

COLUMN DESIGN WITH SEMI-RIGID END FRAME

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ABSTRACT

The strength and the stability of the steel frames is mostly influenced by the connection strength and stiffness. Usually in conventional analysis, the convenient and easy way to design the connection is by using pin or rigid connection. But, in actual the connection is behaved most likely between these two (2) connections. The connection called semi-rigid which possessed a certain degree of rotational restraint. Years ago, numerous studies have been conducted to investigate the behavior of column and developed the design method on semi-rigid connection for braced and unbraced frames. However, semi-rigid connection has not been adopted very enthusiastically by the structural designer due to the lack of confidence about its behavior. They are convenient in using the conventional design method based on BS 5950 [1], Eurocode [2, 3] or AISC. Recently there was a study which proposed the easy way to design column which neglect moment transfer to column known as simplified α_{pin} approach [4, 5]. This study is conducted to look into the column at ultimate limit state with the aid of computer software name LUSAS Analyst and to look the reliability of simplified α_{pin} approach as a straightforward method compared to conventional design. From the result of this study, it found that the column strength is affected by the stiffness of the connection and the simplified α_{pin} approach also reliable to use in design the column without transferring the moment. The value of α_{pin} always give more than unity which mean it is reliable.

ABSTRAK

Kekuatan dan kestabilan struktur keluli biasanya dipengaruhi oleh kekuatan dan kekukuhan sambungan. Lazimnya dalam merekabentuk sambungan, kaedah yang biasa digunakan adalah dengan merekabentuk sambungan secara pin ataupun sambungan tegar. Namun begitu, dalam keadaan sebenar sambungan tersebut berkelakuan di antara sambungan pin dan juga sambungan tegar. Sambungan ini lebih dikenali sebagai sambungan separa-tegar yang mana ia dipengaruhi putaran terhalang yang terhasil akibat daripada pemindahan momen kepada tiang. Sejak beberapa dekad yang lalu, banyak kajian telah dilakukan untuk memahami sifat dan kelakuan sambungan separa-tegar ke atas tiang dan membangunkan kaedah merekabentuk sambungan tersebut bagi kerangka yang dirembat ataupun kerangka tidak dirembat. Walaubagaimanapun, rekabentuk sambungan separa tegar ini kurang di gunakan dikalangan perekabentuk berikutan kurang keyakinan ke atas kelakuan sambungan tersebut. Mereka lebih senang menggunakan kaedah rekabentuk yang lazim digunakan berdasarkan standard BS 5950[1], Eurocode [2, 3], ataupun AISC. Baru-baru ini terdapat penyelidikan yang memperkenalkan kaedah rekabentuk yang dipermudahkan yang mana ia mengabaikan pesongan pada tiang bersambungan separa-tegar yang dikenali sebagai kaedah α_{pin} [4, 5]. Kajian ini dijalankan untuk melihat kesan ke atas kekuatan tiang pada ULS dan juga untuk menentukan kebolehpercayaan kaedah α_{pin} dalam merekabentuk tiang bersambungan separa-tegar. Didapati nilai α_{pin} selalu lebih dari uniti yang mana kaedah tersebut boleh digunakan dalam merekabentuk struktur kerangka keluli dengan sambungan separa-tegar.

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LIST OF SYMBOLS

t_a	-	the thickness of the angle cleat
t_{fc}	-	the thickness of the column flange
E	-	the elastic modulus
S_j	-	the rotational stiffness of a joint;
z	-	the lever arm;
Φ	-	the rotation of a joint
k	-	the stiffness coefficient factor
t_f	-	the flange thickness of an I or H section
l_{eff}	-	the effective length of fillet weld
P_{sr}	-	Ultimate load of semi-rigidly connected column in a frame
P_{pin}	-	Ultimate load of the equivalent axially loaded perfectly pin ended column
λ_{cr}	-	Stability Limit
h	-	the storey height
δ	-	the notional horizontal deflection of the top of storey relative to the bottom of the storey
w kN/m	-	uniform distributed load
L	-	Length
$\{F\}$	-	the external forces at the joints
$[K]$	-	the assembled stiffness matrix of the structure accounting for the initial connection stiffness
$\{\delta\}$	-	the unknown displacement of the joints
F_c	-	the compressive force due to axial force
M_x	-	the nominal moment about the major axis
M_y	-	the nominal moment about the minor axis

