

DEFORMATION ANALYSIS OF A GEODETIC MONITORING NETWORK

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Results of deformation analysis are directly relevant to the safety of human life. Therefore one has to be very careful in assessing the data of a monitoring network to avoid wrong interpretation of the displacements. This paper presents a deformation analysis procedure that consists of network adjustment of individual epochs, trend analysis of the displacement field and modelling of the deformation. During trend analysis, two robust methods and a non-robust method have been adopted and applied in determining the trend of movements for all the common points in a monitoring network. The trend of movements then form a basis for preliminary identification of the deformation models. The developed procedure has been implemented in a program package known as NETDEFAN (NETwork and DEformation ANalysis). A numerical example is also given by using known data of a dam monitoring network.

Introduction

The upper layers of the earth's crust are in constant motion both horizontally and vertically due to factors such as change of ground water level, tectonic phenomena, land slides, etc. Therefore any large man-made structures such as bridges, high rise buildings, dams, etc., which are built on the surface of the earth are subject to deformation. This deformation needs to be monitored continuously for safety assessment purpose.

Generally, the deformation measurement techniques can be divided into geotechnical, structural and geodetic methods. Geotechnical and structural methods are direct measurement methods, which use special equipment to measure changes in length, inclination, relative height, strain, etc. [Teskey and Porter 1988; Chrzanowski 1986]. On the other hand, in the geodetic method there are two basic types of geodetic monitoring networks; namely the reference (absolute) and relative networks [Chrzanowski et al. 1986]. In a reference network, some of the points or stations are assumed to be located outside of the deformable body or object, thus serving as reference points for the determination of the absolute displacements of the object points. However, in a relative network, all surveyed points are assumed to be located on the deformable body. This paper will focus only on the geodetic method using a reference network.

In a geodetic monitoring network, the object or area under investigation is usually represented by a number of points which are permanently monumented or marked. All the points are then observed in two or more epochs of time. The geodetic monitoring network can be either a conventional (terrestrial) network, a photogrammetry (i.e., aerial or close-range) network, Global Positioning System (GPS) network or a combination of these network types.

Deformation analysis using the geodetic method mainly consists of a two-step analysis via independent adjustment of the network of each epoch, followed by deformation detection between the two epochs. During deformation analysis it is important to determine the trend of movements (displacements) for all the common points in a monitoring network. The trend of movements then form a basis for preliminary identification of the deformation models during the modelling of deformation.

Although deformation analysis can be applied on one-dimensional (1-D), two-dimensional (2-D) and three-dimensional (3-D) monitoring networks, this paper will focus only on the 2-D network for the purpose of simplicity and easier understanding.

Network Adjustment

Deformation measurements may consist of a combination of observables such as distances, azimuths, coordinates, directions, coordinate differences, etc. The number of observations usually exceeds the minimum number required to determine the unknown quantities or parameters. In engineering surveying, these unknown quantities are the coordinates of the points. The redundant measurements are useful for checking gross errors or outliers in the measurements, precision of the unknown quantities and quality of the network. The method of least squares estimation (LSE) is an important tool in estimating the unknown parameters from redundant data. The result obtained by LSE is known as the best linear unbiased estimate (BLUE).

The functional model relating the measurements and parameters to be estimated can be expressed as

$$l = f(x) \quad (1)$$

where l is the vector of observations and x is the vector of parameters to be estimated.

In general, equation (1) is non-linear, and it needs to be linearized by using Taylor's theorem. After linearization the observation equation is written as

$$\hat{v} = A \hat{x} + b \quad (2)$$

where, \hat{v} is the vector of residuals, A is the design matrix, \hat{x} is the vector of corrections to the approximate values (x_o) and b is the misclosure vector.

In this study, the datum defect problem is dealt with by fixing a minimum number of parameters or coordinates (i.e., a minimum constraints datum) in order to define the geodetic datum. For a 2-D network there is a maximum of four datum parameters, i.e., two translations, one rotation and one scale. Therefore it is necessary to introduce four independent parameters (for example the coordinates of two points) in order to define the geodetic datum, hence leading to a full rank. However, some of the datum parameters can also be defined by certain observables, for example, the distance defines the scale and the azimuth defines the rotation of the network.

The normal equation with a full rank and the *a priori* variance factor (σ_o^2) is assumed to be known (i.e., $\sigma_o^2 = 1$), can then be written as

$$N \hat{x} + U = 0 \quad (3)$$

where;

$N = A^T W A$, coefficient matrix

$U = A^T W b$

$b = l - l_o$

l = vector of actual observations

l_o = vector of computed observations

$\hat{x} = -N^{-1}U = -(A^T W A)^{-1} A^T W b$

$\hat{x}_a = \hat{x} + x_o$, the updated parameters

$Q_{\hat{x}_a} = (A^T W A)^{-1}$, cofactor matrix of \hat{x}_a

$W = Q_l^{-1}$, the weight matrix

$\hat{\sigma}_o^2 = \frac{\hat{v}^T W \hat{v}}{n - u}$, *a posteriori* variance factor

n = number of observations

u = number of parameters

$Q_{\hat{v}} = W^{-1} - A N^{-1} A^T$, cofactor matrix of the residuals

$\hat{l}_a = l + \hat{v}$, adjusted observations

$Q_{\hat{l}_a} = A N^{-1} A^T$, cofactor matrix of adjusted observations

Although the estimated parameters, \hat{x}_a (i.e., coordinates of the points) and the cofactor matrix, $Q_{\hat{x}_a}$, are datum dependent based on the choice of zero-variance computational base, there exist functions such as \hat{v} , $Q_{\hat{v}}$, \hat{l}_a , $\hat{\sigma}_o^2$ and $Q_{\hat{l}_a}$, which are datum invariant.

During LSE, other important aspects that need to be considered are the global test (Chi-square), local test (Tau test or Baarda Method), precision, accuracy and reliability (internal and external) analysis. For further details on LSE the interested reader are referred to *Wolf and Ghilani* [1997], *Singh* [1997], *Caspary* [1987], *Cooper* [1987], *Singh* [1999], *Setan and Singh* [1999], and *Setan* [1995], to name a few.

Before deformation analysis can be carried out it is important to perform initial checking on the input data and test on the *a posteriori* variance factors of both epochs.

