THE APPLICATION OF DRESSING METHOD ON NONLINEAR EVOLUTION EQUATIONS

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For my beloved wife, daughters and son.

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ABSTRACT

It is well known that Korteweg-de Vries (KdV) equation can be solved by inverse scattering transform (IST). Numerous efforts have been made to extend the range of application of this method. The question now is how wide is the class of equations which are integrable by IST? It would be more useful if it could be extended or generalised to accommodate other equation. That's why we introduced dressing method which was proposed by Zakharov and Shabat (1974). The aim of dressing method is to generate integrable nonlinear equation and simultaneously its solution. We study this method on three integral operators and the differential operators. Besides that, we also study and list down all the properties which will be use together with operators in this method. In this dissertation, we choose only constant coefficient operator and scalar differential operator. We applied it to derive integrable nonlinear KdV and Kadomtsev-Petviashvili (KP) and thereafter we solve for exact solution.

ABSTRAK

Kita telah ketahui bahawa persamaan Korteweg-de Vries (KdV) dapat diselesaikan dengan menggunakan kaedah penyerakan songsang. Pelbagai usaha telah dilakukan untuk memperluaskan penggunaan melalui kaedah ini. Persoalannya ialah berapakah saiz jurang bagi kelas persamaan yang dapat dikamirkan oleh kaedah penyerakan songsang? Ia akan menjadi amat berguna sekiranya dapat dikembangkan atau digeneralisasikan untuk memenuhi persamaan yang lain. Justeru itu, kami memperkenalkan kaedah 'dressing' yang telah dikemukakan dan dipelopori oleh Zakharov dan Shabat (1974). Matlamat kaedah 'dressing' adalah untuk menjana persamaan tak linear yang terkamirkan dan juga penyelesaiannya. Kami mempelajari asas kepada kaedah ini pada tiga operator kamiran dan juga operator pembezaan. Di samping itu, kami juga mengkaji dan menyenaraikan semua sifat-sifat yang akan diguna bersama-sama operator dalam kaedah ini. Dalam dissertasi ini, kami memilih hanya operator pekali malar dan operator pembezaan skalar. Kesemua operator tersebut kami gunakan untuk mengembang serta memperoleh persamaan tak linear boleh kamir KdV dan Kadomtsev-Petviashvili (KP). Seterusnya dengan pilihan yang tepat, kami mencari penyelesaian tepat untuk persamaan KdV dan Kadomtsev-Petviashvili (KP).

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CHAPTER I

INTRODUCTION

In this chapter, we will briefly present about the soliton and its history. Then we also stated briefly the background of the problem of dressing method and its application on nonlinear evolution equation. In the end, we outline the objectives, the scope of study and the organization of the dissertation.

1.1 Soliton and Its History

We give a brief account of the history of soliton theory so that we can understand the relationship more clearly. In 1834, John Scott Russell (1808-1882) the hydrodynamic engineer accidently observed a peculiar water wave while he was riding his horse along a canal near Edinburgh. He called it as 'the great solitary wave', and followed for a few miles before losing it in the meanders of the canal. He devoted about ten years of his life to study this phenomenon. In 1844, Russell reported his discovery in "British Association Reports". However, his discovery was not well received by two scientists who ruin all his expectations. Sir G.B. Airy (1801-1892), the well known astronomer strongly criticized his work in a paper on wave and tide. Airy doesn't agree with the formula derived by John Scott Russell when it compare with his own theory of shallow water waves. G.G.Stokes also concluded that a solitary wave could not exist in a nonviscous fluid.

John Scott Russell deserved recognition of his discovery and achievement when the French scientist Joseph Valentine de Boussinesq (1842-1929) proposed a new theory of shallow water waves, which had solutions which agreed with his observation. A full theoretical understanding of John Scott Russell's observation had to wait until 1895 with studies of Diederik Johannes Korteweg and Gustav de Vries who derived the equation which nowadays bears their names as KdV equation. The KdV equation is one of the prototype equations of soliton theory because it has a remarkable mathematical property which leads to the understanding of the fundamental ideas lies behind the soliton concept.

