THE GPS DATA PROCESSING STRATEGIES FOR THE ENGINEERING STRUCTURE MONITORING SCHEME

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ABSTRACT

Monitoring and detecting deformations within an engineered structure can be determined using a number of geodetic (precise) methods, e.g. geotechnical approach, terrestrial observations and space-based methods. Nowadays, the GPS technology has become the most important tool for estimating the large structural engineering deformation such as buildings, dams, long span bridges, etc. In order to ensure that the engineering structures are exhibiting a safe deformation behavior, a repeated and/or continuous GPS measurements can be employed. The GPS technology provides a quick and precise method of determining 3D movements of a structure over time. Application of GPS technique of deformation detection on a high-rise building is one example of this approach. The appropriate processing strategies of GPS observable to estimate the geodetic parameters of interest is usually carried out in post processing mode, but one may ask 'how does one really tell the quality of GPS solutions?' Two types of adjustments can be used in GPS processing, e.g. minimal constraint and constraint adjustment.. This paper therefore highlights the processing strategies of the GPS data from two epoch observations for monitoring surveys - Case study: KOMTAR building, in Penang, Malaysia. The results of the adjustment in deformation surveys are presented and discussed.

1.0 INTRODUCTION

The need for deformation surveys often arises from concerns associated with environmental protection, property damage and public safety. Deformation refers to the changes a deformable body undergoes in its shape, dimension, and position. The determination and interpretation of the changes are the main goal of deformation surveys. Therefore, as a result, the design, execution and analysis of such surveys are also a matter of considerable practical importance. Various measurement methods may be employed in monitoring surveys. These include structural and geotechnical methods, terrestrial survey techniques and positioning with space-based systems. Various methods of positioning with space-based systems have been employed in the study of crustal dynamics and deformation phenomena such as Global Positioning System (GPS) which offers the greatest accuracy at regional scales, and is the most cost-effective. Thus, the challenge of monitoring surveys using GPS has received growing attention during the last few years, see Kenneth and Behr (1998), and Brown et al. (1999) .

The choice of the procedure for the adjustment of deformation surveys cannot be based only on the available computing facilities but must take into consideration the characteristics of the problem to be treated, for example processing strategy in the deformation networks. Due to the high sensitivity of the least squares estimation method for deformation application, both the pre-adjustment (e.g. gross errors/outlier detections) and post-adjustment data screening techniques (e.g. through statistical testing of the estimated observational residuals) have to be applied, and this is very important for producing reliable results. This paper therefore highlight some computational strategies which can be applied in repeated survey measurements for deformation studies.

2.0 NETWORK DESIGN

The two remaining aspects of a monitoring scheme, which will be considered in details, are the design and analysis stages. The network design is the first step towards establishing a deformation network. A network may be designed to meet specific criteria, before any observations are actually made. In the specific case of a deformation monitoring network, the design may not only be required to meet precision (e.g. variances of point positions or derived quantities) and reliability criteria, but also to be sensitivity to the deformation pattern which is expected to take place. Since a postulated deformation model between two epochs of observations represents in effect a systematic difference between the two sets of measurements, the sensitivity assessment of a network can be regarded as being related to the detection of systematic errors (Othman, 2000).

The design required usually not only needs to solve the problem of meeting precision criteria, but must also be the minimum-cost solution, often referred to as the optimum design. This introduced cost element can be very difficult to quantify, but possible designs are usually assessed subjectively taking regard of previous experience. Once the design problem has been formulated, there are two basic approaches to the solution. Firstly, and most commonly, there is the computer simulation, or pre-analysis method, whereby proposed networks are analysed in turn to see whether they meet the required criteria, being subjectively modified by operator intervention, and using his experience, if the proposed scheme is either too strong or not strong enough. Secondly, the analytical approach attempts to mathematically formulate the design problem in terms of equations or inequalities and then to explicitly solve for the optimum solution.

3.0 NETWORK ADJUSTMENT

All points in a monitoring network are tied to each other by a combination of observable such as coordinates, elevation, etc. The numbers of observation usually exceeds the minimum number required to determine the unknown parameters. The method of least square estimation (LSE) is an important tool in estimating the unknown parameter from redundant data. Generally, the functional model relating the measurements and parameters to be estimated can be expressed in a function of:

$$l = f(x) \qquad [1]$$

where l is the vector of observations and x is the vector of parameters to be estimated. In general, equation [1] is non-linear, and it needs to be linearized by using Taylor's theorem. After linearization the observation equation is written as:

$$\hat{v} = A\hat{x} + b \qquad [2]$$

where $\hat{v}$ is the vector of residuals, A is the design matrix, $\hat{x}$ is the vector of corrections to the approximate values (x_o) and b is the misclosure vector.

