NUMERICAL ANALYSIS OF CONVECTIVE HEAT TRANSFER IN A TRIANGULAR DUCT WITH RADIATION BOUNDARY CONDITIONS

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To my beloved wife and children

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Hassanazhari b. Ahdi
(Abu Luqman)

ABSTRACT

Finite difference has been used to model a laminar, fully developed flow through a isosceles triangular duct with based fully insulated and the other two sides subjected to radiation boundary condition. Taking into account on practical application in the industries, radiation boundary conditions are considered in the analysis as this reflect big potential of application in the heat transfer equipment. Various aspect ratios have been analysed to investigate the behavior of the temperature and velocity profile as well as the mean bulk temperature and the Nusselts number.

Numerical method using finite difference has been selected to solve the governing equations for both momentum and energy equations. SOR (Successive Overrelaxation) and explicit finite difference method have been adopted to solve the momentum and energy equation respectively. These methods are proven and reliable to solve Poisson and parabolic equations as demonstrated in many literatures and give reasonable fast rate of convergence.

The results of mean bulk temperature and Nusselts numbers for three different aspect ratios correspond to 15^o*,* 30^o*dan* 45^o *were presented. The asymptotic Nusselts numbers obtained are 1.9, 7.6 and 13.2 respectively.*

ABSTRAK

Pembezaan terhingga telah digunakan untuk membuat model matematik aliran laminar, terbentuk penuh dalam saluran tiga segi dua sama dengan dasarnya ditebatkan sepenuhnya manakala dua sisi yang lain tertakluk kepada sinaran sebagai syarat sempadan. Memandangkan penggunaannya di dalam industri, syarat sempadan sinaran telah dipertimbangkan. Syarat sempadan jenis ini mempunyai potensi yang cukup besar dalam industri yang melibatkan applikasi peralatan pemindahan haba. Pelbagai nisbah aspek telah dianalisis untuk mengkaji kelakuan profil suhu dan halaju seterusnya menghitung suhu pukal serta nombor Nusselts.

Kaedah berangka pembezaan terhingga telah dipilih untuk menyelesaikan persamaan utama (governing equation) momentum dan tenaga. Kaedah 'SOR' dan Explicit telah digunakan untuk menyelesaikan persamaan momentum dan tenaga. Kaedah SOR dan explicit ini telah banyak digunakan seperti yang telah dilapurkan dalam banyak jurnal dan juga kadar penumpuannya (convergence) adalah pantas.

Keputusan yang telah dihasilkan untuk nisbah aspek, 15^o, 30^o *dan* 45^o *di tunjukkan. Nombor Nusselts yang telah di perolehi ialah 1.9, 7.6 dan 13.2 untuk nisbah aspek seperti yang telah di nyatakan di atas.*

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a	Characteristic length, m
b	Characteristic height, m
C_p	Specific heat, J kg $^{-1}$ K $^{-1}$
D_h	Hydraulic diameter, m
f	Friction factor
$\overline{F}$	Body force
GL	Number of grid line in x and y axis
H1	Axially uniform wall heat flux with peripherally uniform wall temperature condition
H2	Axially and peripherally uniform wall heat flux conditions
k	Thermal conductivity, W m $^{-1}$ K $^{-1}$
m	Number of vertical grid lines
n	Number of horizontal grid lines
Nu_m	Mean Nusselts number
p	Number of iterations
Pe	Peclet Number $\left(= \operatorname{Re} \operatorname{Pr}\right)$
Pr	Prandtl Number $\left(= \dfrac{\nu}{\alpha}\right)$
$\dot{q}$	Heat flux, W m $^{-2}$
Re	Reynolds number $(= \dfrac{\overline{w}}{\nu} D_h)$
t	Time
T	Temperature, K
T	Axially and peripheral uniform wall temperature condition
T_α	Radiation temperature, K
T_w	Circumferential duct wall temperature, K
u, v, w	Dimensional velocity

W	Dimensionless velocity, ($W = \frac{w}{\overline{w}}$)
w^*	Dimensionless velocity, $w^* = \frac{W}{C}$
$\overline{w}$	Mean velocity
X, Y	Dimensionless coordinates
x, y, z	Cartesian coordinates
ρ	Density, kg m^{-3}
μ	Dynamic viscosity, kg m^{-1} s^{-1}
ν	Kinetic viscosity, m^2 s^{-1}
α	Thermal Diffusivity, m^2s^{-1}
Φ	Viscous dissipation
ϑ	Dimensionless temperature
γ	Relaxation factor
λ	Auxiliary parameter
ϑ_b	Bulk temperature, K
ε	Emissivity, for black body $\varepsilon = 1$
σ	Stefan-Boltzmann constant, 5.668 x 10^{-8} W/m^2K^4
ΔX	Horizontal mesh size
ΔY	Vertical mesh size

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CHAPTER 1

INTRODUCTION

Flow inside the ducts in modern engineering play a very crucial role with regards to heat transfer. A wide range of applications have attracted many researchers through out the world investigating numerically, analytically and experimentally to improve and to enrich the literature of heat transfer. Extensive study has been made to increase and optimize the heat transfer rate and this can be achieved by modifying the geometry of the duct to suit any specific application.