M_{bs}	-	the buckling resistance moment for simple column
P_c	-	the compression resistance from 4.7.4
P_y	-	the design strength
Z_y	-	the section modulus about the minor
P_E	-	$\frac{\pi^2 EI}{L^2}$
P_{SQ}	-	$A_g p_y$
I	-	Moment inertia

CHAPTER 1

INTRODUCTION

1.1 General Introduction

The creation or design of a building is complex process and also give challenge to the engineers to create a structure that safe and accomplishes its function. Steel structures have been developed in our society for many centuries. The steel framework is one of the commonly used structure systems in the modern construction. It is effective and widespread for the uses of steel with the development of latest technology nowadays. It also gives advantage to the practicing engineers in designing the structures. The material properties of steel make its good to use in the industries. Beginning from 17th century (Hooke) and 19th century (Euler), they had developed basic constitutive relationship to create impressive steel structure.

The structural steel frame system mainly consist components of beams, columns and connections. Among these three (3) components, the connection between beam to column play important role to the effect of load distribution, strength, stability and constructability of the structure. It also well known that the connections show a variation of behavior in term of strength and stiffness. Usually in conventional method of design, the connection behaves either as a pin

transferring only nominal moment or they are function as a rigid and maintain full moment continuity. Years ago, there were numerous researches and experiment about the joint behavior had been carried out to investigate the truth. From the investigation, they found out that the joint behavior didn't match either pin or rigid connection but lies between those pin and rigid connection. Because of the actual behavior of frame connection always falls between these two extremes, much attention has been focused in the last decades toward a more accurate modeling of such connection. This is because researchers have realized that although the adoption of idealized joint behavior greatly simplifies the analysis and design processes, the predicted response of the idealized structure would be quite unrealistic as compared to that of the actual structure [6, 7]. Many of experimental investigation on actual joint behavior had been done and showed that the effect of connection stiffness is very significant to column capacity [8]. Certain type of this connection in reality should be treated as semi-rigid connection for the purpose of analysis and design. The research conducted during the last decade on structural connection has resulted in considerable progress and understanding of the subject that has prompted change in design provision [9].

Trahair, Bradford and Nethercot [10] define semi-rigid joints as those that had dependable moment capacities and which partially restraint the relative rotations of the member at the joint. The action of this joint in rectangular frame reduce the maximum moment in the beams, so the semi-rigid design method offer potential economic over the simple design method. Semi-rigid construction is recognized by the all major building codes. It was first adopted by American Institute of Steel Construction for Allowable Stress Design (AISC-ASD) as early 1946 and later in the Load and Resistance Design (AISC-LRFD) in 1986. The development of semi-rigid connection contribute to the amendment of some requirement in the British Code 5950 [1] had included the clause 2.1.2.4 which suggested that the stiffness, strength, and rotation capacities of the joints are based on experimental evidence and used to assess the moments and forces in the members. Another code of practice, Eurocode 3 [2, 3] proposed a classification of connection models according to the rigidity and strength.