Initial Checking of Data and Test on Variance Ratio

Initial checking of data is important to ensure that common points, same approximate coordinates and same points names are used in the two epochs. If the number of points in the first epoch is not the same as in the second epoch, then deformation analysis is performed only on the existing common points in both epochs. This can be easily carried out by extracting the common points in both epochs.

The number of datum defects depends on the union of the datum defects in both epochs [Chen 1983; Chen *et al.* 1990]. For example, if the first epoch is a trilateration network which has three datum defects (i.e., two translations and one rotation), while the second epoch is a triangulation network with a datum defect of four (i.e., two translations, one rotation and one scale), then the union of datum defects is equal to the datum defects in the second epoch (i.e., four).

The *a posteriori* variance factors of both epochs are then tested for their compatibility. The null and alternative hypothesis of this test are [Setan 1995; Caspary 1987; Chen *et al.* 1990; Cooper 1987; Singh 1999]

$$H_o: \hat{\sigma}_{o1}^2 = \hat{\sigma}_{o2}^2 \quad (4)$$

and

$$H_a: \hat{\sigma}_{o1}^2 > \hat{\sigma}_{o2}^2 \quad \text{or} \quad \hat{\sigma}_{o2}^2 > \hat{\sigma}_{o1}^2 \quad (5)$$

with $\hat{\sigma}_{o1}^2$ and $\hat{\sigma}_{o2}^2$ being the *posteriori* variance factors for the first and second epochs respectively.

The test statistic is

$$T = \frac{\hat{\sigma}_{oj}^2}{\hat{\sigma}_{oi}^2} \sim F(\alpha, df_j, df_i) \quad (6)$$

with *j* and *i* represent the larger and smaller variance factors, *F* is the Fisher's distribution, α is the chosen significance level (typically $\alpha = 0.05$) and df_i and df_j are the degrees of freedom for epochs *i* and *j* respectively.

The above test is accepted if $T < F(\alpha, df_j, df_i)$ at a significance level α . The failure of the above test may be caused by incompatible weighting between the two epochs or incorrect weighting scheme, and any further analysis should be stopped at this stage.

Trend Analysis

After the test on the variance ratio (equation 6) is accepted, the displacement vector (coordinates differences) and its cofactor matrix can be computed as

$$d = \hat{x}_2 - \hat{x}_1 \quad (7)$$

$$Q_d = Q_{\hat{x}1} + Q_{\hat{x}2} \quad (8)$$

where, \hat{x}_1 and \hat{x}_2 are the estimated coordinates of all the common points in the first and second epochs respectively (with same datum definition), $Q_{\hat{x}1}$ and $Q_{\hat{x}2}$ are the cofactor matrix of the estimated coordinates \hat{x}_1 and \hat{x}_2 , *d* is the displacement vector and Q_d is the cofactor matrix of *d*.

In this study, two well-established rigorous methods known as the robust and congruency testing methods have been applied for estimating the trend of movements for all the common points in a monitoring network.

Robust Methods

Caspary and Borutta [1987] have given a detail explanation about three robust methods namely Danish, M-estimation (Huber) and Least Absolute Sum (LAS) method, for the estimation of the trend of movements. According to Caspary and Borutta [1987], the most robust is the Danish method, followed by LAS and M-estimation (Huber) method.

Chen [1983] has proposed a robust method known as an iterative weighted similarity transformation (IWST). This robust method was developed at the University of New Brunswick, Canada. This paper will focus on two robust methods via the LAS and IWST for estimating the trend of movements of a monitoring network. Such methods were chosen due to their wide application in deformation detection.

The LAS and IWST methods are based on S-transformation (similarity or Helmert transformation) as below [Chen 1983; Chen *et al.* 1990; Setan and Singh 1998b, c; Singh and Setan 1999a, b, c; Singh 1999; Setan and Singh 1999]

$$d^{(k+1)} = [I - G(G^T W^{(k)} G)^{-1} G^T W^{(k)}] d^{(k)} = S^{(k)} d^{(k)} \quad (9)$$

where

I = identity matrix
 k = number of iterations
 d = displacement vector (equation 7)
 S = S-transformations matrix
 W = weight matrix

$$G^T = \begin{bmatrix} 1 & 0 & 1 & 0 & \dots & 1 & 0 \\ 0 & 1 & 0 & 1 & \dots & 0 & 1 \\ y_1^o & -x_1^o & y_2^o & -x_2^o & \dots & y_m^o & -x_m^o \\ x_1^o & y_1^o & x_2^o & y_2^o & \dots & x_m^o & y_m^o \end{bmatrix} \quad (10)$$

where x_i^o , y_i^o are the coordinates of point P_i which are reduced to the centroid or center of gravity of the network, i.e.,

$$x_i^o = x_i - \frac{\left(\sum_{i=1}^m x_i\right)}{m} \quad (11)$$

$$y_i^o = y_i - \frac{\left(\sum_{i=1}^m y_i\right)}{m} \quad (12)$$

with x_i , y_i the approximate coordinates of point P_i and m is the number of common points in the network.

The first two rows of the inner constraint matrix (G^T) take care of the translations in the x and y directions, while the third row defines the rotation about the vertical (z) axis and the last row defines the scale of the network. For a trilateration network, the last row of G^T is omitted [Caspary 1987; Cooper and Cross 1991; Setan 1997; Chen et al. 1990; Singh 1999].

The main difference between the LAS and IWST methods is in forming the weight matrix, W . In the first transformation ($k = 1$) the weight matrix is taken as identity ($W^{(k)} = I$) for all the common points, then in the ($k+1$) transformation the weight matrix is defined as

For the IWST method

$$W^{(k)} = \text{diag} \left\{ \frac{1}{|d_i^{(k)}|} \right\} \quad (13)$$

For the LAS method

$$W^{(k)} = \text{diag} \left\{ \frac{1}{\sqrt{(dx_i^{(k)})^2 + (dy_i^{(k)})^2}} \right\} \quad (14)$$

In equation (13), d is the displacement vector as in equations (7) and (9). However, in equation (14), dx_i and dy_i refer to the displacement components in x and y axes respectively.

It is important to mention that the above weighting schemes (equations 13 and 14) are only applied on the common datum points (i.e., either the reference points for a reference network or a group of points in a stable block for a relative network), whilst for the object points the weight is set as zero (i.e., $W^{(k)} = 0$).

The iterative procedure continues until the absolute differences between the successive transformed displacements of all the common points (i.e., $|d^{(k+1)} - d^{(k)}|$) are smaller than a tolerance value δ (say 0.0001 [m]).

