

SOLVING SHORTEST PATH PROBLEM USING GRAVITATIONAL SEARCH  
ALGORITHM AND NEURAL NETWORKS

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A report submitted in partial fulfillment of the  
requirements for the award of the degree of  
Master of Science (Computer Science)

Faculty of Computer Science and Information Systems  
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APRIL 2010

## **ABSTRACT**

For many years, scientists have aware the importance of conducting research related to actual problems, and Shortest Path Problem (SPP) is one of such examples. SPP is meant to find the shortest path between two given cities or nodes. The travelled distance in such a path obviously depends on the order in which the cities are visited. Hence, it is the problem of finding an optimal ordering of the cities. Therefore, SPP is commonly known as the combinatorial optimization problems. In this study, a hybrid method is proposed that can solve the SPP accurately and fast without being trapped in a local optimum during searching. Gravitational Search Algorithm (GSA), a new optimization algorithm is applied to solve the above problem. In this scenario, decision making is provided by the Neural Network within a short amount of time. The results illustrate that with the precision of the GSA and the relatively high speed of the Neural Network, a very efficient method is obtained accordingly.

## ABSTRAK

Sejak berdekad, saintis telah melihatkan kepentingan menjalankan penyelidikan yang berupaya menyelesaikan masalah sebenar, dan Masalah Perjalanan Jarak Jurujuar (MPJJ) merupakan masalah sebenar yang seumpamanya. MPJJ bertujuan mencari jarak terpendek di antara dua bandar atau nod. Jarak perjalanan bergantung kepada turutan bandar yang dilawati. Oleh yang demikian, MPJJ bertujuan mencari penyelesaian yang optimal berdasarkan turutan bandar tersebut. MPJJ mewakili satu kelas masalah yang besar dan dikenali sebagai satu masalah pengoptimuman kombinatorik. MPJJ merupakan kaedah yang terpenting untuk dijelajahi disebabkan oleh kaedah penyelesaiannya yang mudah. Namun demikian, pendekatan ini agak sukar untuk diselesaikan secara optimum. Kajian ini mencadangkan kaedah hibrid MPJJ yang berupaya menghasilkan keputusan yang tepat, dan pantas tanpa terperangkap dalam optimum setempat semasa proses carian. Pendekatan cadangan ini menggunakan satu algoritma pengoptimuman yang baru, iaitu Algoritma Carian Graviti (ACG). Dalam senario sebegini, rangkaian saraf buatan dilaksanakan bagi memutuskan satu keputusan akhir dalam julat masa yang tersaur. Hasil kajian mempamerkan bahawa kaedah yang dicadangkan telah memberikan keputusan yang memberangsangkan berbanding kaedah setara yang lain.

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## **CHAPTER 1**

### **INTRODUCTION**

#### **1.1 Introduction**

A shortest path problem is for finding a path with minimum travel cost from one or more origins to one or more destinations through a connected network. It is an important issue because of its wide range of applications in transportations, robotics and computer networks. In some applications, it is also beneficial to know the second or third shortest paths between two nodes. For instance, in order to improve the effectiveness of travel information provision, there is a need to provide some rational alternative paths for road users driving in real road network. To meet it, k-shortest path algorithms have been used in general. Yen [1] first proposed k-shortest path searching method, which could generate several additional paths by deleting node on the shortest path. Since Yen, Several k-shortest algorithms have been suggested. Although k shortest path algorithm can provide several alternative paths, it has inherent limit of heavy overlapping among derived paths, which may lead to wrong travel information to the users. This is that significant proportion of links on the first path is overlapped by second and third path calculated from the method, so the drivers on those links may suffer severe congestions if they follow the travel



information. This is the same problem of IIA (Independence of Irrelevant Alternatives) in logic-based stochastic assignment.