In Malaysian context, triangular duct application with associates to nuclear engineering may have a limited opportunity to apply in real life. Nevertheless, these kinds of duct geometry have many applications in agriculture, food and petrochemical industries where triangular duct will act as oil heater and heat exchanger.

1.1 Literature Review

A study of a forced convection heat transfer in triangular ducts is crucial importance in nuclear power plants, heat exchangers and many other heat transfer applications including petrochemical industries, semi-conductors etc.

There is new wave of cost escalation re-emerge substantially in the millennium year due to many uncontrollable factors have made the requirements for downsizing the heat exchangers to reduce cost. Smaller unit means low flowrate and low Reynolds numbers consequently correspond to lower heat exchange efficiency. Various analyses were made to enhance the efficiency with smaller physical size such as modify the flow structure as well as modify the physical geometry [17].

Few attempts have been made to modify internal structure of the triangular duct to increase the heat transfer performance. C.W. Leung *et al* [13] has done an experiment with V-grooved (i.e. orthogonal to the mean flow) inner surfaces and found enhancement of rates of forced convection. C.W. Leung *et al* [14] again have conducted another two experiment having a square ribs orthogonal to mean air flow and roughened the surfaces by milling process. It is found that having fixing the transverse square ribs on the inner surfaces enhances of forced convection over the roughened surface. Based on the results obtained by C.W. Leung *et al* [14], further investigation has been conducted by D.D. Luo *et al* [15] to determine the optimum size of ribs and its internal spacing corresponding to maximize the thermal performance. By adopting the technique of having internal ribs, it has been extended also to rectangular ducts. Experiment has been conducted to study the effect of inclination angle of ribs on flow behavior by Xiufang Gao *et al* [10]. Various technique of internal geometry treatment of the duct such as introducing internal rib, v-grooved, roughening inner surfaces have been studied and evaluated and reported also by various authors [4], [10], [13] and [14].

It can be concluded based on results obtained by C.W. Leung *et al* [14] through his experiment indicates higher rates of heat transfer of forced convection in triangular duct can be achieved by several ways namely imposing artificial roughness on its surface (v-groove, internal ribs or roughened inner surfaces). Nevertheless, such achieve ness was accompanied with some drawbacks such as. higher axial pressure drop along the duct. The friction factor seems to be increasing linearly with v-groove angle and reach the peak when v-groove angle in between 90 to 120 degrees. If v-groove angle bigger than 120 degrees, the friction factor start to decline. From practical aspect from the industries, it is expected that de-scaling of the triangular duct could cause a concern and none of the previous authors has investigated this aspect. Common modes of cleaning used in the industries are either chemical or mechanical/physical cleaning [32]. However, chemical cleaning is always a second option as there are many factors to be resolved prior commence of cleaning. These factors are selection of chemicals, spillage impact on the environment, chemical handling, waste chemical disposal, toxicity, effectiveness, clearance from the authority etc. Mechanical cleaning is always favourable due to its simplicity and effective. Having internal ribs in the ducts will obstruct insertion of cleaning tools. A special cleaning tool has to be developed for de-scaling in the internal ductworks. However, it is forecasted v-groove may not pose a major cleaning concern mechanically.

Extension to the experiment conducted by Xiufang Gao *et al* [10], a numerical analysis of fully developed flow curved square duct with internal fins have been studied by P.K. Papadopoulos *et al* [14]. SIMPLE method in finite volume with staggered grid was used to solve the governing equations. The results indicated increase of friction factor depending on the fin height and the Dean number (De).

Great efforts have been made by E.C. Guyer *et al* [33], for compiling various combinations of triangular ducts performance with regards to Nu_T and $Nu_{H.}$ Other parameters were not presented.

Instead analysing to any particular geometry of the duct, general approach of arbitrary cross sectional of ducting has gained popularity due its universal application as far as geometry of the cross sectional is concern. Nevertheless, the real needs to have arbitrary cross sectional ducting analysis are due to complex in nature of actual geometry and lack of fluid flow and heat transfer correlations for non-circular duct [2], [5], [6] and [11].