It is possible that during the iterations some $d_i^{(k)}$ (or dx_i , dy_i) may approach zero, causing numerical instabilities, because $W^{(k)}$ (equations 13 and 14) becomes very large. There are two ways to solve this problem, either

- (i) Setting a lower bound value (e.g., 0.0001 [m]). If $d_i^{(k)}$ is smaller than the lower bound value, its weight is set to zero, or
- (ii) Replacing equations 13 and 14 as

$$W^{(k)} = \text{diag} \left\{ \frac{1}{|d_i^{(k)}| + \delta} \right\} \quad (15)$$

$$W^{(k)} = \text{diag} \left\{ \frac{1}{\sqrt{(dx_i^{(k)} + \delta)^2 + (dy_i^{(k)} + \delta)^2}} \right\} \quad (16)$$

In equation (16), the tolerance value is only added to those displacement components (either dx_i or dy_i) which are smaller than its lower bound value. In this study, solution (i) was applied to the IWST method, while solution (ii) was applied to the LAS method. This technique was found to be useful in avoiding numerical instabilities.

In this procedure, the LAS method minimizes the sum of the lengths of the displacements (i.e., $\sum \sqrt{(dx_i)^2 + (dy_i)^2} \Rightarrow$

minimum), while the IWST method minimizes the total sum of absolute values of the displacement components (i.e., $\sum |d_i| \Rightarrow$ minimum).

In the final iteration the cofactor matrix of the displacement vector is computed as

$$Q_d^{(k+1)} = S^{(k)} Q_d (S^{(k)})^T \quad (17)$$

The stability information of each common point j can then be determined through a single point test as below [Setan 1995; Setan and Singh 1998c]

$$T_j = \frac{(d_j^{(k+1)})^T (Q_{dj}^{(k+1)})^{-1} d_j^{(k+1)}}{2\hat{\sigma}_o^2} \sim F(\alpha, 2, df) \quad (18)$$

where;

d_j , Q_{dj} = displacement vector and its cofactor matrix respectively for each common point j

$$\hat{\sigma}_o^2 = \frac{[df_1(\hat{\sigma}_{o1}^2) + df_2(\hat{\sigma}_{o2}^2)]}{df}, \quad \text{common}$$

or pooled variance factor

$\hat{\sigma}_{o1}^2$, $\hat{\sigma}_{o2}^2$ = *a posteriori* variance factors of epochs 1 and 2 respectively

df_1 , df_2 = degrees of freedom of epochs 1 and 2

$df = df_1 + df_2$, sum of degrees of freedom of epochs 1 and 2

α = significance level (usually chosen as 0.05)

If the above test passes (i.e., $T_j < F(\alpha, 2, df)$) then the point is assumed to be stable at a significance level α . Otherwise, if the test fails (i.e., $T_j \geq F(\alpha, 2, df)$) then the point is assumed to be deformed (moved).

For further reading on robust methods, Caspary [1987], Caspary and Borutta [1987], Chen [1983], Chen *et al.* [1990], Singh [1999], and Setan and Singh [1998c] are recommended.

Congruency Testing Method

Another rigorous method known as congruency testing has been applied in estimating the displacements of all common points in a monitoring network. As opposed to

the robust methods, congruency testing will iteratively remove one datum point at a time until the congruency test passes.

Basically the adopted procedure of congruency testing consists of the following steps [Setan 1997]

- (1) Transformation of displacement vector (d) and its cofactor matrix (Q_d) respectively, for both epochs into a common datum.
- (2) Determination of stable datum points by congruency testing [Fraser and Gruendig 1985].
- (3) Localization of deformation through single point test, S-transformation and congruency testing.
- (4) Final testing of deformation by a single point test.

Transformation of Both Epochs into a Common Datum

During deformation analysis by the congruency testing (and robust) methods, it is important that the displacement vector, d (equation 7) and its cofactor matrix, Q_d (equation 8) are referred to the same datum or computational base. The initial datum could be either the reference points in a reference monitoring network, or a group of selected points based on *a priori* information in a relative monitoring network.

In this study, the S-transformation has been applied to transform matrix d and Q_d into a common datum definition (either minimum trace or partial minimum trace solutions) [Caspary 1987; Cooper 1987; Fraser and Gruendig 1985; Biacs and Teskey 1990]

$$d_1 = Sd \quad (19)$$

$$Q_{d_1} = S Q_d S^T \quad (20)$$

$$S = I - G(G^T W G)^{-1} G^T W \quad (21)$$

where d_1 and Q_{d_1} are the displacement vector and its cofactor matrix respectively based on the new datum or computational base, G is the inner constraints matrix constructed depending on the union of the datum defects in the two epochs and on the number of common points, and W is the weight matrix with diagonal value of one for datum points and zero elsewhere. Matrix S is symmetric only for the minimum trace solutions

(i.e., all points in the network were defined as datum).

The group of selected datum points is then tested for its stability by using congruency testing.

Congruency Testing on Initial Selected Datum Points

Congruency testing is performed to determine whether a group of selected datum points have significantly moved between the two epochs.

The null and alternative hypothesis for congruency testing are [Setan 1995; Singh 1997; Cooper 1987; Fraser and Gruendig 1985; Biacs and Teskey 1990; Singh 1999]

$$H_0 : E(d'_1) = 0 \quad (22)$$

(no significant deformation for a group of datum points)

and

$$H_a : E(d'_1) \neq 0 \quad (23)$$

(existence of deformation for a group of datum points)

The test statistic is datum independent, i.e.,

$$\omega = \frac{d_1^T Q_{d_1}^{-1} d_1}{h \hat{\sigma}_o^2} \sim F(\alpha, h, df) \quad (24)$$

where

d'_1 , Q_{d_1} = displacement vector and its cofactor matrix of the datum points common in both epochs

$$h = \text{rank}(Q_{d_1})$$

= $(2n-d)$ for a 2-D network with n number of common datum points and d number of datum defects

$\hat{\sigma}_o^2$ = common variance factor

$$Q_{d_1}^{-1} = (Q_{d_1} + G_r G_r^T)^{-1} \cdot G_r (G_r^T G_r G_r^T G_r)^{-1} G_r^T,$$

pseudo inverse

α = significance level, typically $\alpha = 0.05$

The inner constraints matrix G_r is constructed depending on the number of

common datum points and the union of the datum defects in the two epochs. Generally, matrix Q_{d_1} is singular, thus a pseudo inverse is used for the computation of ω .