On the other hand existing shortest path algorithms such as Dijkstra [2], Moore build the optimal path based on the node they reach. However, in the case of considering the network consisting of several turn prohibitions such as restricting left-turn, which are popularly adopted in real world network, it makes difficult for the traditional network optimization technique to deal with. Banned and penalized turns may be not described appropriately for in the standard node/link method of network definition with intersections represented by nodes only. The reason is that Bellman's 'Principle of Optimality' does not hold in that case. Several approaches have been proposed for solving the problem. Among all methods currently available, the most widely used method is network expansion for describing turn penalties, adding extra nodes and extra links to original network and modify the network to be easily implemented the conventional shortest path algorithms. The principal advantage of this method is that it can describe the turn prohibitions perfectly. The method, however, has limitation of expanding network as the size of network increases, so this method could not apply to large networks as well as dynamic case due to its overwhelming additional works.

## **1.2 Problem Background**

As an important branch of graph and network, the traditional Shortest Path Problem (SPP) has extensive applications. That is to say, the SPP is a classical research topic. It was proposed by Dijkstra and has been widely researched. The research works in the past are mostly related to two aspects: The research of time efficiency of algorithms and the research of the derivative problems from SPP. Lately the research of accuracy of algorithms has been paid much attention to also.

Although the Dijkstra algorithm based on the Bellman optimization theory is considered as the most efficient method, it becomes inefficient when needing to repeat much computation or when the network is very big, and it also cannot be implemented in permitted time.

Many new types of algorithms, most of which are in the field of artificial intelligence, have been proposed recently, but most of them overlook an important factor in the solution in one way or the other. Some lack speed and some lack efficiency but the proposed method meets a balance between these two factors.

In this study, by suggesting a new hybrid method, that combines Gravitational Search Algorithm and Artificial Neural Network, many problems of the previous studies like finding an inaccurate solution or the unreasonably low speed, will be solved.

### **1.3 Problem Statement**

Many existing methods that are designed specifically to solve the classic shortest path problem are incomplete and have certain flaws that end in shortcomings in further uses of the problem. Various heuristics and approximation algorithms, which find the solution in a reasonable time but suffer from the lack of accuracy or are accurate but extremely slow, have been designed and proposed already. However, few attempts have been made to improve the overall quality of solutions.

The method that will be proposed in this study, will hopefully find the globally optimal solution in a very short amount of time. Gravitational Search

Algorithm, as mentioned earlier is a new search algorithm that has been proven efficient in solving many problems. In the case of SPP, the main use of GSA would be to obtain a solution close to the global optimum in a short period of time, and later, using an artificial neural network, which will be described in details later in the study, the acceptably precise answer will be found very fast. This new method will address some of the limitations of previous solutions, in particular the lack of reliability in what concerns succeeded, and valid convergence, and the failures in attempts to reduce the time.

#### **1.4 Aim of the Study**

The aim of this study is to present a novel approach to solving the Shortest Path Problem, which can perform better than many of the previous methods in terms of accuracy and speed. The results obtained from the approach will be compared with the ones of the other algorithms to back the claims made in the study.

#### **1.5 Objectives of the Study**

The Shortest Path Problem is concerned with finding the shortest path from a specified starting node (Origin) to a specified ending node (Destination) in a given network while minimizing the total cost associated with the path. The shortest path problem is one of the classical combinational optimization problems, having widespread applications in a variety of settings. The applications of the shortest path

problem include vehicle routing in transportation systems, traffic routing in communication networks, and path planning in robotic systems.

The Shortest Path Problem has been investigated extensively and many modern algorithms have been proposed to solve this problem. Yet, many of the existing algorithms fail to meet some of the requirements of the solutions applicable in the real world.

The main goal of this study is to propose a solution to this important problem by combining two algorithms: Gravitational Search Algorithm and Artificial Neural Networks. The purpose of this study is to compensate the shortcomings in previous methods as much as possible. Many of the algorithms cannot find the accurate answer when the number of nodes increases, because they might find the local optimum and never get the chance to find the optimal solution and many cannot get the answer in a reasonable amount of time. Therefore, in this study the main objectives are:

- I. To optimize the solution to the point where the process of searching slows down, with the help of the GSA.
- II. To reach the precise answer using Neural Network in a short period of time.
- III. To evaluate the proposed method by comparing the results with previous methods.

In this study, a hybrid method for solving Shortest Path is proposed to improve the accuracy and speed of the algorithm to this problem.