The normal equation with a full rank, can be written as:

$$N\hat{x} + U = 0 \qquad [3]$$

where; l = vector of actual observation, l_o = vector of computed observation, $W = \sigma_o^2 Q_l^{-1}$, the weight matrix, n, u = number of observations and parameter.

Other important aspects that need to be considered are global test (Chi-square) and local test (TAU), precision, accuracy and reliability (internal and external) analysis. After the data for each were verified to be free of outliers and have a high degree of reliability, then the deformation analysis can be carried out. The geometrical analysis of deformation surveys involves the reliable determination of changes in the geometrical status of a structure over time (Chrzanowski et al., 1986]. Deformation between subsequent epochs can be inferred directly from a comparison of raw observable, or indirectly from changes in coordinates. Single epoch adjustments allow some leeway for variations in observation scheme between epochs, thus making it possible to utilize all available information in the solution, but introduce the problem of datum dependence. In either case, the use of adjustments has the important advantage that an evaluation of the quality of the observations can be undertaken, and an opportunity for the detection of random outliers and systematic effects is provided. However, it requires that the network be complete and free from configuration defects at each epoch, and that each epoch be referred to a common datum.

In basic approaches to geometrical analysis the displacements at discrete points are directly compared with specified tolerances. In more advanced analyses, the point displacements are assessed for spatial trend, and a displacement field is determined by the fitting of a suitable spatial function. The displacement field may then be transformed into a strain field, which provides a unique description of the overall change in geometric status, by the selection of a suitable deformation model (Chrzanowski et al., 1986).

4.0 NETWORK ANALYSIS

Usually, the analysis of a network observed for the monitoring of deformation is consists of the following tasks :

- Detection of outliers. through various statistical tests.
- the computation of the estimated parameters and associated covariance matrix and unit variance.

However, a monitoring network will be repeatedly measured at various epochs, and a comparison of successive network adjustments must be carried out in an effective manner, so that we can detect any deformations, which have taken place. It is essential to realize that the straightforward difference in the two sets of coordinates does not provide sufficient information to assess whether points have moved or not, since some consideration must be given to the accuracy with which the coordinates have been determined.

The aim of the analysis is to identify stable reference points in the network (if any), and detect single-point displacements, which will later be used to aid in the development of an appropriate deformation model. It is necessary to stress how crucial is the detection of outliers in each of the single epoch adjustments, since errors which escape detection are likely to be assessed as deformations later in the analysis. The first stage of a two-epoch analysis of an absolute network is to assess the stability of the reference points by assuming them to form a relative network and testing whether any points have moved. This may be achieved by carrying out a global congruency test. Similarly in analysing a relative network, the first step is usually to establish whether any group of points in the network has retained its shape between the two epochs, again by use of the global congruency test. If such a group can be identified, then these points may be used as a datum, thus providing an absolute network for the analysis of the other stations. If in either case no group of stable points can be identified, then the resulting relative network must be assessed only in terms of datum invariant criteria.

The test on the variance ratio examines the compatibility of the independent variance factors of the two epochs. The test can either be one-tailed or two-tailed, with the former, as shown below :

$$H_0 : \hat{\sigma}^2_{oi} = \hat{\sigma}^2_{oj} \quad \text{at significance level } \alpha \;;$$

$$H_a : \hat{\sigma}^2_{oi} > \hat{\sigma}^2_{oj} \text{ or } \hat{\sigma}^2_{oj} > \hat{\sigma}^2_{oi} \qquad [4]$$

where $\hat{\sigma}^2_{oi}$ and $\hat{\sigma}^2_{oj}$ are the estimated variance factors of epochs I and j. Let their respective degrees of freedom become df_i and df_j. The test statistic is in the form of a ratio of the variance factors:

$$T = \hat{\sigma}^2_{oj} \Big/ \hat{\sigma}^2_{oi} \sim F_{df_j, df_i} \qquad [5]$$

assuming j and I refer to the larger and smaller variance factor respectively. Their relevant degrees of freedom become df_i and df_j. The outcome of the one-tailed test on variance ratio is:

$T < F_{df_j, df_i, \alpha}$, test passes, accept H_0 or

$T \geq F_{df_j, df_i, \alpha}$, test failed, reject H_0

If H_0 accepted, indicating the two variance factors are statistically equivalent, the variance ratio test is passed and the pooled variance factor $\hat{\sigma}^2_o$ may be computed as:

$$\hat{\sigma}_o^2 = [(\hat{\sigma}_{oi}^2)(df_i) + (\hat{\sigma}_{oj}^2)(df_j)] \big/ df$$

where df = $df_i + df_j$ [6]

If H_0 reject, its indicates improper weighting of observations, and requires the examinations of observational data or the adjustment results. From the results of the two single-epoch adjustments it is possible to calculate the displacement $\hat{\boldsymbol{d}}$ and the associated covariance matrix $Q_{\hat{d}}$ from

$$\hat{\boldsymbol{d}} = \hat{\boldsymbol{x}}_2 - \hat{\boldsymbol{x}}_1 \qquad [7]$$

$$Q_{\hat{d}} = Q_{\hat{x}_2} + Q_{\hat{x}_1} \qquad [\ 8\ ]$$

(assuming $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$ are uncorrelated), and in addition the quadratic form given by

$$\Omega = \hat{d}^T \, Q_{\hat{d}}^{-1} \, \hat{d} \qquad [\ 9\ ]$$

The displacement vectors (equation 7) and its cofactor matrix (equation 8) need to be transform from minimum constraint datum to another datum definitions (i.e. either partial minimum trace of minimum trace datum):

$$d_1 = [I - G(G^T WG)^{-1} G^T W\} d = Sd \qquad [10]$$

$$Q_{d_1} = SQ_d S^T \qquad [11]$$

where; I = identity matrix, d = displacement vector , S = S-transformation matrix, W= weight matrix (with diogonal value of the one for datum points and zero elsewhere). The full components of matrix G^T for a 3-D network can be found in Halim & Ranjit, (2001) .

It is readily shown (Caspary, 1987) that a suitable test of the hypothesis that the points under consideration have remained stable, i.e. F(d) = 0, is

$$\boldsymbol{T} = \frac{\Omega}{\boldsymbol{h}\hat{\boldsymbol{\sigma}}_{\boldsymbol{o}}^2} \qquad [\ 12\ ]$$

where *h* is the rank of $Q_{\hat{d}}$ *(3n-d)* for a 3D network, $\hat{\boldsymbol{\sigma}}_{\boldsymbol{o}}^2 = (r_1 \hat{\boldsymbol{\sigma}}_{\boldsymbol{o}_1}^2 + r_2 \hat{\boldsymbol{\sigma}}_{\boldsymbol{o}_2}^2)/r$, $r = r_1 + r_2$, $\boldsymbol{r_i}$ = degrees of freedom in the adjustment of the i^{th} epoch.

T is tested against the Fisher distribution (F-test) with $F_{h,\ r}$, at an appropriately chosen level of significance. If the test is successful (the hypothesis is not rejected), then the two epochs are assumed congruent, i.e. the points involved have remained stable. If the test is unsuccessful, at least one point has moved, and must be removed from the group of reference points. Several methods exist for identifying which point (or points) should be removed. The simplest of these methods is to identify the point, which has the greatest contribution to Ω. This point is then eliminated from the reference group and the global congruency test repeated. The process is repeated until a stable group of points is identified.

Having determined, by means of the global congruency test, a group of points, which have remained, stable, it is now necessary to calculate coordinates for these stations, as well as for the other, unstable points. There are different solutions to this problem. Firstly, it would be possible to adopt the first epoch estimates for the stable group and use these in a computation of the second epoch observations. However, this is not sensible since the measurements between the stable points in the second set are being ignored. It is also not entirely reasonable to adopt the separate estimates x_1 and x_2, since this would result in stable points having changing coordinates. The most preferable solution is to carry out a *combined adjustment* of the observations from both epochs, with only one set of unknown coordinates being estimated for the stable group of points, and two (one for each epoch) being estimated for the moving points. In fact, the required solution may be obtained without actually carrying out the combined solution since the displacements and covariance matrix of the unstable points can be obtained directly from the information available from the single epoch solutions (Caspary, 1987). The difference in the resulting coordinates for the moving points, (namely the displacements), together with the associated covariance matrix, can then be used in an assessment of the significance of the detected movements.