The null hypothesis is accepted at the α level of significance if the test statistic (equation 24) does not exceeds the critical value of the F -distribution (i.e., $\omega < F(\alpha, h, df)$). The rejection of the null hypothesis indicates the existence of deformation in the group of selected datum points (i.e., accepting the alternative hypothesis). The failure of the test leads to localization of the deformation in locating the datum point whose displacements caused the network to change in shape. The network is then transformed into a new computational base. The localization procedure continues until all the remaining group of datum points were being verified as stable by the congruency test.

Localization of Deformation

If the congruency test (equation 24) is rejected, localization of deformation is then performed. In this study, the developed procedure of localization of deformation consists of the following steps, i.e., the single point test on the datum points, S-transformation into a new computational base and the congruency testing on the remaining common datum points [Setan 1995].

(i) Single Point Test on the Datum Points

The purpose of the single point test is to locate the datum point with the largest statistical value (or unstable datum point), and these point will be eliminated from the computational base or datum.

The single point test for a 2-D network, i.e., neglecting the correlation between the datum points, is [Singh 1997; Setan 1995]

$$T_j = \frac{d_{1j}^T Q_{d_{1j}}^{-1} d_{1j}}{2 \hat{\sigma}_o^2} \quad (25)$$

where d_{1j} and $Q_{d_{1j}}$ are the displacement vector and its cofactor matrix of each datum point j , and $\hat{\sigma}_o^2$ is the pooled variance factor (equation 18).

Instead of the single point test (equation 25), one can also apply the decomposition of the

quadratic form in order to determine the significantly unstable datum point. More details are given in Caspary [1987], Setan [1995], Cooper [1987], and Fraser and Gruendig [1985].

The datum point with the largest T_j is assumed to have caused the change of shape in the network, and it will be eliminated from the computational base. The network is then transformed into a new computational base which is defined by the remaining datum points.

(ii) Transformation of the Network into a New Computational Base

An S-transformation is then carried out to transform d_1 and Q_{d_1} into a new computational base, hence equations 19 to 21 can be written as

$$d_2 = S d_1 \quad (26)$$

$$Q_{d_2} = S Q_{d_1} S^T \quad (27)$$

$$S = I - G(G^T W G)^{-1} G^T W \quad (28)$$

where d_2 and Q_{d_2} are the displacement vector and its cofactor matrix referring to the new datum definition and W is the weight matrix (with diagonal value of one for the remaining datum points and zero elsewhere).

The "retained" datum points are then tested for stability by congruency testing.

(iii) Congruency Testing on a Group of Remaining Datum Points

The statistical test is the same as in equation 24, except that this test is performed only on the remaining datum points. The hypotheses for this test are

$$H_o : E(d'_2) = 0 \quad (29)$$

(no significant deformation for a group of remaining datum points),

and

$$H_a : E(d'_2) \neq 0 \quad (30)$$

(existence of deformation for a group of remaining datum points)

The test statistic then becomes (refer equation 24)

$$\omega = \frac{d_2'^T Q_{d_2}^{-1} d_2'}{(h-2k)\hat{\sigma}_o^2} \sim F(\alpha, h-2k, df) \quad (31)$$

where d_2' and Q_{d_2} are the displacement vector and its cofactor matrix of the remaining datum points, and k is the number of points removed from the datum definition.

If the null hypothesis is rejected (i.e., $\omega \geq F(\alpha, h-2k, df)$), then the localization process will be repeated until the null hypothesis is accepted. Otherwise, if the null hypothesis is accepted (i.e., $\omega < F(\alpha, h-2k, df)$) then the stability information of all the common points in the network is determined through the final testing of deformation.

Final Testing of Deformation by Single Point Test

After the congruency test (equation 31) passes, the single point test is then carried out on all the common points in the network at a significance level α .

The null and alternative hypothesis for this test are

$$H_o : d_{2j} = [dx_{2j} \ dy_{2j}]^T = 0 \quad (32)$$

(no deformation for each common point in the network),

and

$$H_a : d_{2j} = [dx_{2j} \ dy_{2j}]^T \neq 0 \quad (33)$$

(existence of deformation for each common point in the network)

The test statistic (refer equations 18 and 25) is

$$T_j = \frac{d_{2j}^T Q_{d_{2j}}^{-1} d_{2j}}{2\hat{\sigma}_o^2} \sim F(\alpha, 2, df) \quad (34)$$

If the above test passes (i.e., $T_j < F(\alpha, 2, df)$) at a significance level α (typically $\alpha = 0.05$), then the point j is considered as stable (i.e., the vector of displacements lies within its confidence region). Otherwise, the rejection of the above test indicates that the point j is significantly unstable (i.e., the vector of displacements lies outside its confidence region).

All the final datum points should be verified as stable by the single point test. If there still exist any unstable datum points, then the datum point with the largest statistical value (equation 34) should be eliminated from the datum, and the localization process has to be repeated.

For a better interpretation of the displacements of the points, a graphic presentation in the form of displacement vector and error ellipses is used. This graphic presentation is useful in verifying the results of the single point test. Generally, both the graphic presentation and the single point test should provide identical results.

One can see that the formulations and procedures involved for the congruency testing method are much more complex compared to the robust methods. The robust and congruency testing methods are important tools for determining the trend of movements of all the common points in the network, which form a basis for preliminary identification of the deformation models during deformation modelling.

Modelling of Deformation

In this study the procedure for modelling of deformation is based on the well known generalized method developed at the University of New Brunswick, Canada [Chen 1983]. Deformation modelling can be carried out by using either the observation differences approach or displacement (coordinate differences) approach. This paper will focus only on the displacement approach. However, the interested reader can refer to Chen [1983] for a detail explanation of the two approaches.

The generalized method consists of three basic processes

- (1) Preliminary identification of the deformation model
- (2) Estimation of the deformation parameters
- (3) Diagnostic checking of the deformation models and the final selection of the "best" model

Preliminary Identification of the Deformation Model

The identification of the deformation models is based on *a priori* information or on trend analysis of the displacements. In a 2-D analysis the following deformation parameters must be considered

- (i) Two components of the rigid body displacement (a_o and b_o)
- (ii) A rotation parameter in xy axes ($\omega(x,y)$)
- (iii) Two normal strain components in x and y axes ($\epsilon_x(x,y)$ and $\epsilon_y(x,y)$) and shearing strain in xy axes ($\epsilon_{xy}(x,y)$)

The deformation of a block is fully described if a displacement function $d(x,y)$ is given for the whole block. In a general case the displacement function is determined through a polynomial approximation of the displacement field as

$$\begin{aligned} dx &= a_o + a_1x + a_2y + a_3xy + a_4x^2 + \dots \quad (35) \\ dy &= b_o + b_1x + b_2y + b_3xy + b_4x^2 + \dots \end{aligned}$$

where dx and dy are the displacements in the x and y axes, and x and y are the coordinates of the common points (i.e., either approximate or estimated coordinates).