Realising the importance of non-Newtonian fluids in the industries, big coverage of research is also being carried out to ascertain certain crucial parameters required by the industries. An experimental study on steady developing laminar for equilateral triangular cross section was done by S. Gh. Etemad *et al* [11]. It has been reported also numerous number of experiment and numerical analysis were made to reflect the importance of non-Newtonian fluid in the real application [29] and [30]. One of the major finding is the Prandtl number correspond to lower the heat transfer in the developing region of the duct [31].

There is another interesting shape of ducting presented by Chiu-Chia Su *et al* [1] i.e. divergent ducts of rectangular cross-section where the application can be found such as in Stirling cycle machine. This works basically as an extension of a straight rectangular duct which has been investigated extensively by many peoples and solving channels flow problem successfully presented by Patankar [24].

In general it is not a big challenge to transform the physical problem into mathematical model, nevertheless, to formulate the boundary conditions sometimes create some confusion. Any error in the boundary conditions will have a significant mistake in the final solution. There are three basic approaches to obtain the final results for any physical domain defined. As elaborated in details by Ghosdastidar [28], the basic approach are experiment, analytical and numerical method. Numerical method nowadays has become very popular due to advancement in computing speed, bigger memory and low in cost [27] and [28]. For a numerical method, there are four basic method of descretising the differential equation as described by Ghosdastidar namely as Finite Difference Method, Finite Element Method, Spectral

Method and Control Volume Formulation. Justification or which method to be employed is very much dependent on final form of the governing equations.

Numerical solution for right triangular duct has been solved by H.Y Zhang *et al* [2] to obtain distribution of the mean bulk temperature and mean Nusselt no. in the region of thermal development using Method of Line (MOL) hydride methodology for various degrees of right triangular cross section. The results indicates that the predicted $N_{um,T}$ value using MOL technique are in good agreement with analytical solution via Runge-Kutta algorithm. H.Y. Zhang *et al* [2] have presented the results obtained for mean bulk temperature and mean Nusselt number are good agreement with analytical results with less than 0.5% relative error for right–triangular angle less than 30 degrees. If the right-triangular angle is more then 30 degrees then the relative error exhibit nearly 3%. Regardless of any relative error, the results pattern has shown that the asymptotic Nusselts number will decrease parabolically with duct length increase. H.Y Zhang *et al* [2] have use report from results using Galerkin method for comparison.

Z.F. Dong *et al* [3] have presented a solution for triangular cusped duct by using the numerically generated boundary fitted coordinate system where the complex domain in the physical plane is transformed into a regular square domain in the computational plane. Finite difference analysis on the control volume is used to discretise the transformed governing equations. To validate the results obtained Z.F. Dong *et al* [3] have applied the method to some other geometry duct which the results can be obtained from open literature. Final results found to be an excellent agreement between the new numerically generated boundary system fitted coordinate and any other conventional finite difference.

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Any irregular physical plane the partial differential equations can be transformed from physical domain to square shape computational domain by adopting elliptic grid generation technique. Transforming the partial differential equations for physical domain into computational domain is done by ordinary chain rule of partial differentiation. The major advantage of transforming into square shape computational domain is that 'overshoot' of any element during griding is eliminated and substantial error reduced in the final results. A technique of overrelaxation numerical method was adopted to compute velocity and temperature distribution in the duct and the results obtained have a very good agreement with other results found in the open literature. Therefore, it can be concluded that elliptic grid generation is an appropriate method for irregular boundary duct solution. Ibrahim Uzun *et al* [5] have presented a case study for both uniform wall temperature and uniform wall heat flux boundary conditions. As presented this technique show an appropriate results and the accuracy of the outputs very much dependent on the grid spacing, smaller the grid will increase in accuracy.

Before the strength of computer came as a close friend to engineers, a fully developed laminar combined of a free and forced convection through non-circular duct have been studied and an approximation solution have been obtained through variational calculus and finite difference procedure. Not only that an exact solution for forced connection have been presented by M. Iqbal *et al* [31]. Although classical analytical solution like this play very important role but may not be popular and appropriate in this millennium years as numerical approximation promise faster results, high accuracy, less tedious and less chances of computing mistake compare to the former.

Another method which has gained popularity due to advancement of computer technology is the Finite Element Method (FEM). A. L. Nayak *et al* [6] has studied the problem of combined free and forced convection in a fully developed laminar through vertical ducts with constant axial heat flux and uniform peripheral wall temperature on a boundary condition. A triangular element has been selected for solution algorithm due to more flexible to approximate arbitrary region with greater fidelity [6]. For triangular ducts the results obtained have a good agreement with the figure obtained from open literature [6].

