# DETERMINATION OF THE FLOW CURVE OF NECKING TENSILE SPECIMEN

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Specially dedicated to my parents En Razali, Puan Che Nah, family members, Along, Abg KF, Amin, Didi, friends and my love Mohd Irwan.

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## ABSTRACT

The main objective of this project is to study and to determine the flow curve of necking specimen by using finite element analysis and to validate the approximation formulae of equivalent plastic stress and strain introduced by Bridgman and Davindekov-Spiridonova. This research is done for 3 tensile test specimens with different types of hardening which are ideal plasticity, linear hardening and non linear hardening. In this project, the finite element method is applied for high accurate simulation of tensile tests. The obtained results are discussed in the context of approximation formula and previously known results. Different numbers of element have been carried out in order to study the influence of meshing on the results. From the study, it was found that larger number of elements give stable results, thus, larger number of elements are been choose for simulation. Computer simulation has been done to verify the assumption of the approximation formula and to recognize the possible error. From the results interpretation, it was stated that the error connected with application of the simple formula can be estimated as 10 % in comparison with the numerical simulations, which was considered as the reference solution. The results shows that, the Davidenkov Spiridonova approximation formula give better compared to the Bridgman formula.

## ABSTRAK

Objektif kajian yang dijalankan adalah untuk mengkaji dan mendapatkan flow curve'bagi spesimen ujian tegangan dengan menggunakan kaedah unsur terhingga dan membuat perbandingan antara von-Mises equivalent stress dan 'equivalent stress' formula yang diperkenalkan oleh Bridgman dan Davidenkov-Spiridonova. Kajian ini dijalankan keatas 3 jenis spesimen ujian tegangan yang mempunyai sifat pengerasan yang berbeza iaitu, ideal plastik, pengerasan linear, dan pengerasan tidak linear. Dalam projek ini, kaedah unsur terhingga digunakan untuk mendapatkan keputusan simulasi bagi ujian tegangan yang tepat. Beberapa model yang berbeza jumlah unsur dikaji untuk untuk menentukan pengaruh jumlah unsur kepada hasil keputusan simluasi. Dari kajian, didapati bahawa jumlah unsur yang besar memberikan hasil keputusan yang stabil, dengan demikian, jumlah unsur yang besar ini telah dipilih untuk simulasi ujian tegangan. Dari tafsiran keputusan, didapati bahawa ralat yang diperolehi daripada 'approximation formula' adalah sebanyak 10% dibandingkan dengan simulasi numerical, yang dianggap sebagai penyelesaian rujukan. Keputusan menunjukkan bahawa, approximation formula Davidenkov-Spiridonova memberikan keputusan yang lebih baik berbanding dengan approximation formula Bridgman.

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## SYMBOL

## DESCRIPTION

ρ	curvature of the longitudinal stress trajectory
а	radius at necking area
R	Radius of curvature
$\bar{\sigma}_z$	Equivalent stress
F	Force
Α	Area
d	Diameter
$\varepsilon_N$	natural strain
$\mathcal{E}_{c}$	engineering strain
σ	true stress
Ε	Young's Modulud
darepsilon	increment of strain
$d\varepsilon_e$	elastic strain increment
$darepsilon_p$	plastic strain increment
$d\sigma$	stress increment
$E_t$	tangent modulus
ν	Poisson's ratio
$\varepsilon_p$	plastic strain
$\sigma_Y$	yield stress
k	characteristic constants

Ν	characteristic constants
$\sigma_{v_M}$	von-Mises equivalent stress
$\sigma_{T_r}$	Tresca equivalent stress
$\sigma_{ heta}$	circumferential stress
$\sigma_r$	radial stress
$ au_{max}$	maximum shear