However, in this study only the linear model has been taken into consideration and equation (35) can be further simplified as

$$\begin{aligned} dx &= a_o + a_1x + a_2y \quad (36) \\ dy &= b_o + b_1x + b_2y \end{aligned}$$

or

$$\begin{aligned} dx &= a_o + \epsilon_x x + \epsilon_{xy} y - \omega y \quad (37) \\ dy &= b_o + \epsilon_{xy} x + \epsilon_y y + \omega x \end{aligned}$$

Equation (37) can be written in matrix form as (ignoring rotation parameter ω)

$$d = B \hat{c} \quad (38)$$

or

$$\begin{bmatrix} dx_i \\ dy_i \end{bmatrix} = \begin{bmatrix} 1 & 0 & x_i & 0 & y_i \\ 0 & 1 & 0 & y_i & x_i \end{bmatrix} \begin{bmatrix} a_o \\ b_o \\ \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \end{bmatrix} \quad (39)$$

where d is the vector of displacements (equation 7), B is the design matrix and \hat{c} is the vector of

unknown deformation parameters (or coefficients of the polynomials). Equation (38) is well known as the deformation model.

Some examples of the typical deformation models can be found in *Chrzanowski et al.* [1983], *Chen* [1983], and *Chrzanowski et al.* [1986]. In this study, six typical deformation models have been adopted and applied i.e.,

- (1) No global deformation ($dx = 0$, $dy = 0$)
- (2) Stable points ($dx_i = 0$, $dy_i = 0$)
- (3) Single point movement ($dx_i = a_i$, $dy_i = b_i$)
- (4) Rigid body movement ($dx = a_0$, $dy = b_0$)
- (5) Homogeneous strain ($dx = \epsilon_x x + \epsilon_{xy} y$, $dy = \epsilon_{xy} x + \epsilon_y y$)
- (6) Rigid body plus homogeneous strain ($dx = a_0 + \epsilon_x x + \epsilon_{xy} y$, $dy = b_0 + \epsilon_{xy} x + \epsilon_y y$).

Estimation of the Deformation Parameters

The deformation model equation (38) is based on the Gauss-Markov model which can be written as

$$d + \hat{v} = B \hat{c} \quad (40)$$

where \hat{v} is vector of residuals.

In order to determine the vector \hat{c} , the number of known displacements of the common points must be at least equal to the number of unknown deformation parameters in the deformation model. If the number of known displacements d is larger than that of the unknown deformation parameters, then the vector \hat{c} is determined through LSE.

The solution is then obtained from

$$\hat{c} = (B^T P_d B)^{-1} B^T P_d d \quad (41)$$

where P_d is the weight matrix and is calculated as [*Chen et al.* 1990; *Chen* 1983; *Singh and Setan* 1999b, c; *Singh and Setan* 1998; *Singh* 1999]

- (i) for all the common points in the network:

$$P_d = [S Q_d S^T + G(G^T G)^{-1} G^T]^{-1} G(G^T G)^{-1} G^T \quad (42)$$

or

$$P_d = N_1(N_1 + N_2 + G G^T)^{-1} N_2 \quad (43)$$

with

$$S = I - G(G^T G)^{-1} G^T \quad (44)$$

where, Q_d is given in equation (8) and N_1 and N_2 are the coefficient matrix (i.e., full matrix) for epochs 1 and 2 respectively. Matrix N_1 , N_2 , G and Q_d are formed with respect to all the common points in the network.

- (ii) for a portion of common points in the network:

$$P_d = [S_r Q_{d_r} S_r^T + G_r(G_r^T G_r)^{-1} G_r^T]^{-1} G_r(G_r^T G_r)^{-1} G_r^T \quad (45)$$

or

$$P_d = N_{1r}(N_{1r} + N_{2r} + G_r G_r^T)^{-1} N_{2r} \quad (46)$$

with

$$S_r = I - G_r(G_r^T G_r)^{-1} G_r^T \quad (47)$$

$$N_1 = \begin{bmatrix} N'_{11} & N'_{12} \\ N'_{21} & N'_{22} \end{bmatrix} \quad (48)$$

$$N_2 = \begin{bmatrix} N''_{11} & N''_{12} \\ N''_{21} & N''_{22} \end{bmatrix} \quad (49)$$

$$N_{1r} = N'_{22} - N'_{21} N'^{-1}_{11} N'_{12} \quad (50)$$

$$N_{2r} = N''_{22} - N''_{21} N''^{-1}_{11} N''_{12} \quad (51)$$

where matrix S_r , Q_{d_r} , G_r , N'_{11} and N''_{11} are formed only for a portion of common points.

The cofactor matrix of \hat{c} is

$$Q_{\hat{c}} = (B^T P_d B)^{-1} \quad (52)$$

The procedure for calculation as shown above should be followed in order to avoid any numerical problems. According to *Chen* [1983] the rotation parameter ω can be omitted (considered as being zero) from the deformation model (equation 40) if the weight matrix P_d is calculated as in equations (42) or (43) and (45) or (46). Only in this case the omission of ω is

justified when no external orientation of the network is included in the observables. The rotation parameter ω only plays the role of a additional or nuisance parameter without any practical meaning. More details are given in Chen [1983].

Diagnostic Checking of the Deformation Model and Selection of the "Best" Model

The global appropriateness of the deformation model can be tested using the quadratic form of the residuals \hat{v} . The global test is based on the following hypotheses

$$H_o : \hat{\sigma}_{oc}^2 = \hat{\sigma}_o^2 \quad (53)$$

and

$$H_a : \hat{\sigma}_{oc}^2 \neq \hat{\sigma}_o^2 \quad (54)$$

where $\hat{\sigma}_o^2$ is the common variance factor and $\hat{\sigma}_{oc}^2$ is calculated as [Chen 1983; Chrzanowski et al. 1983; Kuang 1996; Secord 1985; Singh and Setan 1998; Singh and Setan 1999b, c; Singh 1999]

$$\hat{\sigma}_{oc}^2 = \frac{\hat{v}^T P_d \hat{v}}{df_c} \quad (55)$$

where

$\hat{v} = B\hat{c} - d$, vector of residuals, equation (40)

P_d = weight matrix (equation 17)

$df_c = 2n - d - m_c$, with n number of points in forming matrix P_d , d is number of datum defects and m_c is number of parameters to be estimated, \hat{c} .