## 1.6 Scopes of the Study

The scopes of this study are as follows:

- I. In order to find an almost accurate answer in a reasonable amount of time, the new “Gravitational Search Algorithm” will be used.
- II. Neural Networks are applied to find the optimal solution very fast.
- III. Randomly generated graphs are used to evaluate the method,
- IV. Matlab v.2009a is used to implement the method.

The Gravitational Search Algorithm that is used in this study, is a new algorithm proposed by Rashedi in 2008, and is an attempt to establish a new population-based search algorithm based on gravity rules. One of the important advantages of GSA is its ability to find the global optimum in a shorter period of time in comparison with previous optimization algorithms. However, the high speed of the algorithm shrinks dramatically when we get closer to the answer in the search environment. Therefore, an Artificial Neural Networks is used to speed up the final stages of the process and find the accurate answer in a short amount of time, even for very large problems.

It is worth mentioning that hopefully the neural network that will be used in this study will solve or provide solution to the Shortest Path Problem faster than would be possible with any algorithmic solution, relying on the neural networks parallel architecture.

## **1.7 Significance of the Study**

This study is aimed to propose a new hybrid solution to a classical problem that has been applied to many real-life applications. The method presented in this study takes advantage of a recently-proposed soft-computing method, namely the Gravitational Search Algorithm. The study can contribute both to the level of performance of the Shortest Path problem solving methods and an evaluation of the Gravitational Search Algorithm

## **1.8 Organization of Report**

The report for this study consists of four chapters. Chapter 1 presents an introduction to the study, problem background, objective, scope and significance of this study. Chapter 2 reviews the Shortest Path Problem Types and Applications, classic methods to solving the Shortest Path Problem, heuristic algorithms for the Shortest Path Problem, and the Gravitational Search Algorithm, which are the fundamental parts in the method proposed in this study. Chapter 3 discusses the methodology used in this study. Chapter 4 presents the results and evaluation of the results. And finally Chapter 5 is the Conclusion.

## REFERENCES

1. J. Y. Yen. *Finding the K Shortest Loopless Paths in a Network*, Mgt. Sci., 17 (1971) 712-716.
2. E. W. Dijkstra. *A Note on Two Problems in Connection with Graphs*. Numer. Math., 1:269-271, 1959.
3. Moore, E.F. *The shortest path through a maze*. pp. 285-292 in Proceedings of an International Symposium on the Theory of Switching (Cambridge, Massachusetts, 2-5 April, 1957), Harvard University Press, Cambridge
4. R. K. Afmja, K. Mehlhorn, J. B. Orlin, and R. E. Tarjan. *Faster Algorithms for the Shortest Path Problem*. Technical Report CS-TR-154-88, Department of Computer Science, Princeton University, 1988.
5. R. E. Bellman. *On a Routing Problem*. Quart. Appl. Math., 16:87-90, 1958.
6. D. P. Bertsekas. *The Auction Algorithm for Shortest Paths*. SIAM J. Opt., 1:425-447, 1991.
7. T. H. Cormen, C. E. Leiserson, and R. L. Rivest. *Introduction to Algorithms*. MIT Press, Cambridge, MA, 1990.
8. R. B. Dial. *Algorithm 360: Shortest Path Forest with Topological Ordering*. Comm. ACM, 12:632-633, 1969.
9. L. R. Ford, Jr. and D. R. Fulkerson. *Flows in Networks*. Princeton Univ. Press, Princeton, NJ, 1962.
10. H. N. Gabow and R. E. Tarjan. *Faster Scaling Algorithms for Network Problems*. SIAM J. Comput., pages 1013-1036, 1989.
11. G. Gallo and S. Pallottino. *Shortest Paths Algorithms*. Annals of Oper. Res., 13:3-79, 1988.
12. V. Goldberg, H. Kaplan, and R. F. Werneck. *Reach for A\*: Efficient Point-to-Point Shortest Path Algorithms*. In Proc. 7th International Workshop on

