NUMERICAL SIMULATION OF ADIABATIC ACOUSTIC WAVES IN A CLOSED RECTANGULAR CHAMBER

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To my wife Wan Rozaida Mazlina and my daughter Nur Qaisara Batrisyia

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ABSTRACT

This study of acoustic wave was motivated by the thermoacoustic refrigerator. Normally acoustic wave is used to create a temperature gradient across the stack in a resonant tube. Numerical simulation of adiabatic acoustic waves in a closed two-dimensional rectangular chamber is considered for this case. The waves are generated by a membrane boundary condition on one wall. The Navier-Stoke system is solved here by assuming constant thermophysical properties of the compressible Newtonian fluid and perfect gas. The results for adiabatic and non-adiabatic cases for velocity profile, temperature distribution and pressure distribution are discussed.

ABSTRAK

Kajian gelombang akustik ini didorong oleh sistem penyejukan termoakustik. Kebiasaanya gelombang akustik ini digunakan bagi menjana perubahan suhu apabila melalui susunan plat di dalam tiub resonan. Simulasi berangka bagi gelombang akustik yang adiabatik di dalam kebuk segiempat tepat tertutup dua dimensi dipertimbangkan untuk kes ini. Gelombang diuja oleh keadaan sempadan membran pada satu dinding. Penyelesian sistem Navier-Stoke telah dibuat dengan menganggap bahawa sifat termofizik bendalir Newtonian boleh mampat adalah malar dan gas adalah unggul. Keputusan untuk kes adiabatik dan bukan adiabatik bagi profil halaju, taburan suhu dan tekanan dibincangkan.

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Roman

c_p - Isobaric specific heat

H - Height of resonator

k - Thermal conductivity

L - Length of the resonator

L/H - Aspect Ratio

M - Mach number

p - pressure

Pe - Peclet number

Pr - Prandtl number

R - Specific gas constant

Re - Reynolds number

t - Time

T - Temperature

u,v - Velocity component

U_o - Forcing amplitude

V - Volume

Greek

α - Thermal diffusivity $\left(\equiv \frac{k}{\rho c_p} \right)$

ϕ - Non-linear terms

Φ - Viscous Dissipation

γ - Specific heat ratio

λ - Wave length

μ - Dynamic viscosity

ν - Kinematic viscous

ρ - Density

τ - Viscous stress tensor

ω - Forcing frequency

∇ - Vector differential operator

Subscript

i,j - Grid location in x, y direction

m - Mean

Superscript

$*$ - Denote a non-dimensional quantity

‘ - Fluctuating part

CHAPTER 1

INTRODUCTION

A brief overview of major theoretical, experimental and numerical study advancements in thermoacoustic and acoustic wave is presented in this section. This will serve as a general motivation for this research. A review of the theory will be given first, followed by a summary of experimental investigation and ended with numerical and simulation study. Currently few researchers have developed numerical interpretation of the thermoacoustic effects (Aranha, Yue and Mei 1982; Wolikar and Knio, 1996; Normah, 2001).

The numerical simulation of acoustic wave in a closed rectangular chamber is very important in order to ensure the phenomena of acoustic effects. The acoustic wave is widely used in thermoacoustic refrigerators and thermoacoustic engines.

1.1 History

The use of thermoacoustic effects started over a century ago. The fundamentals on thermoacoustics can be referred to in Rott's review article and Swift's review article. Here we expand upon the representative results of those works.

Rott is generally considered the initiator of the field, being the first to write down a full theoretical description of the thermoacoustic effect (N. Rott, 1980). However, he attributes the beginnings of the theoretical work to Kirchhoff who modified the result of the Helmholtz-Rayleigh theory of sound attenuation in ducts to consider the heat transfer between the gas and the isothermal duct wall. In his review, Rott summarized the results obtained over more than a decade ago by him and his colleagues which originally intended to explain theoretically how Taconis vibration occurs. The momentum, continuity and the energy equations, all in the small-oscillating limit, were considered. Ideal gasses were assumed. He showed how a temperature difference can arise in the walls of a narrow tube due to the time-averaged entropy flow in a gas forced into oscillation and found the expression of second order heat flux associated with these oscillations.