A typically bell-shaped, plane wave pulse which translates in one direction without changing its shape is called as solitary wave. In another words, a solitary wave is a single isolated symmetrical hump travelling wave of unchanged shape. More formally, it is a localized wave packet that propagates with steady speed and does distort in shape. The idea has remained with us and we use it for the meaning of this. A solitary wave which translating with speed v along x can be said as any bell-shaped function $u(x - vt)$(R.K.Bullough and P.J.Caudrey, 1980). The study about solitary wave ran aground because it has been thought as unstable waves. This as a result, someone suspected whether the shapes of two solitary waves should be completely destroyed since KdV equation is a nonlinear differential equation and does not satisfy the superposition principal of solutions. So to study them in physics is meaningless.

Until the 1950s, a new situation appeared which can be considered as a new development of soliton theory. It is due to the work of famous physicists Enrico Fermi (1901-1954), John R. Passta (1918-1981) and Stanislaw M. Ulam (1909-1984) (FPU) on an experiment to study the thermalisation of a solid by using one of the very first computers, the 'MANAIC'. First they thought that, energy introduced in a single mode (mode k=1) would slowly drift to the other mode after steady state had been reached. To their surprise, after a long period of the mode k=1, almost all the energy was back to the lowest frequency mode. The remarkable discovery of FPU was made in 1953. This remarkable result was known as FPU paradox and published in 1955 in a classified Los Alamos Laboratory report.

Their study inspired Norman J. Zabusky and Martin D. Kruskal to analyze the KdV equation which has been arisen from the FPU problem. In 1965, they investigated and analyzed the nonlinear simulation in detail and obtained more complete result, which confirmed that the solitary waves do not change the shapes after the interaction. They named them “soliton’ because the solitary waves have the unchangeable property like the collision of particles. Their work was an important milestone in the history of the soliton theory. The concept of the “soliton” introduced by them has been accepted in general because they had correctly revealed the substance of solitary waves. From this moment, the study of soliton theory had been developed more rapidly and caused worldwide study. This is about more than 130 years after the discovery of the solitary waves by John Scott Russell.

In 1967, Gardner, Green, Kruskal and Miura (GGKM) using the idea of direct and inverse scattering transform to derived method of solution for nonlinear evolution equation such as KdV equation. In 1968, Lax made the generalization of their results and introduced the concept of Lax pair. Ryogo Hirota published an article about new method called ‘the Hirota direct method’ to find exact solution of the KdV equation for multiple collisions of solitons, in 1971.

The discovery and development of the soliton theory has a close relationship with modern physics. It shows great vitality in combining, interdepending, permeating and promoting between the severity of mathematics and the practicality of physics. It had been one of the important characters in the developments of modern natural sciences. Combination of the theory and the experimentation will be another character for this purpose (Guo.B, 1995).

1.2 Background of the Problem

Since 1967, when Gardner, Greene, Kruskal and Miura first found the inverse scattering transform for KdV equation, numerous efforts have been made to extend the range of application of the method. The important question is, how wide is the class of equations which are integrable by the inverse scattering transform? It would be even more useful if it could be extended or generalized to accommodate other equations. Indeed, we have already hinted that this is possible. Lax (1968) developed a formalism or we call it as Lax pair, which show the way to an answer to this question. This is because Lax suggested that the first equation other than the KdV equation found to be integrable by the Zakharov & Shabat inverse scattering scheme was the nonlinear Schrodinger equation (see Zakharov & Shabat 1972, 1973).

