1)^{-1}P\Phi^TAB(B^TB)^{-1}B^TA^T\Phi P + (\delta\gamma_1^2)I_{n-m} < 0 \end{aligned} \tag{14}$$

where γ_1 is given in assumption (A2). Moreover, the matrix S can be proposed to be as follows:

$$S = (B^TB)^{-1}B^T(I_n + (\delta\gamma_1^2)^{-1}A^T\Phi P\Phi^T) \tag{15}$$

Proof See (Husain et al., 2008b).

Theorem 1 provides a very important result in which it indicates that the system is able to achieve asymptotic stability with the present of mismatched system uncertainty. The surface S can be designed by solving the Riccati-like equation or it can be formulated as LMI and solved efficiently by using LMI solver. The detail of both techniques are outlined in (Husain et al., 2008b) and intentionally not covered in this work.

3.2 Surface design with imbalance presents

When the assumption *E2*) is relaxed which implies that the imbalance is present, Theorem 1 fails and in order to achieve system stability, the surface needs to be redesigned. To continue with the design, the following assumptions are introduced.

A4) The linear model mismatch is supposed to belong to a convex polytope in parameter space such that:

$$\Delta A = \sum_{i=1}^{k}\alpha_i\Delta A_i, \quad \sum_{i=1}^{k}\alpha_i = 1, \; \alpha_i \geq 0 \; \forall i \in 1,\cdots,k.$$

Maintaining the sliding surfaceas (10), the define an output variable as

$$z(t) = Lx(t) \tag{16}$$

Due to the mismatched condition, the transfer function from the exogenous input vector $w(t)$ to this output $z(t)$ is given by

$$Z(s) = H(s)W(s) \tag{17}$$

where $Z(s)$ and $W(s)$ are the Laplace transform of $z(t)$ and $w(t)$ respectively and $H(s)$ is valid for $t > \tau$. When the system in the sliding motion, the H_2 norm of the transfer function (17) for the closed-loop system is defined as follows:

$$\|H(s)\|_2^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} tr[H(j\omega_s)^* H(j\omega_s)]d\omega_s \tag{18}$$

From this, as highlighted in (Takahashi and Perez, 1999), the main objective is to find the optimal sliding surface C_{opt}, such that the upper bound of the H_2 norm (18) over all mismatched ΔA such that:

$$C_{opt} = \arg\min_{C}\max_{\Delta A}\Omega(C,\Delta A),\; \Omega(C,\Delta A) \geq \|H(s)\|_2 \tag{19}$$

The minimization of the upper bound of $\Omega(C,\Delta A)$ is the solution to the sliding surface. The design of the sliding surface is carried by transforming the system dynamic into a regular form and this is in-line with the step outlined in (Spurgeon and Edwards, 1998). A new transformation matrix T is chosen such that

$$TB=\begin{bmatrix}0\\ \bar{B}_2\end{bmatrix},\ TT^T=I \tag{20}$$

where matrix $\bar{B}_2 \in \Re^{m\times m}$ is nonsingular. Then applying this transformation to system (9), a new system representation is obtained as:

$$\begin{aligned}&\bar{x}=Tx\\ &\dot{\bar{x}}=T(A+\Delta A)T^{-1}\bar{x}+TBu+TEw\\ &CT^{-1}\bar{x}=0\\ &z=LT^{-1}\bar{x}\end{aligned} \tag{21}$$

Thus, the system states can be partitioned as follows:

$$\bar{x}=\begin{bmatrix}\bar{x}_1\\ \bar{x}_2\end{bmatrix},\ \bar{x}_1\in\Re^{n-m},\ \bar{x}_2\in\Re^{m} \tag{22}$$

Then, the matrices of the system can be partitioned to form the following new matrices:

$$\begin{aligned}&\bar{A}=TAT^{-1}=\begin{bmatrix}\bar{A}_1 & \bar{A}_2\\ \bar{A}_{21} & \bar{A}_{22}\end{bmatrix},\\ &\overline{\Delta A}=TAT^{-1}=\begin{bmatrix}\overline{\Delta A}_1 & \overline{\Delta A}_2\\ \overline{\Delta A}_{21} & \overline{\Delta A}_{22}\end{bmatrix},\\ &\bar{E}=TE=\begin{bmatrix}\bar{E}_1\\ \bar{E}_2\end{bmatrix},\ \bar{C}=CT^{-1}=\begin{bmatrix}\bar{C}_1 & \bar{C}_2\end{bmatrix}\\ &\text{and } \bar{L}=LT^{-1}=\begin{bmatrix}\bar{L}_1 & \bar{L}_2\end{bmatrix}\end{aligned} \tag{23}$$