In his review, Swift (1988) took a decisive step toward implementing Rott's theory of thermoacoustic phenomena into creating practical thermal engines. He considered and extended the fundamental theory of Rott's to calculate the heat and work flux in a stack as a collection of individual narrow thermoacoustic elements, placed in a standing-wave resonator. The efficiency of a thermoacoustic device, either as a prime-mover or as a refrigerator, was calculated. The derivations were done in increasing steps of complexity. The limiting cases, like that of zero viscosity, illustrate the basic concepts in a more clear way and make the theory more accessible. Also, the pictorial representation of the moving parcel of gas in Lagrange frame of reference has helped many beginners in the field to gain a more intuitive understanding of the phenomenon. He also derived the equation for absence of plate wherein the acoustic wave is adiabatic.

Another important theoretical approach in thermoacoustic was presented in the paper by Keller and Millman (1979). They studied on compressible wave travelling in a rigid cylindrical waveguide. This paper created the governing equation by ignoring the effect of viscosity and heat conduction.

Gogate and Munjal (1992) did the analytical solution of the laminar mean flow wave equation in a line or unlined two-dimensional rectangular duct. This analysis used the Newton-Raphson technique to solve the boundary conditions. The analysis was limited to the laminar flow only.

Experimental study on thermoacoustic was done by Merkli and Thomann (1975). They described an experiment that tests quantitatively the predictions of thermoacoustic theory. They measured the thermoacoustic time average heat flux along the walls of a tube driven to resonance by a piston. Heating at the close end (high pressure compression) and cooling at the velocity antinodes (low pressure-expansion), were observed. The results of thermal measurement in air (standard temperature and pressure in equilibrium) were compared with calculations made on the same line with the earlier results of Rott. The study was limited to the calculation of the heat flux entering the tube wall because it only solved first-order quantities of differential equation.

Another experimental work on thermoacoustic was done for a flow through a thermoacoustic refrigerator (R.S. Reid and G.W. Swift, 2000). The focus on this experiment was done on a standing-wave thermoacoustic refrigerator with parallel superimposed steady flow. The flow was moved by a piston driver.

Numerical simulation was used to overcome the problem created by experiments in thermoacoustic. By using a numerical method we can predict the result and it is also inexpensive to develop the model compared to an experimental rig. The Worlikar and Knio review article (1996), the first numerical work on thermoacoustic refrigerator concentrated on unsteady adiabatic flow around the stack. They used central finite difference methodology on rectangular grids. Their numerical study however covered only the region enclosed by two plates and without oscillating flow anywhere in their computational domain. They also neglected the thermal diffusion.

Ahmad Zakaria *et al.* (2000) used the central finite difference scheme to model propagation in time domain of acoustic wave in shallow water. Their numerical method was fourth order and second order accurate in time. They showed that the finite difference scheme on this acoustic wave problem is stable. However, their scope is on incompressible fluid. Another numerical study on nonlinear acoustic wave was done by Aranha et al. and S. Lichter and J. Chen (1982). They showed that in their study the nonlinear initial boundary value problems were solved using a semi-implicit finite difference scheme of the Crank-Nicolson.

The numerical study on acoustic wave was performed by Mohd Ghazali, Normah (2001) on a rectangular chamber. The effects of chamber with a plate and without a plate were simulated. The finite difference spatial discretization and semi-implicit time marching procedures was used in the numerical study.

However the study considered the acoustic wave as non-adiabatic. This study is an extension of the work of Mohd. Ghazali, Normah but an adiabatic case is now being considered. The purpose is to see if there are any differences in the two cases which may or may not justify the extra terms added for the non-adiabatic case to the controlled equation modelling the physical domain.