In 1972 (1971 in Russian), Zakharov & Shabat published the inverse scattering transform for nonlinear Schrodinger equation, $iu_t + u_{xx} + u|u|^2 = 0$. Shortly thereafter they extended the technique to other equations (Shabat, 1973, Zakharov & Shabat, 1974). This extensions which we shall call as Zakharov & Shabat scheme, essentially takes the Lax method and recasts it in a matrix form, leading directly to a matrix Marchenko equation. Ablowitz, Kaup, Newell and Segur (AKNS) (1974) subsequently gave their generalization of the Zakharov & Shabat scheme and found more classes of equation (this included the Sine – Gordon equation) to be integrable by the generalized scheme. The good accounts of these developments can also be found in some excellent books, for example Echaus (1981), Lamb (1980), Eilenberger (1981), Newell (1985) and Drazin and Johnson (1989).

Applying the above mentioned method to solve nonlinear evolution equation, one invariably has to solve the corresponding inverse spectral problem, which generally speaking is not an easy task. With a different approach, Zakharov & Shabat (1974) partly or can be said almost overcame this difficulty and also gave a clear indication of the method of calculating the exact solution. Their method had been further developed by themselves (see Zakharov & Shabat 1979, Zakharov 1980, Zakharov & Manakov 1979, and Novikov *et al.* 1984). Chowdhury & Basak (1984) applied this method to obtain the soliton solution of the Hirota – Satsuma coupled system of KdV equation. In 1984, Kuznatson, Spector and Fal'kovich used this method to study stability problem for stationary periodic waves in s weakly dispersive medium described by the Kadomtsev-Petviashvili (KP) equation. This method is now known as the dressing method.

The fundamental and basic idea about the dressing method is as following. Linear operators with variable coefficient can be obtained by means of transformation operators from operators with constant coefficients. If two such constant coefficient operators which have a common spectrum are simultaneously transformed, the condition of compatibility assumes the form of a nonlinear equation. Since the procedure is then to 'dress up' the constant coefficient operator to obtain the constant or variable coefficient operators, so we called this method as 'dressing method'. Actually we can also 'dress up' variable coefficient operators to obtain variable coefficient operators.

Zakharov & Shabat (1974), in their first paper regarding the dressing method, only 'dress up' constant coefficient operators. Although thereafter (see Zakharov 1980, Novikov *et al.* 1984) they suggested and allow their method to be use to dress up variable coefficient operators. Actually no one has really used the variable coefficient version of this method except Kuznatson, Spector and Fal'kovich (1984) used this variable coefficient version of the dressing method to study the stability problem. By using the variable coefficient version of the dressing method, we found that we can make an extension to the original Zakharov & Shabat's method and more nonlinear equations can be integrated.

1.3 Statement of the Problem

The research project wishes to study and master the dressing method with constant coefficient operators. The aim is to use dressing method to solve and obtain solution for nonlinear evolution equation such as KdV equation and Kadomtsev-Petviashvili (KP) equation.

1.4 Objectives

1. To study the dressing method which had been discovered by Zakharov and Shabat.

2. To apply dressing method to KdV equation and Kadomtsev-Petviashvili (KP) equation.

1.5 Scope

This research will focus on constant coefficient version and scalar differential operators in dressing method on KdV and KP equation.

1.6 Organisation of the Dissertation

In Chapter II, we present the literature review regarding the inverse scattering transform (IST), the Lax formulation and Zakharov and Shabat Scheme (dressing method). We discuss a few important basic properties on each method.

In Chapter III, we discuss in detail on the integral operators and differential operators. Here, we write out the properties which are useful to this method and derive all the calculation in detail.

In Chapter IV, we use the constant coefficient operators together with scalar differential operators to derive the nonlinear KdV and Kadomtsev-Petviashvili (KP) equations. At the same time, by choosing the right choice for $K_+(x, x; t)$, we find solution for the KdV and Kadomtsev-Petviashvili (KP) equations.

Finally, in Chapter V, we made conclusion on our dissertation and mention some suggestions for future study or research.

In the appendices, we include the proving of differential operators and commuting operators.