Then, the surface equation becomes

$$\bar{C}_1\bar{x}_1+\bar{C}_2\bar{x}_2=0 \tag{24}$$

By knowing that $\det(\bar{C}_2)\neq 0$, this will lead to

$$\bar{x}_2=-\bar{C}_2^{-1}\bar{C}_1\bar{x}_1 \tag{25}$$

To simplify the derivation, the following terms are introduced which are:

$$\begin{aligned}&F\triangleq\bar{C}_2^{-1}\bar{C}_1,\\ &\Phi\triangleq\bar{A}_1+\overline{\Delta A}_1,\\ &\Gamma\triangleq\bar{A}_2+\overline{\Delta A}_2.\end{aligned} \tag{26}$$

From (23), (24) and these new terms, the new representation of the reduced-order closed-loop system is obtained as follows:

$$\begin{aligned}&\dot{\bar{x}}_1=\Phi\bar{x}_1+\Gamma\bar{x}_2+\bar{E}_1w\\ &\bar{x}_2=-F\bar{x}_1\\ &z=\bar{L}_1\bar{x}_1+\bar{L}_2\bar{x}_2\end{aligned} \tag{27}$$

Thus, viewing this closed-loop system, it is obvious that the design of the sliding surface is equivalent to designing feedback gain, F, for the reduced-order system with uncertainties. Looking at the assumption *A2)*, the matrices $\Phi\triangleq\bar{A}_1+\overline{\Delta A}_1$ and $\Gamma\triangleq\bar{A}_2+\overline{\Delta A}_2$ also belongs to a polytope-type set with known vertices. The set can be defined as follows:

$$\wp\triangleq\left\{\langle\Phi,\Gamma\rangle=\sum_{i=1}^{k}\alpha_i\langle\Phi_i,\Gamma_i\rangle,\sum_{i=1}^{k}\alpha_i=1,\ \alpha_i\geq 0\right\} \tag{28}$$

The vertices of this polytope (28) corresponds to he vertices of the uncertain defined in *A4)* in obtained by using the transformation matrix T.

By following the design step outline in (Takahashi and Perez, 1998; Husain et. al. 2008b), an assumption on the matrix L in (16) is introduced in which the assumption will guarantee that the H_2 cost function optimization for the sliding surface design is nonsingular. The assumption is as follows:

$$\bar{L}_2^T\bar{L}_2>0 \tag{29}$$

Then, with the constraint of (29) a new variable is introduced as follows:

$$e(t)=(\bar{L}_2^T\bar{L}_2)^{-1}\bar{L}_2^T\bar{L}_2\bar{x}_1+\bar{x}_2 \tag{30}$$

The reduced-order system (27) may be written in term of $\bar{x}_1$ and e as

$$\begin{aligned}&\dot{\bar{x}}_1=(\Phi-\Gamma(\bar{L}_2^T\bar{L}_2)^{-1}\bar{L}_2^T\bar{L}_1)\bar{x}_1+\Gamma e+\bar{E}_1w\\ &z=(\bar{L}_1-\bar{L}_2(\bar{L}_2^T\bar{L}_2)^{-1}\bar{L}_2^T\bar{L}_1)\bar{x}_1+\bar{L}_2e\end{aligned} \tag{31}$$

Furthermore, the system can be simplified by defining the following terms:

$$\begin{aligned}&\bar{\Phi}=\Phi-\Gamma(\bar{L}_2^T\bar{L}_2)^{-1}\bar{L}_2^T\bar{L}_1\\ &\Lambda=\bar{L}_1-\bar{L}_2(\bar{L}_2^T\bar{L}_2)^{-1}\bar{L}_2^T\bar{L}_1\end{aligned} \tag{32}$$

Then, the transformed uncertainty polytope (28) can be recast as:

$$(\overline{\Phi},\Gamma)\in\aleph \Leftrightarrow (\Phi,\Gamma)\in\wp$$

$$\overline{\Phi}_i = \Phi_i - \Gamma_i \overline{L}_2(\overline{L}_2^T\overline{L}_2)^{-1}\overline{L}_2^T\overline{L}_1,\ \ \forall i=1,\cdots,k \tag{33}$$

