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# MATHEMATICAL MODELLING OF EARTHQUAKE INDUCED VIBRATIONS ON MULTISTORY BUILDINGS

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ALLAH

Always in my heart…

Abah and Mak

Love you always…

Abang Sufian and Abang Shahriman

My inspiration…

Junaidizal

My truly supporter…

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## ABSTRACT

Logically, the taller building will be collapsed with a small period. We will give the reason in mathematical ways according to our study. The ability to make predictions about the vibrations from the earthquake could enable scientist to evaluate or plans and may have a significant effect on the period of time for the building collapsed. The modelling of unforced and forced vibrations on multistory buildings are used as a tool to study the mechanisms of vibration that caused by an earthquake. Both models are used to calculate the natural frequencies, $\omega$ and period, P of the building vibrations. Furthermore, the maximal amplitude of the building vibrations also can be analysed. For this research, eight, eleven and fifteen floors building have been applied. In our analysis, eigenvalues method is used via Maple 13 package. The graph of forced frequency versus amplitude and period versus maximal amplitude are plotted and discussed.

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# ABSTRAK

Secara logik, bangunan yang tinggi akan runtuh pada jangka masa yang singkat. Kami akan buktikan mengikut kaedah matematik seperti kajian yang telah kami jalankan. Keupayaan mengagak getaran daripada gempa bumi membolehkan para saintis mengkaji dan merancang sekiranya terdapat signifikan akibat kesan masa bangunan akan runtuh. Model tanpa daya dan model dengan daya getaran terhadap bangunan digunakan untuk mengkaji mekanisma getaran yang di sebabkan oleh getaran. Kedua-dua model boleh mengira frekuensi natural, $\omega$ dan masa,P di mana bangunan akan membuat satu getaran yang lengkap. Tambahan lagi, amplitud maksimum getaran pada bangunan juga boleh dianalisa. Untuk kajian ini, lapan, sebeles dan lima belas bangunan bertingkat telah diaplikasikan. Dalam kajian ini, kaedah eigenvalues telah digunakan melalui pakej Maple 13. Graf bagi frekuensi lawan amplitud dan masa lawan maksimum amplitud diplot dan dibincangkan.

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## LIST OF SYMBOLS

| | |
|---|---|
| $=$ | Equal |
| $+$ | Plus |
| - | Minus |
| $m$ | Mass |
| $F$ | Force |
| $x_1, x_2 \ldots x_n$ | Displacement of the floors |
| $f(t)$ | External force |
| M | The mass matrix |
| K | The stiffness matrix |
| $n$ | Number of floors |
| $\omega$ | Frequencies |
| **P** | Period |
| $\lambda$ | Eigenvalue |

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# CHAPTER 1

# RESEARCH FRAMEWORK

## 1.1 General Introduction

Mechanical and electrical vibrations are routinely covered in differential equations (DE's). Mathematical models of the real world [Blancard, 1998] can be formulated in term of differential equation. We used differential equations to study and get a better understanding of vibrations caused by earthquake [Marchand, Timothy and Mcdevis, 1999]. It is relate to earthquake induced of vibrations on multistory buildings.

This research takes full advantage of analytic, numerical, and qualitative techniques to address a large range of mathematical concepts. Throughout, familiarity with Maple13 programmed is assumed to prevent from using technology as a crutch instead of as a tool, a primary goal of the project is to determine the appropriate use of the available technology.

There are many of research regarding the earthquake focus on engineering systems such as seismic evaluation and retrofit of lifeline systems, seismic retrofit of acute care facilities and mitigation of infrastructure against multi-hazard extreme

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events [The Multidisciplinary Center for Earthquake Engineering Research]. However, the research of the earthquake corresponds to mathematical system is not as many as in engineering system.

There are many questions on the mathematical contextual for the vibrations of an earthquake. For example, which building will collapse first compare to the other building? How long the period of will produce the destructive resonance vibration on the building? Why an earthquake will demolish one building but leave the one next door untouched? Hence, all the questions above can be answered by extends the study of ODE concept to model of the vibrations of multistory building.

The purpose of this research is to introduce formulated a mathematical modelling of mechanical vibrations of multistory building, caused by earthquake and then we will use the solution of the method to discuss the relationship between resonance and amplitude of the vibration.