The statistic then becomes

$$T = \frac{\hat{\sigma}_{oc}^2}{\hat{\sigma}_o^2} = \frac{\hat{v}^T P_d \hat{v}}{df_c \hat{\sigma}_o^2} \sim F(\alpha, df_c, df) \quad (56)$$

The test passes or the null hypothesis is accepted if the statistic does not exceed the critical value (i.e., $T \leq F(\alpha, df_c, df)$) at a significance level α (typically $\alpha = 0.05$).

Otherwise the null hypothesis is rejected if the statistic exceeds the critical value (i.e., $T > F(\alpha, df_c, df)$).

The deformation model is acceptable if the null hypothesis is not rejected at a significance level α .

The significance of the individual parameter \hat{c}_i or a group of parameters \hat{c}'_i which is a subset of \hat{c} (equation 41), is given by the following hypotheses [Chen 1983; Kuang 1996; Singh 1999]

$$H_o : \hat{c}_i = 0 \text{ and } H_a : \hat{c}_i \neq 0 \text{ (individual parameter)} \quad (57)$$

$$H_o : \hat{c}'_i = 0 \text{ and } H_a : \hat{c}'_i \neq 0 \text{ (a group of parameters)} \quad (58)$$

The local test becomes [Chen 1983; Kuang 1996; Chrzanowski et al. 1983; Chrzanowski et al. 1986; Secord 1985; Singh and Setan 1999b; Singh 1999]

$$T_i = \frac{\hat{c}_i^2}{\hat{\sigma}_o^2 q_{\hat{c}_i}} \sim F(\alpha, 1, df) \text{ (individual parameter)} \quad (59)$$

$$T_g = \frac{\hat{c}_i^T Q_{\hat{c}_i} \hat{c}_i}{\hat{\sigma}_o^2 u_i} \sim F(\alpha, u_i, df) \text{ (a group of parameters)} \quad (60)$$

where

$q_{\hat{c}_i}$ = the i th diagonal element of matrix $Q_{\hat{c}}$

\hat{c}_i = individual parameter

$\hat{\sigma}_o^2$ = common variance factor

df = sum of degrees of freedom of epoch 1 and epoch 2

\hat{c}'_i = subset of \hat{c}

u_i = number of parameters in \hat{c}'_i

$Q_{\hat{c}'_i}$ = submatrix of $Q_{\hat{c}}$

α = significance level, usually $\alpha = 0.05$.

The null hypothesis (equations 57 or 58) will be rejected at a significance level α if

$$T_i > F(\alpha, 1, df) \text{ or } T_g > F(\alpha, u_i, df) \quad (61)$$

The tested parameters are expected to be statistically significant. If the parameters are not significant, then a new deformation model needs to be selected and re-evaluated [Kuang 1996]. This process is repeated until the global test passes and all the parameters are statistically significant.

Since the behavior of the deformable body is usually not completely known, there is often more than one possible model that may be appropriate. The “best” model is selected based on the following criteria

- (i) The model passes the global test and all the parameters involved are statistically significant
- (ii) If more than one model satisfies criteria (i), the model with the fewest parameters is then selected
- (iii) On the basis of the estimated trend of movements or a priori or other information (makes “mechanical” sense)

Program Package NETDEFAN

The adopted procedure (i.e., network adjustment, trend analysis and modelling of deformation) has been implemented in a program package known as NETDEFAN (NETwork and DEformation ANalysis) developed at the Center for Industrial Measurement and Engineering Surveying, Universiti Teknologi Malaysia [Singh 1999]. NETDEFAN is developed using FORTRAN77, and can handle only 2-D monitoring networks. The program package consists of three modules known as COMPUT2, DEFORM2 and STRANS2 [Setan and Singh 1997; Setan and Singh 1998a; Singh 1997].

Program COMPUT2 is designed for the network adjustment of each epoch, with three main options, i.e.,

- (i) LSE of each epoch
- (ii) Pre-analysis
- (iii) Variance component estimation

Program DEFORM2 is designed for deformation analysis, and consists of seven main options, i.e.,

- (i) Trend analysis by congruency testing method
- (ii) Trend analysis by IWST method
- (iii) Trend analysis by LAS method

- (iv) Trend analysis by direct coordinate differences [Singh and Setan 1999a]
- (v) Sensitivity analysis for estimating the minimum detectable deformation of a monitoring network
- (vi) Modelling of deformation
- (vii) Strain analysis

Only option (i) of program COMPUT2 and options (i), (ii), (iii) and (vi) of program DEFORM2 are mentioned in this paper.

Program STRANS2 is an additional program for S-transformations of LSE results of a single epoch from one datum definition to another.

The input for program COMPUT2 are the observations and the approximate coordinates files of each epoch. Program COMPUT2 produces LSE summary files, deformation files and plot files for epochs 1 and 2. The deformation file of each epoch contains information of *a posteriori* variance factor, degrees of freedom, number of datum defects, types of datum defects, approximate coordinates, estimated coordinates and the lower triangle of the cofactor matrix of the estimated coordinates.

The deformation files of epoch 1 and 2 are then used as input for program DEFORM2 for deformation analysis. The deformation file for each epoch is also used as input for program STRANS2 for transformation of LSE results into a new datum definition.

All the above programs provide plot files which are used as input for a special program called Graphical Presentation for Deformation Survey Version 2 (GPDSV2), for graphical display of the displacement vectors and their standard ellipses respectively [Singh 1999].

Deformation Analysis of a Dam Monitoring Network

A known data of a dam monitoring network (refer Figure 1), taken from Caspary [1987], have been used to test the developed procedure and the program package.

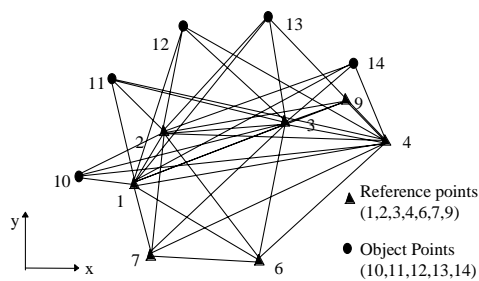


Figure 1: A dam monitoring network [Caspary 1987].