From (27) and (30), the transformed gain matrix is

$$\begin{aligned} \phi(t) &= (-F + (\overline{L}_2^T\overline{L}_2)^{-1}\overline{L}_2^T\overline{L})_2\overline{x}_1 \\ &= K\overline{x}_1 \end{aligned} \tag{34}$$

Consider now the set Ξ of the symmetric positive-definite matrices X such that:

$$\Xi:\{X\in\Re^{(n-m)\times(n-m)}\mid X=X^T>0;\ (\overline{\Phi}-\Gamma K)X+X(\overline{\Phi}-\Gamma K)^T+\overline{E}_1\overline{E}_1\le 0,\ \forall(\overline{\Phi},\Gamma)\in\aleph\} \tag{35}$$

For an arbitrary but fixed pair of $(\overline{\Phi},\Gamma)\in\aleph$, the H_2 norm of system (31) is bounded by:

$$\|H(s)\|_2^2 \le tr((\Lambda-\overline{L}_2K)X(\Lambda-\overline{L}_2K)^T),\ \forall X\in\Xi. \tag{36}$$

Let a new matrix $W\in\Xi$, then a new variable can be defined as

$$Z=KW,\ \therefore K=ZW^{-1} \tag{37}$$

Thus, the problem of minimization of the upper bound of the H_2 norm (36) can be represented as

$$\min\ tr(\Lambda W\Lambda^T+\overline{L}_2ZW^{-1}Z^T\overline{L}_2^T)$$

s. t..:

$$\overline{\Phi}W-\Gamma Z+W\overline{\Phi}^T-Z^T\Gamma^T+\overline{E}_1\overline{E}_1^T\le 0,\ \ \forall(\overline{\Phi},\Gamma)\in\aleph. \tag{38}$$

Obviously, this problem can be expressed as LMI problem. Defining the objective function in (38) as a new variable as follows:

$$Q\ge \Lambda W\Lambda^T+\overline{L}_2ZW^{-1}Z^T\overline{L}_2^T \tag{39}$$

Taking the Schur complement of (39), then, the problem can be represented as and LMI optimization problem:

$$\Omega^2=\min_{W,Z,Q}\ tr(Q)$$

$$\text{s. t. :}\begin{cases}\begin{bmatrix} W & Z^T\overline{L}_2^T \\ \overline{L}_2Z & Q-\Lambda W\Lambda^T\end{bmatrix}\ge 0 \\ W>0 \\ \overline{\Phi}_iW+W\overline{\Phi}_i-\Gamma_iZ-Z^T\Gamma_i^T+\overline{E}_1\overline{E}_1^T\le 0, \\ \qquad\qquad \forall i=1,\cdots,k.\end{cases} \tag{40}$$

If the solution of (40) is feasible, then the sliding surface parameter that guarantees the existence of the optimal upper bound H_2 norm can be obtained. With the values of W and Z determined, as stated in the theorem, the gain K in (37) can be obtained and lead to the optimal surface values as follows:

$$C_{opt}=\left[\overline{C}_2((\overline{L}_2^TL_2)^{-1}\overline{L}_2^T\overline{L}_1-K)\ \ \overline{C}_2\right]T \tag{41}$$

Notice that the matrix $\overline{C}_2$ does not have any influence in the reduced order system and can be chosen freely provided it is full rank and $\overline{C}_2=I$ is a convenient as stated in (Spurgeon and Edwards, 1998; Takahashi and Peres, 1998).

3.3 Control Law design

The control law development is not shown in detail in this work, instead, the controller for sliding surface with and without imbalance from (Husain et al., 2008a; Husain et. al., 2008b) are used as shown below.

Control law for system without imbalance
(Husain et al., 2008a)

$$u(t)=-SAx-\{[\gamma_1\|S\|]\|x\|+\beta\}sat(\frac{\sigma}{\Pi}) \tag{42}$$

where S is obtained from (15) and Π is the thickness of the boundary layer.

Control law for system with imbalance
(Husain et al., 2008b)

$$u(t)=-(CB)^{-1}CAx(t)-\rho\ sign(Cx(t)) \tag{43}$$

where C is obtained from (41).

4. SIMULATION RESULT AND DISCUSSION

The following settings and calculated result are used to obtain the shown result.