1.2 Problem Statement

Although there are numerous research reported about the methods and advantages [20] of semi-rigid connection in the design, but there is still no orderly absorption by structural designer due to lack of confident about its behavior [9, 11]. According to Ahmed [21], the semi-rigid nature of the connection affects the frame behavior in that the distribution of internal forces and moments in the beams and columns are different from those of the standardized curves. Needless to say, frame analysis neglecting the true behavior of the connection will result in unreliable prediction of frame response. Rigorous tools for analyzing the semi-rigid frames have been available for quite some time, but the main bottleneck in treating semi-rigid design as a viable design alternative, was the lack of a simple hand method. Simplified methods for analyzing semi-rigid frames were available in BS5950 [1] treated semi-rigid connections of a range of stiffness as pinned and so failed to take account of the moments being transferred to the column. To overcome this, Ahmed[21] proposed a simplified analysis technique of semi-rigid frames using computer software to study the behavioral pattern of non-sway semi-rigid frames.

In 1990's, Gibbons [4] had proposed simplified method known as α_{pin} after investigation using full scale test. This method is neglecting the transfer of moment to column. But, this method had been introduced was known to satisfy for cases where the columns were subjected to monotonically loaded to failure. However, in contrast of the fact that column may subjected to variable loading and unloading behaviour. So, in year 2000, Shahrin [5] done further research to study the column subjected to variable loading and unloading behaviour. He had proposed a condition such that α_{pin} values to be always greater than unity after study about 1107 columns behavior. From that α_{pin} approach ready to used as practical column design method. Hence, this study will look into the column strength with the aid of computer software name LUSAS Analyst and the reliability of the simplified α_{pin} approach.

$$\alpha_{pin} = \frac{P_{sr}}{P_{pin}} \geq 1 \quad \text{equation} \quad 1.0$$

Where;

P_{sr} : Ultimate load of semi-rigidly connected column in a frame

P_{pin} : Ultimate load of the equivalent axially loaded perfectly pin ended column, [$A_g P_c$]

1.2.1 Definition

The following terms were identified as especially relevant to this study. A preliminary as well as comprehensive examination of the materials related to the study and similar works by other researchers, suggested that the following terms appear almost invariably in the related reports. In the present research too, these terms were used in various phases of the research, including the review of the literature, validation and analysis parts. Most of the definitions of the terms are taken from Eurocode 3 [2]– the new European standard for design of buildings in steel.

Rigid connection ensures that there is no relative rotation between connected beams and columns and hence the bending moment can be completely transferred from a beam to the adjacent column

Pinned connection ensures that the bending moment cannot be transferred at all from a beam to the adjacent column and hence relative rotation occurs between these two elements

Semi-rigid end connection, also known as partially-restrained (PR) connection, has a moment capacity between rigid and flexible connections. It ensures that there is relative rotation between adjacent beams and columns and the bending moment is transferred only partially between these elements

Limit State, A criterion beyond which a structure or structural element is judged to be no longer useful for its intended function (serviceability limit state) or beyond which it is judged to be unsafe (ultimate limit state)

Limit states design, A design method that aims at providing safety against a structure or structural element being rendered unfit for use.

Buckling is the primary disadvantage of steel structures subject to compression. It essentially arises because the steel component attains a more favourable equilibrium position when it buckles or moves out of the plane of loading. Buckling of the steel component usually exhausts its strength and results in catastrophic failure of a composite member. Hence, means must be established to ensure that buckling does not occur. There are several types of buckling modes for structural members: Euler buckling, torsional buckling, lateral-torsional buckling, local plate buckling, and their combinations

Eurocode 3, hereafter referred to as EC3, was published in draft form in 1984 and then a European pre-standard, reference no. DD ENV 1993-1-1:1992, in September 1992. EC3 had been developed and published as a European standard in 2003 and is expected to replace existing national codes such as BS5950 by March 2010 [2, 3]

LUSAS is a feature modeller that is associative in nature. In this software package, the geometry of any particular model is entered in terms of features. In order to get accurate analysis, the features in turn are discrete in nature i.e. they are further divided into various finite elements.

