FINITE ELEMENT METHOD FOR TWO-DIMENSIONAL ELASTICITY PROBLEM

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ABSTRACT

The purpose of this study is to develop a system which can be used to solve two-dimensional elasticity problem. A manual solution has been made to solve this two-dimensional elasticity problem based on the heat problem. In the first step, the main component is solved to get the stiffness matrix. Next, the initial strain and load vector, and the boundary vector are computed to complete the solution. The solution is computed and verified using MATLAB.

ABSTRAK

Kajian ini bertujuan untuk membangunkan satu sistem yang boleh digunakan bagi menyelesaikan masalah kekenyalan dua dimensi. Satu penyelesaian secara manual untuk menyelesaikan masalah kekenyalan dua dimensi ini telah dibentuk dalam disertasi ini berdasarkan masalah haba. Dalam langkah pertama, komponen-komponen utama yang diselesaikan bagi mendapatkan matrik stiff. Seterusnya, nilai awal kekenyalan dan vektor limpahan serta vektor sempadan diselesaikan bagi menyempurnakan penyelesaian. Setiap penyelesaian dikira dan ditentusahkan menggunakan perisian MATLAB.

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LIST OF SYMBOLS

$\mathbf{K}$	-	Stiffness matrix
$\mathbf{B}$	-	Matrix differential operator for shape function
$\mathbf{D}$	-	Constitutive matrix
$\mathbf{f_b}$	-	Boundary vector
$\mathbf{f_l}$	-	Load vector
$\mathbf{f_o}$	-	Initial strain vector
$\mathbf{N}$	-	Shape function
$\boldsymbol{\varepsilon}_\mathbf{o}$	-	Initial strain
$\boldsymbol{\sigma}$	-	Stress
$\boldsymbol{\varepsilon}$	-	Strain
$\boldsymbol{\alpha}$	-	Thermal expansion coefficient
$\Delta\mathbf{T}$	-	Change in temperature
℃	-	Temperature
$\mathbf{t}$	-	Traction vector
$\mathbf{n}$	-	Normal vector
$\mathbf{b}$	-	Body force
$\mathbf{E}$	-	Young’s modulus
$\mathbf{v}$	-	Poisson’s ratio
$\mathbf{u}$	-	Displacement vector

GLOSSARY OF TERMS

FEM	-	Finite Element Method
FDM	-	Finite Difference Method
BEM	-	Boundary Element Method
dof	-	Degree of freedom

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CHAPTER 1

RESEARCH FRAMEWORK

1.1 Introduction

Historically, Kevin and his friends such as Lame, Boussinesq, Cerruti and others are one of the first experimentally observed about the theory of elasticity. They mentioned that the rigorous to some boundary value problem of elasticity theory for the regions bounded by the surfaces prescribed by a single parameter have been obtained. However, Saint-Venant was reported his observation in the books of “On torsion of prisms” and “On bending of prisms” in which “the semi-inverse method” and Saint-Venant’s principle were suggested. It should be proclaimed as the origin of elasticity theory as an applied discipline. Thus the elasticity problems represent, not a flux and temperature, but the traction vector and the displacement vector need to be found.

The problem of finding a flux and temperature at certain nodes in heat problem is such a way in giving ideas about elasticity problem. Some problems in elasticity field such as torsion need detail calculations and a system must be used if it comes to a big number. It contains some crucial parts and need to spend more times when doing manually. The system developed here is just for certain problem using certain techniques in finite element method (FEM).

By using this finite element method, all the calculations contain such as the differentiation and integration parts in most calculation and solving a matrix system using Gauss elimination system. These entire things are the main parts in solving this type of problem. Elasticity is just one of the applications in finite element method. The methods were in action when some forces are given to a solid that make changes in their shape. When this occurs, the shape for that solid are not permanently in their new shape but still have a chance to back normal but it depends on the force that given on it.

1.2 Background of the problem

The elasticity problem is common problem that many engineers faced it. They give the problem to model and solve it numerically. This problem will be solved using finite element method for grid rectangle shape function. It depends on the value given such as the area, coordinate of the model and the flux at beginning. For instance, it does not give directly, but need to find based on the model graphically. However, the process are the same as heat flow problem except only for certain part such as the value of load vector, boundary vector and followed by initial strain vector. Each such solution has different shape function based on the number of element. Recent comparisons with every element show the different flux and temperature for every nodal point with surprising accuracy. The stress and strain value needs to be found by doing some other calculation before go through to the real problem. This success suggests that more complicated of three-dimensional problem might provide accurate physical models of more complex elasticity phenomena or even though with other shape function.

1.3 Statement of the problem

Given a two-dimensional domain for an axially loaded elastic bar with a constant traction vector along the boundary by assuming a rectangular disk. The forces and the traction vector are given. In the same time, the mass density and the acceleration due to gravity are provided. The problem is to determine the finite element formulation of elasticity for one element by getting the value of the boundary vector, the load vector and the initial strain vector. The result shows that one-fourth of this total force is distributed to each nodal point. The same results are obtained when considering the other elements and this implies that the uniform traction is equivalent to the nodal forces.

1.4 Objectives of the study

The main objectives of this research are to:

1. Study the characteristics and problems of two-dimensional elasticity from previous.
2. Derive finite element formula for two-dimensional elasticity for grid rectangle problem.
3. Determine the solvability of the formulated finite element method.
4. Provide a finite element technique of two-dimensional elasticity for grid rectangle problem using software MATLAB.

1.5 Scope of the study

This research will only consider the study on two-dimensional elasticity problem which is the extension from finite element method. A two-dimensional elasticity problem will be solved using finite element method and Gauss elimination is a technique to solve the matrix part. Apart from that, several requirements will only get from the problem itself or will be given directly. The solution of two-dimensional elasticity problem will only focus on the problem which is using a grid rectangle as a method of solution in order to get their shape function.

1.6 Outline of the research

This dissertation has five chapters including the preceding introductory part that have been discussed in the earlier sections. Chapter 2 will provide with some information on the literature studies related to this research. This chapter starts with the history of elasticity problem followed by some introductions on finite element method. Then, there will be a section that discusses on two-dimensional elasticity problem and finally some brief introduction on finite element method.

In **Chapter 3** will focus on the computational methods. This chapter starts with the process in order to complete the programming parts that have been used in some research by the other researchers. Some journal about using the elasticity or some parts in elasticity will be referred. All solutions are obtained by using MATLAB computer programming.

Chapter 4 is mainly about the elasticity problems in details and some explanations about the problem. The manual solutions are used to compare the solutions using programming system. Most of the problems are nearly same as heat problem because it has a strong connection between both problems. There are conditions for elasticity to occur in heat problem such as for one-dimensional and two-dimensional problem. Manual solutions will be produced in this section.

Finally, the last chapter will be the conclusion part that summarized the whole research. There are also some suggestions regarding related research area will be proposed in the last chapter, as well as the interesting applications of two-dimensional elasticity problem in daily physical problems. Attachments have been provided in order to complete this research. **Appendix A** is a research methodology based on waterfall model. **Appendix B** is a Gantt chart that shows the timeline and milestones of the project that reflects with the **Appendix A**. While, the **Appendix C** shows the detail for calculations inside and the **Appendix D** shows the algorithm for the calculations.

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