AMB without imbalance

1. Sliding surface, S

$$S=10^{-4}\begin{bmatrix} 3 & 23 & 5 & 8 & 1 & 7 & 0 & -3 \\ -1 & -7 & -1 & 10 & 0 & 0 & 15 & 1\end{bmatrix}$$

2. Rotor speed, ω = 10krpm

The result for the X and Y trajectories for the system without imbalance is as shown in Fig. 3. Obviously the asymptotic stability is achieved and the states approach zero in finite time which implies the rotor does not experience and whirling effect. The chattering-free input is shown in Fig. 4 for two selection of boundary layer thickness.

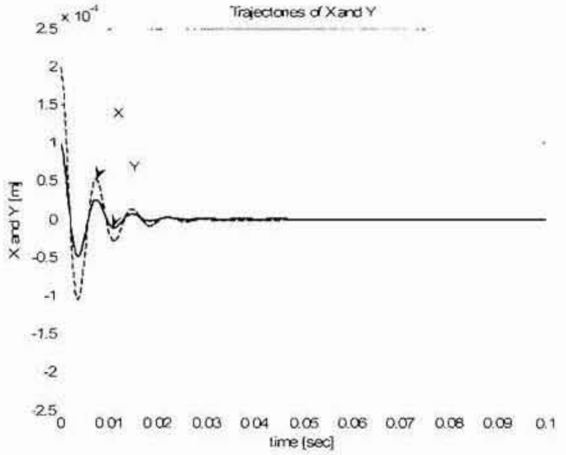

Fig. 3. X and Y trajectories with $\omega = 10$krpm

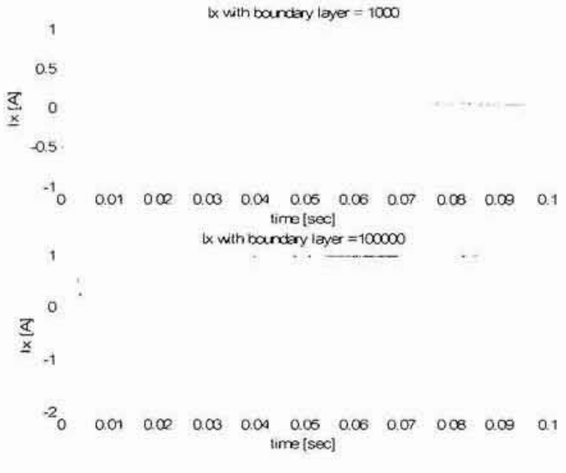

Fig. 4. Current I_x with $\Pi = 1000$ (top) and $\Pi = 100000$ (bottom)

AMB with imbalance

1. Chosen output matrix, L.

$$L = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

2. Calculated Sliding surface, C

$$C = \begin{bmatrix} -5.6019 & -5.4063 & 2.7145 & -0.0031 & -5.4236 & 0 \\ 0.6659 & -0.1302 & -94.1392 & -5.3830 & 0 & 0 \end{bmatrix}$$

$$\begin{matrix} 0 & 0 \\ -5.4224 & 0.000 \end{matrix}$$

3. Rotor speed, $\omega = 6$krpm and 10krpm

To compare the performance of the controller, the result obtained by (Sivrioglu and Nonami, 1998) as shown by Fig. 5 is used as the benchmark. For rotor speed, $\omega = 6$krpm, the critical speed of AMB system, the rotor orbit is about 50 μm. Then, under the controller (43) with the settings outlined above, at rotor speed $\omega = 6$krpm and 10krpm, the trajectories of X and Y reach almost zero in which it exhibits that the rotor whirling effect is unnoticeable as shown by Fig. 6 and Fig. 7.

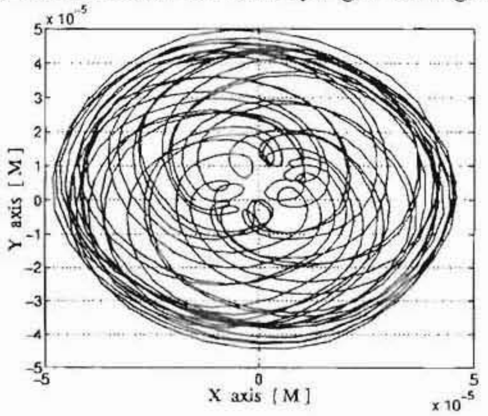

Fig. 5. Rotor orbit at $\omega = 6$krpm for controller by (Sivrioglu and Nonami)