The movement $\hat{\boldsymbol{d}}$ calculated for an unstable point can be tested for significance by comparing it with the appropriate elements $(Q_{\hat{d}_i})$ of the associated covariance matrix. The test statistic, which is most commonly used, is

$$T = \frac{\hat{d}_i^T Q_{\hat{d}_i}^{-1} \hat{d}_i}{\hat{\sigma}_o^2} \qquad [13]$$

and it is tested against the Fisher distribution $2F_{2,r}$ at the chosen level of significance. In a similar fashion to the computation of absolute point error ellipses it is possible to compute a point displacement ellipse by using the appropriate sub-matrix of $Q_{\hat{d}}$ in place of the sub-matrix of $Q_{\hat{x}}$. This ellipse may then be plotted along with the displacement vector for a graphical representation of the significance of the movement.

5.0 RESULTS AND ANALYSES [13]

The GPS network consist three references points, namely P314, P288 and DCA02. Their corresponding Cartesian coordinates are summarized Table 1.

Table 1: Cartesian coordinates for the reference stations

Station	X (m)	Y (m)	Z (m)
P 314	-1140193.1164	6246578.6211	598604.6758
P 288	-1143708.2813	6244748.2409	610775.9046
DCA02	-1132315.6122	6249421.9207	583780.3523

The results for the coordinates repeatability using the minimum constraint method for the fixed station P314, P288 and DCA02 shows the daily standard deviation for all points of the both epochs. Generally, the standard deviation of all object points is less than ± 6 mm for horizontal and ±4 mm for vertical component of the both epochs, respectively.

The results for the displacement calculations for both epochs are shown in Table 2 – 4, respectively. Table 2, shows the differences of all object points for Day 1 and Day 3 of the fixed station P314 and the differences of adjusted coordinates are within ± 1.5 cm in horizontal and ± 0.5 cm in vertical component. Figure 1 shows the different adjusted coordinates of the fixed station P314 for Day 1 and Day 3 of the both epoch.

Table 2: Different of adjusted coordinate for the fixed station P314

Station	1st epoch			2nd epoch		
	ΔX (m)	ΔY (m)	ΔZ (m)	ΔX (m)	ΔY (m)	ΔZ (m)
KT1	0.0032	0.0036	0.0001	-0.0011	-0.0031	0.0028
KT2	-0.0004	0.0117	-0.0052	-0.0026	-0.0009	-0.0021
KT3	0.0062	0.0153	-0.0026	0.0052	0.0103	-0.0015
KT4	0.0095	0.0019	0.0028	-0.0015	-0.0002	0.0080
KT5	-0.0015	0.0165	0.0036	0.0147	0.0246	0.0077
KT6	0.0051	-0.0059	0.0032	0.0227	0.0149	-0.0008

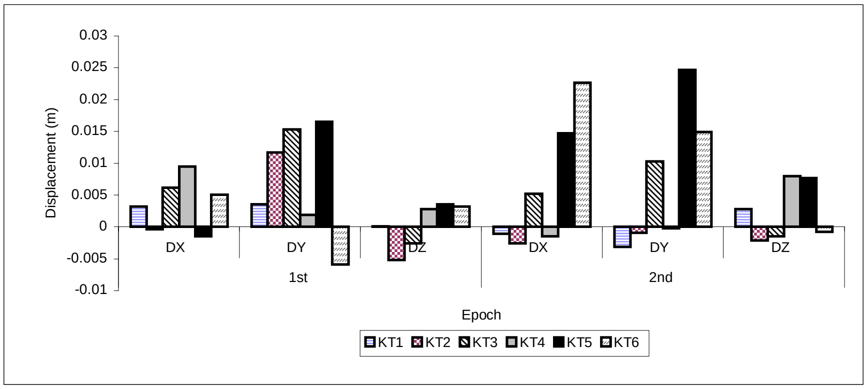

Figure 1: Different of adjusted coordinates for Day 3 and Day 1 of both epoch (fixed station P314)