The monitoring network consists of two epochs of observations with 12 points i.e., 7 reference points (1, 2, 3, 4, 6, 7 and 9) and 5 object points (10, 11, 12, 13 and 14). Points 10 through 14 are located on the crest of the dam thus represent the object under investigation. Each epoch consists of 49 directions (standard error of 1 second) and 6 horizontal distances (standard error of 0.3 [mm]). Simulated deformation were given to points 3, 11, 12 and 13 as shown in Table 1.

Table 1: Simulated deformation [Caspary 1987].

Point	dx (mm)	dy (mm)
3	-0.60	-0.50
11	-0.75	0.60
12	0.50	1.10
13	0.30	1.00

Adjustment of the Network

Network adjustment of each epoch is carried out by using program COMPUT2 by fixing coordinates x_1 , y_1 and x_2 (minimum constraints solution) and leading to 29 degrees of freedom and 26 parameters (i.e., 21 coordinates and 5 orientation parameters). The significance levels for the Chi-square and Tau tests were chosen as 0.05. The criteria for convergence was set to 0.0001 [m], and both epochs converged at the second iteration.

The estimated variance factors were 0.829 and 0.937 for the first and second epoch respectively. The combined variance factor is 0.883. The LSE results of each epoch passed both the global and local tests. The average redundancy number is 0.53 for both epochs, thus indicating that the network possesses a high degree of reliability.

Trend Analysis of the Displacement Field

The trend of movements of the monitoring network are determined by using program DEFORM2 with three solutions as below

- Trend analysis by the congruency testing method with points 1, 2, 3, 4, 6, 7 and 9 defined as a datum
- Trend analysis by IWST method with points 1, 2, 3, 4, 6, 7 and 9 defined as a datum
- Trend analysis by LAS method with points 1, 2, 3, 4, 6, 7 and 9 defined as a datum.

The significance level for deformation detection is specified as 0.05. The tolerance value and the lower bound value were taken as 0.0001 [m] and 0.000001 [m] respectively. The test on the variance ratio passes at 0.05 significance level (i.e., $1.130 < 1.861$), thus indicating the compatibility between the two epochs.

The congruency test (equation 24) failed for solution (i), thus confirming the existence of deformation for a group of selected datum points. The localization process then removes unstable datum points from the computational base one at a time until the congruency test passes. This procedure removed point 3 and 6, resulted in 5 datum points (i.e., 1, 2, 4, 7 and 9) for final computation of the displacements. All datum points passed the single point test (equation 34) and were confirmed as stable. Solution (i) verified points 3, 6, 11, 12 and 13 as significantly deformed and the rest of the points as stable.

Solutions (ii) and (iii) converged at the second iteration. Solution (ii) verified points 3, 12 and 13 as moved and the rest of the points as stable. While solution (iii) verified points 3, 6, 12 and 13 as moved and the rest of the points as stable. The coordinate differences obtained for the three solutions are shown in Table 2.

Table 2: Estimated displacements for solutions (i) to (iii)
(* Point verified as moved).

point	solution (i)			solution (ii)			solution (iii)		
	dx (mm)	dy (mm)	disp. vect. (mm)	dx (mm)	dy (mm)	disp. Vect. (mm)	dx (mm)	dy (mm)	disp. vect. (mm)
1	0.1	0.1	0.2	0.3	0.0	0.3	0.3	0.1	0.3
2	0.0	0.0	0.0	0.2	-0.1	0.2	0.2	0.0	0.2
3	-0.9	-0.5	1.0	-0.7	-0.6	0.9	-0.7	-0.5	0.9
4	-0.2	0.0	0.2	0.0	0.0	0.0	0.0	0.0	0.0
6	-0.5	0.2	0.5	-0.3	0.1	0.3	-0.3	0.2	0.4
7	0.2	0.0	0.2	0.4	-0.1	0.4	0.4	0.0	0.4
9	-0.2	-0.2	0.3	0.0	-0.2	0.2	-0.1	-0.1	0.1
10	0.1	0.1	0.1	0.3	0.0	0.3	0.2	0.1	0.3
11	-0.7	0.4	0.8	-0.5	0.3	0.6	-0.5	0.4	0.7
12	0.5	1.3	1.4	0.6	1.3	1.4	0.6	1.3	1.5
13	0.1	1.1	1.1	0.3	1.1	1.1	0.3	1.1	1.2
14	-0.1	0.0	0.1	0.0	0.0	0.1	0.0	0.0	0.0

Selection of the “Best” Displacements

The stability information (i.e., stable and moved points) varies between the three solutions (i.e., congruency testing, IWST and LAS). The “best” displacements is then selected based on the results of modelling of single point movement. Three models were selected as below

- Points 3, 6, 11, 12 and 13 are experiencing single point movement in separate blocks, while the rest of the points as a stable block
- Points 3, 12 and 13 are experiencing single point movement in separate blocks, while the rest of the points as a stable block
- Points 3, 6, 12 and 13 are experiencing single point movement in separate blocks, while the rest of the points as a stable block.

The global test (equation 23) failed for models (ii) and (iii). Model (i) was therefore selected as the “best” model because the global test passed and all the group of parameters are statistically significant. The results of single point movement are shown in Table 3. Figure 2 depict the “best” displacements via the congruency testing method (model i).