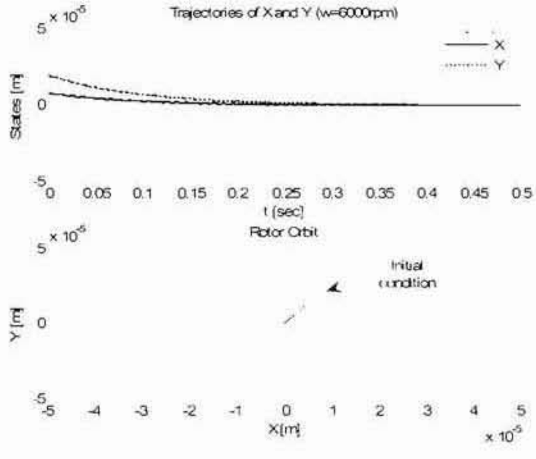

Fig. 6. X and Y trajectories (top) and rotor orbit (bottom) using L_s at $\omega = 6$krpm.

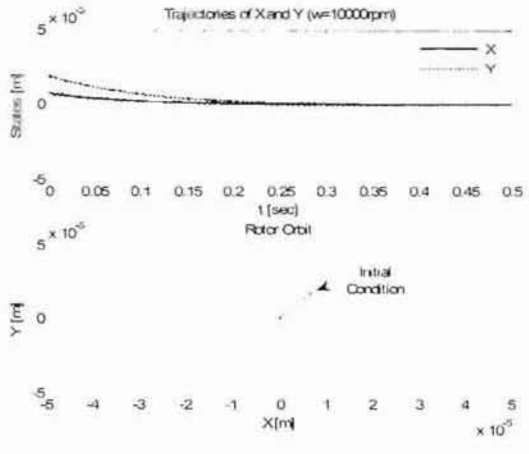

Fig. 7. X and Y trajectories (top) and rotor orbit (bottom) using L_s at ω = 10krpm.

5. CONCLUSION

The control of AMB system by using SMC techniques is demonstrated based on LMI and H2 theory. Both the mismatched gyroscopic and imbalance effects are overcome effectively to meet the system required stability.

REFERENCES

Husain A. R., Ahmad M. N. and Yatim A. H., Application of H2-based Sliding Mode Control for an Active Magnetic Bearing System, Int. Jour. Mech. Sys. Science and Engineering, vol.2, no. 1, pp. 1-8, 2008a.

Husain A. R., Ahmad M. N. and Yatim A. H., Asymptotic Stabilization of an Active Magnetic Bearing System using LMI-based Slidin Mode Control, Int. Jour. Mech. Sys. Science and Engineering, vol.2, no. 1, pp. 9-16, 2008b.

Komori M., Kumamoto M., and Kobayashi H., A Hybrid-Type Superconducting Magnetic Bearing System with Nonlinear Control, *IEEE Trans. on Applied Superconductivity*, vol. 8, no.2, pp. 79–83, June 1998.

Lee J. H., Allaire P. E., Tao G., Decker J., and Zhang X., Experimental Study of Sliding Mode Control for a Benchmark Magnetic Bearing System and Artificial Heart Pump Suspension, *IEEE Trans. on Contr. Sys. Tech.*, vol. 11, no. 1, pp. 128–138, Jan. 2003.

Losch F., Gahler C., and Herzog R., Low Order μ-Synthesis Controller Design for a Large Boiler Feed Pump Equipped with Active Magnetic Bearing, *Proc. of IEEE Int. Conf. on Contr. Appl.*, pp. 564–569, August 1999.

Mukhopadhyay S. C., Ohji T.,Iwahara M., and Yamada S., Modeling and Control of a New Horizontal-Shaft Hybrid-Type Magnetic Bearing, *IEEE Trans. On Ind. Elec.*, vol. 47, no. 1, pp. 100–108, Feb. 2000.

Osman J. H. S., and Roberts P. D. Two Level Control Strategy for Robot Manipulators. *Int. Journal of Control* vol. 61, no. 1, pp. 1201 – 1222, 1995.

Sivrioglu S., and Nonami K. Sliding Mode Control With Time-Varying Hyperplane for AMB Systems. *IEEE/ASME Trans. on Mechatronics* 3(1), pp. 51-59, 1998.

Spurgeon S., and Edwards E. *Sliding Mode Control: Theory and Applications*. London: Taylor and Francis, 1998.

Takahashi R. H. C. and Peres P. L. D., H_2 Guaranteed Cost-switching Surface Design for Sliding Modes with Nonmatching Disturbances, *IEEE Trans. on Auto. Contr.*, vol. 44, no. 11, pp. 2214-2218, 1999.

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