Sway and non-sway frames is depend on geometry and load cases under considerations as well as the influenced of $P\Delta$ effect. For sway frame, the change of geometry (2nd-order effect) is significant, but it negligible for non-sway frame. As specified in BS 5950 [1], clause 2.4.2: stability limit state stated that under vertical load only, it should satisfy $\lambda_{cr} \geq 10$ for non-sway frame. Meanwhile for sway sensitive; $\lambda_{cr} \leq 4$. The calculation of λ_{cr} :

$$\lambda_{cr} = h / 200\delta \quad \text{equation} \quad 1.1$$

Where

- h is the storey height
- δ is the notional horizontal deflection of the top of storey relative to the bottom of the storey

The scope of this study is narrowed down to non-sway frames which use horizontal support to cater horizontal load.

1.2.2 Beam to Column Connection Philosophy

Before discussing the topic in details, basic philosophy of the connection will be discussed in this section.

In simple design, as a result to pin connection which allow connection flexibility, the beam is free to rotate and able to develop full rotation at the beam end. The beam will carry full moment at the mid-span with no transferring moment to the beam end. The formula known for pin connection at mid span is $wL^2/8$. (Figure 1.1 (a))

For rigid connection, most of the moment will be transferred to the beam end. There is no rotation allow at beam end. The formula for moment at beam end is $wL^2/12$, meanwhile at the mid span the moment is $wL^2/24$ less than moment at mid span for pin connection. (Figure 1.1 (b))

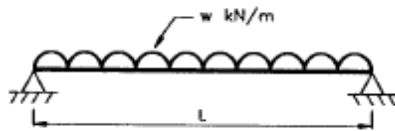
But, the reality of the connection behaviour lies between the two (2) idealised pin and rigid connection. It means that, for rigid connection possesses certain degree of rotation; on the other hand, pin connection does receive a certain amount of bending moment at beam end. Therefore, for mid span moment the value more than $wL^2/24$ but less than $wL^2/8$. Meanwhile for beam end moment the value will be less than $wL^2/12$. (Figure 1.1(c))

The moment at beam end known as hogging moment and the mid span moment acknowledge as sagging moment. The values of hogging and sagging moment depend on the type of connection. For semi-rigid that lies between the idealised pin and rigid connection may lead to the saving of the beam size/weight.

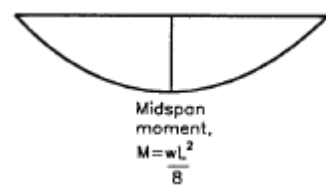
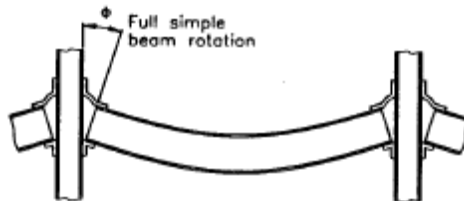
Where:

w kN/m : uniform distributed load

L : Length

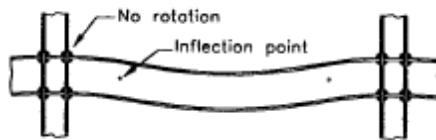


Bending moment diagram

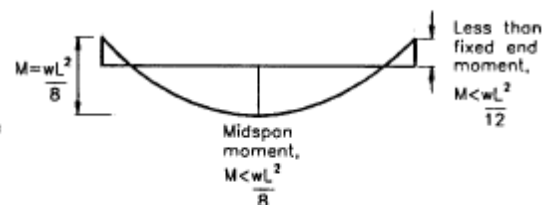
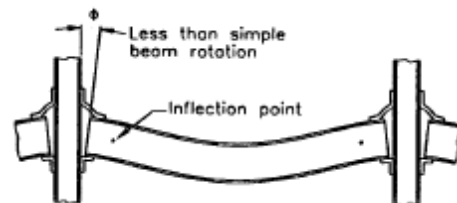


No end moment,
 $M=0$

a. Pin Connection



b. Rigid Connection



c. Semi-Rigid Connection

Figure 1.1 Classes of Connections

1.3 Objective and scope of the study

The main objective of this study can be described as follow:

- To study the column strength in semi-rigid frame at ultimate limit state with the aid of computer software name LUSAS Analyst.
- To study the reliability of simplified α_{pin} approach for column design with semi-rigid frame.