- Algorithm Engineering and Experiments. SIAM, 2006.
13. Moffat and T. Takaoka. *An all pairs shortest path algorithm with expected time  $o(n^2 \log n)$* . SIAM J. Comput., 16:1023-1031, 1987.
  14. E. W. Dijkstra: *A note on two problems in connexion with graphs*. In Numerische Mathematik, 1 (1959), S. 269–271.
  15. Dimitri P. Bertsekas. *A Simple and Fast Label Correcting Algorithm for Shortest Paths*. Networks, Vol. 23, pp. 703-709, 1993.
  16. Robert Sedgewick. *Algorithms in Java*. Third Edition. ISBN 0-201-36121-3. Section 21.7: Negative Edge Weights.
  17. Floyd, Robert W. (June 1962). "Algorithm 97: Shortest Path". Communications of the ACM **5** (6).
  18. Johnson, Donald B. (1977), *Efficient algorithms for shortest paths in sparse networks*, Journal of the ACM 24 (1).
  19. Mischa Scharwtz. *Telecommunication Networks*. Addison-Wesley Publishing Company, 1987.
  20. J.J. Hopfield and D.W. Tank. *Neural computation of decisions in optimization problems*. Biological Cybernetics, pages 533-541, 1986
  21. Herbert E. Rauch and Theo Winarske. *Neural networks for routing communication traffic*. IEEE Control Systems Magazine, pages 26-31, April 1988.
  22. L. Zhang and S. C. A. Thomopoulos. *Neural network implementation of the shortest path algorithm for traffic routing in communication networks*. In Proceedings of International Conference Neural Networks, page 591, 1989.
  23. Mustafa K. Mehmet Ali and Faouzi Kamoun. *Neural networks for shortest path computation and routing in computer networks*. IEEE Transactions on Neural Networks, 4(5):941-953, November 1993.
  24. Dong-Chul Park and Seung-Eok Choi. *A neural network based multi-destination routing algorithm for communication network*. IEEE, pages 1673-1678, 1998.
  25. Dorigo, M. *Optimization, Learning and Natural Algorithms*, PhD Thesis, Dipartimento di Elettronica, Politecnico di Milano, Italy, 1992.
  26. Dorigo, M. and Gambardella, L. *Ant Colony System: A Cooperative Learning Approach to the Traveling Salesman Problem*, IEEE Transactions on Evolutionary Computing, 1, pp. 53-66, 1997.



27. Dorigo, M and Gambardella, L. *Ant Colonies for the Traveling Salesman Problem*, Biosystems, 43, pp. 73-81, 1997.
28. Dorigo, M. and Di Caro, G. *The Ant Colony Optimization Metaheuristic*, in New Ideas in Optimization, Corne, D., Dorigo, M. and Glover, F. (eds), McGraw-Hill, pp. 11-32, 1999.
29. Dorigo, M., Maniezzo, V. and Coloni, A. *The Ant System: Optimization by a Colony of Cooperating Agents*, IEEE Transactions on Systems, Man and Cybernetics - Part B, 26, pp. 29-41, 1996.
30. Stutzle, T. and Dorigo, M. *ACO Algorithms for the Traveling Salesman Problem*, in Evolutionary Algorithms in Engineering and Computer Science, Miettinen, K., Makela, M., Neittaanmaki, P. and Periaux, J. (eds), Wiley, 1999.
31. Fukuyama, Y. *Fundamentals of particle swarm optimization techniques*. In: IEEE PES Tutorial on Modern Heuristic Optimization Techniques with Application to Power Systems, ch. 5, January 2002.
32. Coloni, A., Dorigo, M., Maniezzo, V. *Distributed Optimization by Ant Colonies*. In: Proceeding of First European Conference on Artificial Life, pp. 134–142. MIT Press, Cambridge 1991.
33. Kennedy, J., Eberhart, R.: *Particle Swarm Optimization*. In: Proceedings of IEEE International Conference on Neural Networks (ICNN), vol. IV, Perth, Australia, pp. 1942–1948 (1995).
34. Eberhart, R.C., Kennedy, J.: *A New Optimizer using Particle Swarm Theory*. In: Proceeding of the Sixth International Symposium on Micro Machine and Human Science, Nagoya, Japan, pp. 39–43 (1995).
35. D. Awduche, A. Chiu, A. Elwalid and X. Xiao, *Overview and Principles of Internet Traffic Engineering*, IETF RFC3272, 2002.
36. D. Awduche, J. Malcolm, J. Agogbua, M. O'Dell, and J. McManus, *Requirements for Traffic Engineering Over MPLS*, IETF RFC2702, 1999.
37. S. Vutukury and J. J. Garcia-Luna-Aceves, *A Traffic Engineering Approach based on Minimum Delay Routing*, Proc., IEEE ICCCN, 2000.
38. Y. Wang, and Z. Wang, *Explicit Routing Algorithms for Internet Traffic Engineering*, Proc., IEEE ICCCN, 1999.
39. Fortz and M. Thorup, *Internet Traffic Engineering by Optimizing OSPF Weights*, Proc., IEEE INFOCOM, 2000.