Table 3: Different of adjusted coordinate for the fixed station P288

Station	**1st epoch**			**2nd epoch**		
	ΔX (m)	**ΔY (m)**	**ΔZ (m)**	**ΔX (m)**	**ΔY (m)**	**ΔZ (m)**
KT1	-0.0085	0.1154	0.2936	-0.0029	0.0152	0.0350
KT2	-0.0025	0.0076	-0.0107	0.0254	-0.0417	-0.0205
KT3	0.0484	0.6551	-0.5167	0.0405	0.0154	0.0142
KT4	-0.0417	0.0771	-0.0892	0.0344	0.0284	0.0007
KT5	0.1492	-0.3391	0.2423	-0.2561	0.6742	-0.3255
KT6	-0.2163	-0.1878	-0.0230	0.0010	-0.1279	-0.0132

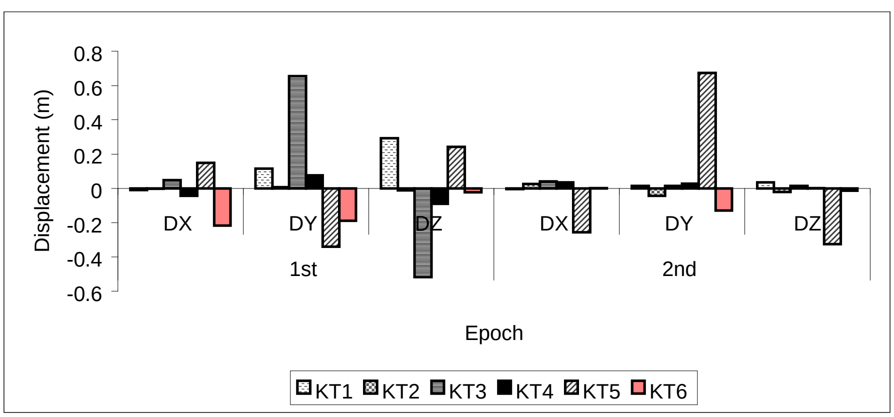

Figure 2: Different of adjusted coordinates for Day 2 and Day 3 of both epoch (fixed station P288)

Table 3, shows the differences of all object points for Day 2 and Day 3 of the fixed station P288. As been seen on Figure 2, station KT3 has a bigger value, where the different are varies within 65.50 cm and 4.80 cm in horizontal and –52.00 cm in vertical component and station KT5, has a different value within 67.4 cm and –25.61cm in horizontal and –32.55 cm in vertical component

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Table 4: Different of adjusted coordinate for the fixed station DCA02

Station	1st epoch			2nd epoch		
	ΔX (m)	**ΔY (m)**	**ΔZ (m)**	**ΔX (m)**	**ΔY (m)**	**ΔZ (m)**
KT1	-0.0039	-0.0345	-0.0041	0.0095	0.0426	0.0046
KT2	0.0186	-0.0565	0.0142	0.0184	0.0154	-0.0084
KT3	-0.0266	0.0088	0.0340	-0.0395	-0.0456	-0.0350
KT4	-0.0986	0.2368	0.0800	1.6426	0.3459	-0.2271
KT5	0.0990	-1.0610	0.1997	0.0146	-0.0054	-0.0283
KT6	0.0419	-1.0611	-0.5172	2.5478	1.6915	0.1475

Figure 3: Different of adjusted coordinates for Day 2 and Day 1 of both epoch (fixed station DCA02)

Table 4, shows the differences of all object points for Day 1 and Day 2 of the fixed station DCA02. As can been seen from the Figure 3, station KT5 and KT6 for the first epoch has the different within –106.10 cm and 90.90 cm in horizontal and –0.200 cm in vertical component. While at the second epoch station KT4 and KT6 has a bigger value within 34.59 cm to 254.78 cm in horizontal and –22.71 to 14.75 in vertical component. Overall result from the minimal constraint adjustment shows that the fixed station P288 and DCA02 has a bigger differences at the stations KT3, KT4, KT5 and KT6 than station P314, this is believe that it is due to the “noise” in observation data at the station.