To achieve the objective mentioned above, London University Structure Analysis Software (LUSAS) is used for analysis. The programme is developed by London University to solve the problem using finite element method. Other than that, the result from experiment and previous research using Semi-Rigid Nonlinear Finite Element (SRI-NOFA) developed by Lee Choon Siang [8] is used to validate the keying data in LUSAS. After that the parametric study is conducted to see the reliability of the method.

The scope of this study is narrowed down as mentioned below:

- Non-sway frame
- Two dimensional plane frame using LUSAS
- Only major axis bending of columns are considered
- Non-linear behaviour of semi-rigid steel frame
- Only I section are involved in this analysis

The column in minor axis and lateral torsional buckling is excluded from this study.

1.4 Outline of Thesis

In order to achieve the objectives of this study, the scopes of the thesis as follow:

- Chapter 2 will discussed literature review which related to the study by previous researchers.
- In chapter 3, the methodology of this study will be presented.
- Meanwhile in chapter 4, the validation of keying data in LUSAS will be conducted before continue with the parametric study in chapter 5.
- The result from chapter 5 will be compared with other method and the new approach of designing the column will be proposed in chapter 6
- Chapter 7 will summarised the overall conclusion of the study and also recommendation for further study will be presented.

Bibliography

1. BS 5950-1:2000, "Structural use of steelwork in building- Part 1: Code of practice for design- Rolled and welded section".
2. BS EN 1993-1-1:2005, "Eurocode 3: Design of steel structure- Part 1-1: General rules and rules for building".
3. BS EN 1993-1-8:2005, "Eurocode 3: Design of steel structure- Part 1-8: Design of joints".
4. Gibbons, C (1990), "The Strength of Biaxially Loaded Columns In Flexible Connected Steel Frames." University of Sheffield: Ph.D. Thesis.
5. Shahrin Mohammad (2000), "Column in Semi-Rigidly Jointed Steel Frames Under Variable Loading." University of Sheffield: Ph.D. Thesis
6. D.A. Nethercot, T.Q.Li & B.Ahmad (1998), " Unified Classification System for Beam-to-Column Connections", *Journal of Construction, Steel Res.* Vol.45, No.1, pp. 39-65
7. Chen, W.F, (2000), "Practical Analysis for Semi-Rigid Frame Design", London: World Scientific
8. Chen, W.F. and Liu, E.M. (1991). "Stability Design of Steel Frames" CRC Press Inc.
9. Burns, S.A., (2002), "Recent Advances in Optimal Structural Design", Massachusetts: ASCE Publications
10. Trahair, N.S., Bradford, M.A., Nethercot, D.A., (2001), "The Behaviour and Design of Steel Structures to BS5950", 3rd edition: New York: Taylor & Francis
11. Ahsan, R., Ahmed, I., Ahemd, B., (2003), "Lateral Drift of Semi-Rigid Steel Frames-1", *Journal of Civil Engineering: The Institution of Engineers, Bangladesh*, Vol. CE, No. 2
12. Mazzolani, F.M., (2003), "Behaviour of Steel Structure in Seismic Areas", Lisse: Taylor & Francis
13. Kishi, N., Goto, Y., Chen, W.F., Hasan, R., (1994), "Revision of Semi-rigid Connection Database", I-12, n.d.,
Webpage: <http://library.jsce.or.jp/jsce/open/00057/1994/50-0048.pdf>