40. A. Sridharan, R. Guerin and C. Diot, *Achieving Near-Optimal Traffic Engineering Solutions for Current OSPF/IS-IS Networks*, Proc., IEEE INFOCOM, 2003.
41. Z. Wang, Y. Wang, and L. Zhang, *Internet Traffic Engineering without Full Mesh Overlaying*, Proc., IEEE INFOCOM, 2001.
42. K.M. Alzoubi, P. Wan, O. Frieder. *New Distributed Algorithm for Connected Dominating Set in Wireless Ad Hoc Networks*. Proceedings of 35th Hawaii International Conference on System Sciences, Hawaii 2002.
43. Bevan Das and Vaduvur Bharghavan. *Routing in Ad-Hoc Networks Using Minimum Connected Dominating Sets*. IEEE International Conference on Communications (ICC '97), (1) 1997: 376-380.
44. S. Guha and S. Khuller. *Approximation algorithms for connected dominating sets*. Algorithmica, Vol 20, 1998.
45. M.Q. Rieck, S. Pai, S. Dhar. *Distributed Routing Algorithms for Wireless Ad Hoc Networks Using d-hop Connected d-hop Dominating Sets*. Proceedings of the 6<sup>th</sup> International Conference on High Performance Computing: Asia Pacific, December 2002.
46. Jie Wuan d Hailian Li. *A Dominating-Set-Based Routing Scheme in Ad Hoc Wireless Networks*. *Special issue on Wireless Networks in the Telecommunication Systems Journal*, Vol. 3, 2001, 63-84.
47. Liang and Z. J. Haas. *Virtual Backbone Generation and Maintenance in Ad Hoc Network Mobility Management*. Proc. 19th Ann. Joint Conf. IEEE Computer and Comm. Soc. INFOCOM, vol. 3, pp. 1293-1302, 2000.
48. Subhankar Dhar, Michael Q. Rieck and Sukesh Pai. *On Shortest Path Routing Schemes for Wireless Ad-Hoc Networks*. 10th International Conference on High Performance Computing (HiPC '03), December 2003.
49. G. Cybenko, *Approximation by superposition of a Sigmoidal function*, Mathematics of Control, Signals and Systems, Vol. 2, (1989), pp. 492–499.
50. C.K. Looi, *Neural network methods in combinatorial optimization*. Computers and Operations Research, 19 (3/4), (1992), pp. 191–208.
51. S. Kumar, *Neural Networks: A Classroom Approach*. International Edition, McGraw-Hill, Singapore, (2005).

52. J.J. Hopfield, *Neural networks and physical systems with emergent collective computational abilities*, Proceedings of the National Academy of Sciences 79, (1982), pp.2554–2558.
53. T. Kohonen, *Self-organized formation of topologically correct feature maps*, Biological Cybernetics 43, (1982). pp.59–69.
54. D.H. Ackley, G.E. Hinton, T.J. Sejnowski, *A learning algorithm for Boltzmann Machines*, Cognitive Science, 9, (1985), pp.147–169.
55. K. Gurney, *An introduction to neural networks*, Taylor & Francis e-Library, (2005).
56. Rosenblatt, F. (1958) *The perception: a probabilistic model for information storage and organization in the brain*. Psychological Review, 65, 386–408.
57. Rumelhart, D.E. and McClelland, J.L. (1986) *Parallel Distributed Processing: Volume 1 Foundations*. The Massachusetts Institute of Technology.
58. Rashedi E., Nezamabadi-pour H. and Saryazdi, S. *GSA: A Gravitational Search Algorithm*. Information Sciences. 2009. 179(13): 2232-2248.