The output for the two fixed points shows that Day 1 and Day 2 has a bigger the standard deviations value which is, vary from 0 mm to 170 mm in horizontal and 0 mm to 90 mm in vertical, while Day3 vary from 0 mm to 9 mm in horizontal and 0 mm to 6 mm in vertical component for both epoch. The results also shows that all residuals are accepted since the compute Tau-max-test value is smaller than value from the Tau-max table, 3.03 and the residuals are smaller than the standard deviation of the observations. The internal reliability is varies from 0.05 to 18.52 in horizontal and 0.05 to 13.59 in vertical component. While the external reliabilty also small which varies from 0 to 1.457 in horizontal and 0.717 in vertical element. The results show that the network has a good reliability. The confidence region, shows that all stations has a smallest value vary from 0.0448 m to 0.1005 m for the semi-major axis and 0.0312 m to 0.0995 m for the vertical. The average size of semi-major axis is 0.6853 m for Day1, 0.8459 m for Day2 and 0.0589 m for Day3. While, the vertical is 0.5851 m for Day1, 0.8393 m for Day2 and 0.0507 m for Day3. From the results it can be see that the Day3 of both epochs has a smallest confidence region for the network. It shown that the quality of the Day3 data is very good.

Comparison from two fixed stations adjustment also has been summarized in Table 6 – 8 and Figure 4 - 5. From Table 6, for the fixed stations P314 and DCA02, shows that the biggest value is at station KT5, whereby the value is 0.1652 m and –0.475 m for the horizontal and –0.2875 for the height. While, station KT2 has a smallest value of 0.0449 m and –0.0396 m for the horizontal and 0.0022 m for the vertical component (see Figure 4).

Table 6: Different of adjusted coordinate for the fixed stations P314 and DCA02

Station	ΔX (m)	ΔY (m)	ΔZ (m)
KT1	0.0799	0.0910	0.0501
KT2	-0.0396	0.0449	0.0022
KT3	-0.0823	0.0477	-0.0278
KT4	0.3386	0.0142	0.0679
KT5	-0.475	0.1652	-0.2875
KT6	0.7076	-0.2334	0.022

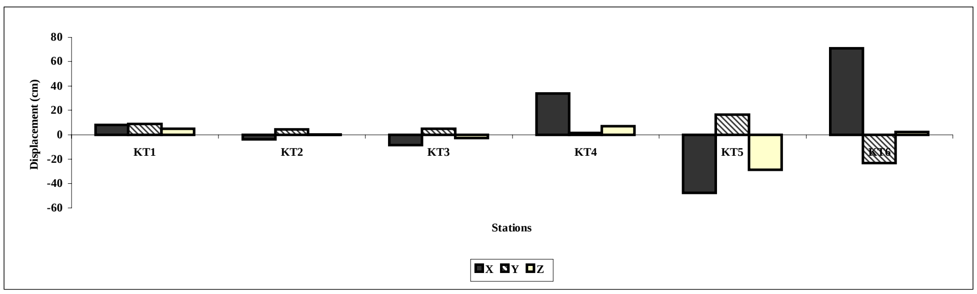

Figure 4: Different of adjusted coordinate for the fixed stations P314 and DCA02 for the Day 1 (Epoch 2 – Epoch 1)

As be seen in Table 7 and Figure 5, the difference of adjusted coordinate of the fixed stations P288 and DCA02 for all stations, varies from -0.7433 m to 0.8882 m in horizontal and –0.5196 to 0.1073 in vertical element. It believed that this due to the effect of some bias and noise at the reference points DCA02.

Table 7: Different of adjusted coordinate for the fixed stations P288 and DCA02

Station	ΔX (m)	ΔY (m)	ΔZ (m)
KT1	0.0286	0.8882	-0.0263
KT2	0.1134	-0.7433	0.1002
KT3	-0.7065	0.1951	-0.5196
KT4	-0.0289	-0.2960	0.1073
KT5	-0.043	-0.1941	0.074
KT6	-0.4286	-0.2423	0.0190