## 1.2 Background of the problem

Logically, the taller building will be collapsed with a small period. We will give the reason in mathematical ways according to our study. The ability to make predictions about the vibrations from the earthquake could enable scientist to evaluate or plans and may have a significant effect on the period of time the building will demolish. The modelling of unforced vibrations on multistory buildings and the modelling of forced vibration on multistory buildings are the tools which have been used to study the mechanisms of vibration that cause by an earthquake.

Analysis of the frequencies, periods and amplitude of natural oscillations will give some ideas related to earthquake induced of vibrations. Therefore, the study is focus on tall building and the seismic wave caused by an earthquake.

Mathematical modelling is of considerable importance in the study of mechanical resonance because it may provide understanding of the underlying mechanisms which influence the vibration and may suggest the prevent strategies. The earliest account of mathematical modelling of vibration was carried out in 1999 by Rich Marchand and Timothy J. McDevitt. Marchand and McDevitt created a mathematical model for a single floor building. From this model, the ODE can be used in build the mathematical modelling of the vibrations on multistory building [[Marchand, Timothy and Mcdevis, 1999].

Following Marchand and McDevitt, other mathematicians contribute to the mathematical modelling is C. H. Edwards and D. E. Penny. They had applied the second order differential system in a case of earthquake induced vibration on sevent floor building. The calculation from the model showed that different buildings have different natural frequency of vibration.

## 1.3 Statement of Problem

The problem of an earthquake induced vibration on multistory building is to studied the mathematical model that formulate depends upon the assumption made about the structure and the restrict motion to the horizontal direction which is the displacement of the floors. This mathematical model is used as tools in analyzing the forced frequency, periods of natural oscillations and the amplitude as a function of period.

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## 1.4 Objectives of Study

The main purposes of this research are to:

1. Using existing model to apply on earthquake induced vibrations on multistory buildings.

2. Analyse the frequencies and periods of natural oscillations of eight floors building, eleven floors building and fifteen floors building.

3. Analyse and discuss the maximal amplitude as a function of period for eight floors building, eleven floors building and fifteen floors building.

4. Compare the maximal amplitude as a function of period for multistory of eight floors building, eleven floors building and fifteen floors building.

## 1.5 Significant of Study

This research will give many benefits to scientist. In other way, it will provide some idea for mathematician to conduct further research about this field. We show that differential equation can be used in terms of investigate the vibration of mathematical system as analogy of an earthquake to multistory buildings.

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## 1.6 Scope of Study

This research will discuss about the application of differential equation to earthquake induced vibration on multistory buildings, we will focus on two models, which are the unforced vibrations of a multistory buildings model and forced vibrations of a multistory buildings model. This study will concentrate on eight, eleven and fifteen floors building.

In this study, will used a data taken from the Earthquake Induced Vibration of Multistory Buildings (Edwards and Penny, 1993). This event was an earthquake that happened in Mexico City. The data that we used is obtained from the event (September 19, 1985).

## 1.7 Management of the Research

In recent years, mathematics has made a considerable impact as a tool with which to model and understand mechanical resonance phenomena such as vibrations and so on. In general, this study actually discuss about some knowledge about vibration cause by earthquake. Other than that, the basic mathematical modelling of unforced vibrations of a multistory buildings model and forced vibrations of a multistory buildings model also been discuss to make sure there are some contribution in earthquake knowledge which is related with mathematics.

In this chapter, the introductory about this research are reviewed, including the important tools such as background and problem statement, some objectives, significant and scope of this research also have been discussed.

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This introductory chapter is followed by a study. Chapter 2 presents the foundation of earthquake, including definition and information of the field. Next, this chapter reviews on the earthquake induced vibration on multistory building at Mexico City.

Theoretical implementation is discussed in Chapter 3. This chapter discussed about the basic concept of vibration cause by an earthquake propose by Edwards and Penny. This chapter also included some mathematical studies of unforced vibrations and forced vibrations. In applying the vibration model, the analytical study is devoted on Chapter 4. The chapter analysed and discussed a method on solving second order differential equation. Beside that, the eigenvalue method for homogeneous systems is used to solve the model which is nonlinear systems. In plotting the behaviour of the vibration models, Maple 13 is used.