Table 3: Results of modelling of single point movement for model (i) to (iii).

model no.	estimated deformation parameters and their statistical testing for a group of parameters			global test
i	$dx_3 = -0.8$ $dx_6 = -0.5$ $dx_{11} = -0.8$ $dx_{12} = 0.4$ $dx_{13} = 0.2$	$dy_3 = -0.5$ $dy_6 = 0.1$ $dy_{11} = 0.5$ $dy_{12} = 1.4$ $dy_{13} = 1.2$	$34.47 > 3.13$ (significance) $4.52 > 3.13$ (significance) $12.07 > 3.13$ (significance) $19.25 > 3.13$ (significance) $11.55 > 3.13$ (significance)	passed $0.55 < 1.96$
ii	$dx_3 = -0.7$ $dx_{12} = 0.5$ $dx_{13} = 0.2$	$dy_3 = -0.5$ $dy_{12} = 1.3$ $dy_{13} = 1.1$	$35.10 > 3.13$ (significance) $18.63 > 3.13$ (significance) $10.63 > 3.13$ (significance)	failed $2.64 > 1.84$
iii	$dx_3 = -0.9$ $dx_6 = -0.5$ $dx_{12} = 0.5$ $dx_{13} = 0.2$	$dy_3 = -0.5$ $dy_6 = 0.0$ $dy_{12} = 1.3$ $dy_{13} = 1.1$	$39.23 > 3.13$ (significance) $4.74 > 3.13$ (significance) $18.14 > 3.13$ (significance) $9.43 > 3.13$ (significance)	failed $2.32 > 1.89$

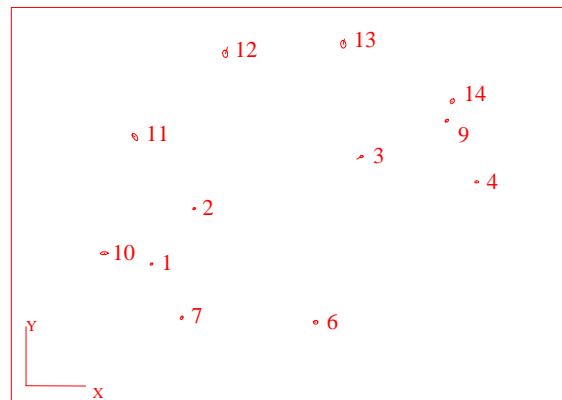


Figure 2: Graphical display of the “best” displacements together with their 95% confidence ellipses for model i (not to scale)

Modelling of Deformation

A trend analysis of the displacement field (refer Figure 2) leads to a choice of several possible deformation models as below

- No global deformation model on all the points in the network
- Points 10, 11, 12, 13 and 14 are experiencing homogeneous strain
- Points 3, 6 and 11 are experiencing single point movement in separate blocks, points 12 and 13 are experiencing rigid body movement and the rest of the points as a stable block
- Points 3 and 6 are experiencing single point movement in separate blocks, points 10, 11, 12, 13 and 14 are experiencing rigid body movement plus homogeneous strain and the rest of the points as a stable block.

The results of the selected deformation models are shown in Table 4. All models (except model iii) failed the global test. Model (iii) was then selected as the “best” model because it passes the global test and all the group of parameters are statistically significant.

Table 4: The estimated deformation models.

model no.	Estimated deformation parameters and their statistical testing for a group of parameters			global test
i				failed 1.85 > 1.49
ii	$\epsilon_x = 20.98 \mu\text{strain}$ $\epsilon_y = -10.05 \mu\text{strain}$ $\epsilon_{xy} = 7.37 \mu\text{strain}$ 6.70 > 2.70 (significance)			failed 1.65 > 1.51
iii	$dx_x = -0.8 \text{ mm}$ $dx_y = -0.5 \text{ mm}$ $dx_z = -0.8 \text{ mm}$ $a_x = 0.3 \text{ mm}$ $b_x = 1.4 \text{ mm}$	$dy_x = -0.5 \text{ mm}$ $dy_y = 0.1 \text{ mm}$ $dy_z = 0.5 \text{ mm}$	35.91 > 3.13 (significance) 4.21 > 3.13 (significance) 12.46 > 3.13 (significance) 26.60 > 3.13 (significance)	passed 0.60 < 1.89
iv	$dx_x = -1.1 \text{ mm}$ $dx_y = -0.6 \text{ mm}$ $a_x = -0.9 \text{ mm}$ $\epsilon_x = 8.1 \mu\text{strain}$ $\epsilon_y = 19.7 \mu\text{strain}$ $\epsilon_{xy} = -2.4 \mu\text{strain}$ 7.50 > 2.75 (significance)	$dy_x = -0.5 \text{ mm}$ $dy_y = 0.1 \text{ mm}$ $b_y = -1.8 \text{ mm}$	72.15 > 3.13 (significance) 6.21 > 3.13 (significance) 10.75 > 3.13 (significance)	failed 4.36 > 1.92

Therefore it can be concluded that the body was experiencing rigid body movement for points 12 and 13, while points 3, 6 and 11 are experiencing single point movement in separate blocks. The results for model (iii) (refer Table 4) are similar to the simulated deformation (i.e., Table 1). The “best” model is graphically presented in Figure 3.

Caspary [1987] only estimated the trend of movements for this monitoring network (i.e., using congruency testing and robust (LAS) method) and modelling of deformation was not performed. Therefore no further comparison can be made with Caspary [1987] regarding the results of modelling (refer Table 4).

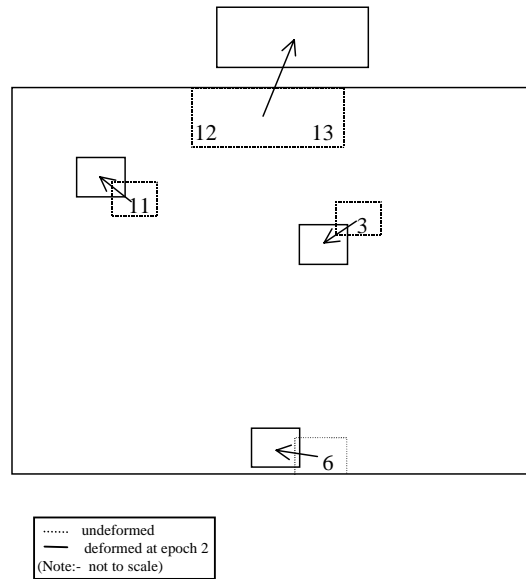


Figure 3: Graphical presentation of the “best” deformation model.

Conclusions

This paper has presented a procedure of deformation analysis, which comprises of the network adjustment of each epoch, trend analysis and the modelling of deformation. The developed procedure provides a systematic step-by-step analysis and it can be applied to either reference or relative monitoring network.

The six typical deformation models applied in this study were found to be optimal enough in determining the behavior of the body or object under investigation. However, the second and third order models (equation 35) should also be taken into consideration in order to obtain a deeper insight of the behavior of the body.

During trend analysis of the displacement field, several methods have been compared in order to avoid wrong interpretation of the trend of movements of the object under investigation. The “best” displacements are then selected based on the results of modelling of single point movement before deformation modelling is performed.

The results obtained for the dam monitoring network show that the developed procedure and the program package are applicable for the geometrical analysis of deformation.

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