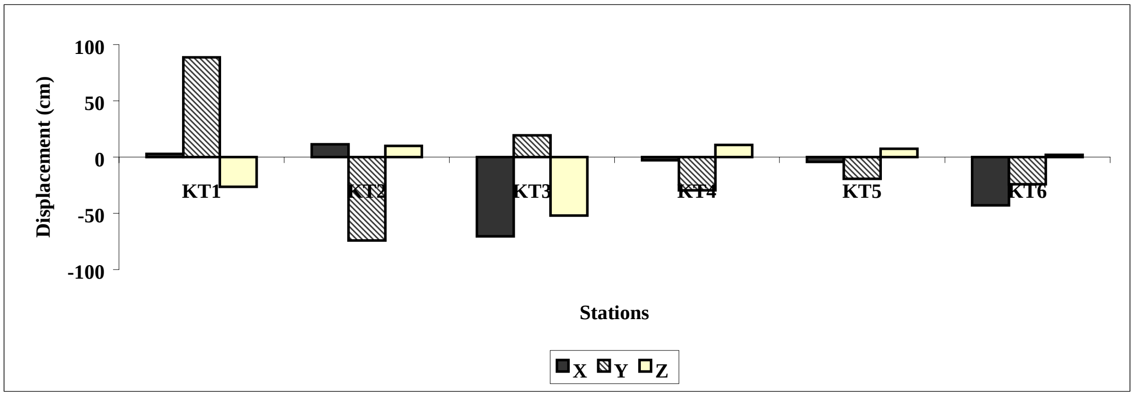

Figure 4.26: Different of adjusted coordinate for the fixed stations P288 and DCA02

Results from Table 8, shows that the differences of adjusted coordinates for the fixed stations P314 and P288 are small compared to the other fixed stations. The differences are varies from –0.0463 m to 0.0577 m in horizontal and –0.0394 m to –0.0025 m in vertical. The comparison shows that the data of Day3 has a good quality.

Table 8: Different of adjusted coordinate for the fixed stations P314 and P288

Station	ΔX (m)	ΔY (m)	ΔZ (m)
KT1	0.0013	0.0167	-0.0025
KT2	-0.0022	-0.0283	-0.0032
KT3	-0.00185	0.0320	-0.0114
KT4	-0.0035	0.0137	-0.0394
KT5	-0.0102	0.0577	-0.0106
KT6	-0.0088	-0.0463	-0.0289

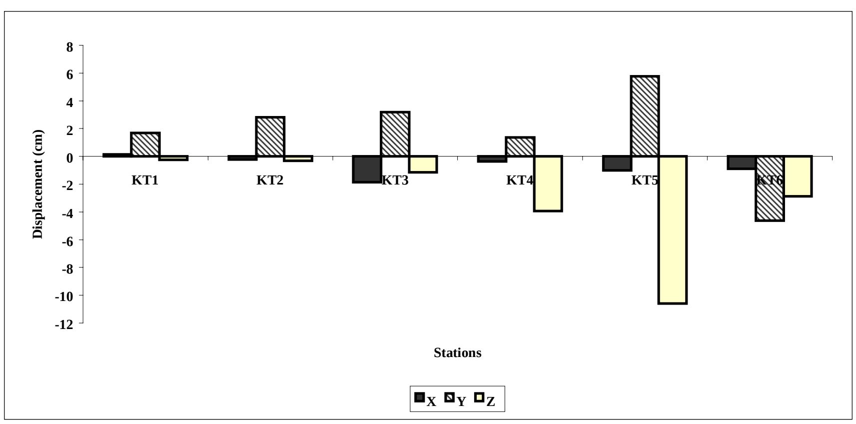

Figure 4.27: Different of adjusted coordinate for the fixed stations P314 and P288

In order to determine whether significant movements occurs between the two epochs for the GPS observation, a statistical test known as the congruency test is performed. The result of the stability determination with two fixed point shows that the variance ratio test at significance level 0.05 is pass for all three days, where the value is smaller than the critical value (1.204 < 2.405). All the datum points and object points has also passed the single points test at significance level 0.01 and the global congruency test at significance level 0.05 is also pass, for all three days, whereby all stations were confirmed stable.

6.0 CONCLUSION

The objective of this project was to develop methods and processing techniques (strategies) to be able to detect high rise building deformation (if any) over time at the required accuracy. The monitoring network is properly adjusted and analyses before the results are used in the deformation analysis. From the minimally constraint solution, its can be seen that the fixed station P314 has a good quality of observations compared to reference stations P288 and DCA02. The contraint least squares adjustment with two fixed points show that the data quality for Day1 and Day2 observations for both epochs have a lot of noise, compared to Day3 observations. A congruency test is also performed in this experiment. The corresponding results (with two fixed points) have shown that the variance ratio test at significance level 0.05 is passed for all observations. From the analysis, we can see that all reference and object points are in stable conditions, i.e. there was no movement of the KOMTAR building. Finally, the overall results from the GPS observations have shown that the contraints adjustment have give more information and analysis than the minimum constraints and therefore it is more appropriate to be adopted for engineering structure monitoring schemes.

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