## **CHAPTER 1**

#### **INTRODUCTION**

The tensile test is one of the important standard engineering procedures to characterize some important elastic and plastic variables which are related to the mechanical behavior of materials. The mechanical properties that can be determined from the tensile test include yield stress, ultimate tensile stress, Young's modulus and Poisson's ratio of the material. The engineering stress-strain curve does not give a true indication of the deformation characteristics of a metal because it is based entirely on the original dimensions of the specimen, and these dimensions change continuously during the test. Furthermore, due to the non uniform stress and strain distributions existing at the neck for high levels of axial deformation, it has been long recognized that significant changes in the geometric configuration of the specimen have to be consider in order to properly describe the material response during the whole deformation process up to the fracture stage. Although in many engineering applications the design of structural parts is restricted to the elastic response of the material involved, the knowledge of their behavior beyond the elastic limit is relevant since plastics effect usually large deformations take place in many manufacturing procedures such as metal forming. Other important applications of elastoplastic models for metal are crashworthiness, impact problems, inelastic buckling of thin-walled structures, and superplastic forming. Analytically the derived formula serves in practical use to evaluate the complex stress and strain state in the necking region of tensile test. Among the others, the formula proposed by Bridgman and Davindenkov-Spirinova are well-known and considered in this research. This research project will analyze the flow curve behavior of ductile material by using the FEM to validate the approximation formula which is introduced by Brigdman and Davindenkov-Spirinova

## 1.1 Objective of the Project

The main objective of this research project is to find the flow curve of a necking specimen which is subjected tensile load. Most of the current research is focused on the material behavior under tensile load which finally breaks in a ductile or brittle manner. The procedure of the current research includes analyzing the equivalent stress and strain distribution e.g by applying the von Mises equivalent stress, the surface metallography investigation, the critical analysis of plastic material properties and carrying the tensile test to compare the result with analytical and numerical solution. In this research project, the flow curve of tensile specimens will be analyzed by using the finite element method.

The major steps of this research project for both simple plate and round cylinder specimen are define as below:

- To investigate the influence of parameters (Radius of curvature and ratio of deformation <sup>σ<sub>ν</sub></sup>/<sub>σ</sub>) on the equivalent stress and strain for the tensile test.
- To validate the approximation formulae of equivalent plastic stress and strain introduced by Bridgman and Davindekov-Spiridonova with finite element method analysis and estimate the involved error.
- Study the necking of the material
- Analyze stress distribution after necking

• Analyze the stress and strain state in all points of the material

#### 1.2 Scopes of the project

In this work, the main objective is to investigate the flow curve after necking of a tensile specimen. After necking a multiaxial stress state exits which finally results in the failure of the specimen. The project will be carried out by using Marc Mentat (Finite Element Analysis software). The FEA results will be used to validate some approximation formulae e.g (Bridgman 1964). A round cylinder specimen will be analyzed in this research project.

#### **1.3 Methodology of the Project**

The goals are mainly achieved by finite element method analysis which are then compared the results with some approximation formulae and estimate the error. This research covers the study of equivalent stress and strain distribution. The numerical method of analysis is carried out by using the finite element method Marc Mentat (MSC) software. The research methodology flowcharts systematically highlighting the major work of the study are shown in Figure 1.1.



Figure 1.1: Research methodology flowcharts

#### **1.4 Thesis Organization**

The thesis consists of six chapters. The current chapter discusses the problem definition, justification for carrying out the research, objective, scopes and the research methodology of the project.

Chapter 2 reviews some of the previous researches on the tensile analysis of equivalent plastic stress and strain. A brief description and discussion of the basic fundamentals of stress-strain relationship are introduced in Chapter 3. Besides that, the fundamental concepts and theories that are related to the research are reviewed in this chapter.

Numerical investigations using finite element models are given in Chapter 4. The problems are solved by using Marc Mentat MSC finite element software. Chapter 5 will be discussing about the results of equivalent stress and strain evaluate by using some of approximation formulae and the results will be compare with the finite element analysis. The results and discussion in details will be state in this chapter.

The conclusions are stated in Chapter 6 together with the summary of the findings of the research and suggestions for other areas of additional research.