G. Yang *et al* [12] have made further advancement by analyzing heat transfer in arbitrary shaped ducts which having interaction of forced convection and radiation. Due to complexity of the governing equation, G. Yang *et al* [12] has limits the study to circular duct. They have adopted methods of moment and method of lines for radiation effect and convection respectively. The results obtained found to be good agreement with previous published data for pure convection. As for combined heat transfer, they found that conduction-radiation parameter, optical thickness and wall emissivity are major parameters that control the overall performance. However, results obtained were not able to compare as non-existence of data on the open literature.

To add to the richness for open literature Rajashankar Sadasivam *et al* [7] have studied numerically fully developed forced connection through trapezoidal and hexagonal ducts. This is to capture duct shape encountered in lamela type compact heat exchangers used in the pulp and paper, alcohol and other chemical industries [7]. Similar to the technique adopted by many other researchers [8], [9], [11], [12] i.e. finite difference, Rajashankar Sadasivam *et al* [7] has modeled their double-trapezoidal duct with both H1 and T thermal boundary condition. Similar technique as adopted by Ibrahim Uzun *et al* [5], Rajashankar transformed the partial differential equations of physical domain into rectangular computational domain to avoid half-grid cell which will results higher numerical errors. For computational methodology, finite difference of central differencing was employed for discretisation. The descretised equations were solved by point-iterative Gauss-

Seidel method. To verify the results obtained they have set the angle to 90° which means rectangular duct then verification with other published results on rectangular duct can be made and found good agreement. The results obtained indicated that the duct geometry and flow distribution have great influence on temperature field. The major finding is that the core temperature seems to be increasing with increase in aspect ratio.

Extensive studies have been made with regards to triangular ducts. Various arrangement of triangular ducts have been investigated as those arrangement have specific usage in heat transfer application in various industries which includes nuclear plant, petrochemical, electronics, pulp and paper, alcohol etc. Among the arrangement of the triangular investigated are right-triangular, equi-triangular, isosceles triangular, cusped triangular and sinus triangular as found in open literature [2], [3], [5], [6], [11], [12] and [13].

Constant wall temperature seems to be very popular boundary condition as this reflects big coverage in the industries [2], [3], [5], [6] and [10]. Nevertheless, combined boundary conditions of a constant wall temperature and constant heat flux input also have been studied by many researchers [5], [6] and [10]. Very little works done on the interaction of forced convection and radiation of triangular ducts. G. Yang *et al* [12] have studied numerically using differential method, the method of moment with combined convection-radiative heat transfer in the hydrodynamically and thermally developed region [12].

In the era of millennium years rate of convergence seems do not pose a major concern to any numerical technique employed. There is no specific threat by convergence rate reported in the year of 1990s till today but rather number of discrete area assumed is a concern due to accuracy of the final results obtained.

1.2 Current Problem

As the whole world changing rapidly every aspect of life has to follow the rhythm of the world including technology. Space constraint can pose a major hindrance especially for offshore facilities such as oil platform or shipping industries. Compact heat exchanger always hot focal points for heat transfer application as this piece of equipment has extensive application especially in the industries mentioned above and space can be translated directly into capital expenditure. Data for rate of heat transfer for triangular ducts with radiation as a boundary conditions for two sides and insulated at bottom side is still a vacuum in open literature. The optimum aspect ratio with corresponding angle is the parameter to be studied including the behavior of the streamline in the ducts. These factors are important from heat exchanger designer's point of view as these have great impact on the cost and constructability of the exchangers.

It is also crucial for the thermal designer to know for limiting parameters in design aspect such as fouling factor, temperature, pressure drop, etc. especially those related to thermal design parameters. For any limiting parameters known to the designer this will lead to a faster conclusion of feasibility study conducted which normally include but not limited to material selection, space availability, geometrical selection, constructability, handling, maintenance and cost.

The other important aspect of current problem that need to be addressed is the solution approach. Various leading expert in the field [22], [23], [24], [25], [26] and [27] have described in details various method and technique used to solve the physical domain and they are all agreed that nobody can claim their method is the best.

It is hope that with solution obtained at later part will conclude the most appropriate solution method with respect to accuracy and computing speed to this triangular duct problem in particular.

1.3 Scope And Objective

This study will cover only laminar flow in the ducts with hydrodynamically fully developed flow and temperature is developing. The flow medium assumed to be incompressible fluid, viscous with low Pandlt number.

The main objectives are to determine the temperature distribution in the duct and to perform parametric study on the behaviour of the streamlines in the duct. Various aspect ratio will be studied and this will give the relationship or rather indicates on how the temperature and velocity behave with various aspect ratio.

One of the prime interest figure is to obtain the Nusselt number based on final results obtained from numerical solution as the Nusselt number will ultimately conclude the heat transfer coefficient 'h' of the duct which will be the governing factor of any thermal design of heat exchanger.