14. Barakat, M., (1989), “ Simplified Design Analysis of Frames with Semi-rigid Connections”, Phd dissertation, School of Engineering, Purdue University, West Lafayette
15. Levy, R., Spillers, W.R., (2003), “Analysis of Geometrically Non-linear Structure”, 2nd edition, Massachusetts: Kluwer Academic Publishers
16. Chan, S.L., Chui, P.P.T., (2000), “Non-linear static and Cyclic Analysis of Steel Frames with semi-rigid Connections”, Oxford: Elsevier Science Limited
17. Chen, W.F. and Nethercot, D.A. (1988), “Effects of Connections on Column.” *Journal of Construction Steel Research*, 201-239
18. Carr, J.F and Gibbons, C. (1993), “A Simplified Approach to the Design of Semi-rigidly Connected Columns in Multi-storey Non-sway Steel Framed Building”, University Sheffield: M.Sc. Thesis
19. Lee Choon Siang, (2003), “Behaviour of Column with Semi-Rigid End Restraint in Non-Sway Frames,” Universiti Teknologi Malaysia: Master Thesis
20. Mahmood, Shahrin and Abdul Kadir Marsono (1999), “Economic Aspect of Partial Strength Design on Multi-storey Braced Steel Frame”
21. Ahmed, I. (1996), “Approximate analysis of Semi-rigid Steel Frame”, Dept. Of Civil Engineering, Bangladesh University of Engineering and Technology Dhaka, Bangladesh, pg 227
22. Ivanyi, M., Baniotopoulos, C.C (2000), “Semi-rigid Joints in Structural Steelwork”, Italy: Springer, pg. 5
23. Bjorhovde, R., Brazetti, J., Colson, A. (1990), “ A Classification System for Beam-to-Column Connection”, *Journal of Structural Engineering*, ASCE: Vol. 1116, No. ST11, pg 3059-3076
24. Chen, W.F., Goto, Y., Liew, J.Y.R (1996), “Stability Design of Semi-rigid Frame”, New York:Wiley-IEEE, pg 18-19
25. Goto, Y., Miyashita, S. (1995), “Validity of Classification system of Semi-rigid Connection in Behaviour of Flexible Joints and Dynamic Analysis of Steel Frames with Semi-rigid Connection”, edited by S.L., Chan, Special Issue of *Journal of Engineering Structure*, London: Butterworth

26. Davies, J.M., Brown, B.A (1996), "Plastic Design to BS 5950", 3rd edition, Oxford: Wiley-Blackwell, pg 209
27. Chen, W.F., Kim, S.E. (1997), "LFRD Steel Design Using Advanced Analysis", Florida:CRC-Press, pg125
28. Blognoli, M., Gelfi, P., Zondonini, M. (1998), "Optimal Design of Semi-rigid Braced Frames Via Knowledge Based Approach", Journal of Constructional Steel Research: 46: 1-3; Paper no. 79, Elsevier Science Ltd, pg 1-2
29. Mroz, Z., Stayroulakis, G.E. (2005), "Parameter Identification of Materials and Structures", New York: Springer, pg 34
30. Goverdham, A.V. (1984), "A collection of Experimental Moment Rotation Curves and Evaluation of Prediction equations for Semi-rigid Connection", Doctoral Dissertation, Vanderbilt University, Nashville, Tennessee
31. Nethercot, D.A. (1985), "Steel beam to column connections – A review of test data and their application to the evaluation of joints behaviour on the performance of steel frames", CIRIA Report, RP 338
32. Chen, W.F. and Kishi, N. (1989), "Semi-rigid steel beam-to-column connections: Data base and modelling", Journal of Structural Engineering, ASCE, Vol.115, No.1, pp 105-119
33. Faella, C., Piluso, V., Rizzano, G. (1999), "Structural Steel Semi-rigid Connection Theory, Design and Software", Florida:CRC Press, pg 3
34. Colsan A. And Louveau, J.M. (1983), "Connection Incidence on the elastic Behaviour of steel structure", EuroMech Colloquium, No.174
35. Mac Donald, B.J. (2007), "Practical Stress Analysis with Finite Element", Dublin:Glasnevin Publication, pg 247
36. Wriggers, P. (2008), "Nonlinear Finite Element Methods", Heidelberg: Springer, pg 258