A NOVEL SOLUTION TO TRAVELING SALESMAN PROBLEM USING FUZZY SETS, GRAVITATIONAL SEARCH ALGORITHM, AND GENETIC ALGORITHM

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To my beloved parents, thank you for always being there for me, supporting me and encouraging me to be the best that I can be.

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ABSTRACT

Throughout the history, many researchers have come across a problem in their real lives, called the Traveling Salesman Problem (TSP). This problem is to find the shortest route of a traveling salesperson that starts at a home city, visits a prescribed set of other cities and returns to the starting city. The distance travelled in such a tour obviously depends on the order in which the cities are visited and, thus, the problem is to find an optimal ordering of the cities. TSP is a representative of a large class of problems known as the combinatorial optimization problems. TSP is among the most important areas to explore, since it is very easy to describe, but very difficult to solve. There are several real-life applications of the TSP in several areas of knowledge including mathematics, computer science, operations research, genetics, engineering, and electronics. Many of the existing methods by which the TSP solved, suffer from considerable problems, such as lack of accuracy and speed. In this study, a hybrid method of TSP is proposed that can yield accurate results and relatively fast, without being trapped in a local optimum during the searching. In this approach, suitable fuzzy matrices are used to illustrate the details of the problem. A new optimization algorithm, called the Gravitational Search Algorithm (GSA) is applied to solve TSP as precise as possible, and finally the Genetic Algorithm (GA) finds the final answer in an acceptable amount of time. The results have illustrated that the proposed method are convincing compared to other related methods.

ABSTRAK

Sorotan sejarah telah memperlihatkan bahawa banyak penyelidikan yang dijalankan berkait langsung dengan kehidupan sebenar bagi mencari satu penyelesaian; antaranya adalah Masalah Perjalanan Jarak Jurujual (MPJJ). MPJJ bertujuan mencari jarak terpendek bagi seorang jurujual yang memulakan perjalanan dari bandar asal, seterusnya ke bandar yang lain sebagaimana yang tercatat dalam senarai set pra-lawatan bandar, dan akhirnya kembali semula ke tempat asal. Jarak perjalanan bergantung kepada turutan bandar yang dilawati. Oleh yang demikian, MPJJ bertujuan mencari penyelesaian yang optimal berdasarkan turutan bandar tersebut. MPJJ mewakili satu kelas masalah yang besar dan dikenali sebagai satu masalah pengoptimuman kombinatorik. Namun demikian, pendekatan ini agak sukar untuk diselesaikan secara optimum. Terdapat banyak penggunaan sebenar MPJJ, dan ini termasuklah dalam bidang matematik, sains komputer, penyelidikan operasi, genetik, kejuruteraan dan elektronik. Namun begitu, kebanyakkan kajian terdahulu MPJJ mempunyai banyak kelemahan seperti tahap ketepatan yang terpinggir dan kelajuan yang amat perlahan. Oleh yang demikian, kajian ini mencadangkan kaedah hibrid MPJJ yang berupaya menghasilkan keputusan yang tepat, dan pantas tanpa terperangkap dalam optimum setempat semasa proses carian. Pendekatan cadangan ini mengunakan matriks kabur yang bersesuaian untuk memperlihatkan keperincian MPJJ. Satu algoritma pengoptimuman yang baru, iaitu Algoritma Carian Graviti (ACG) dilaksanakan terhadap masalah MPJJ bagi mendapatkan keputusan yang setepat mungkin, manakala Algoritma Genetik (AG) dilaksanakan bagi memutuskan satu keputusan akhir dalam julat masa yang tersaur. Hasil kajian mempamerkan bahawa kaedah yang dicadangkan telah memberikan keputusan yang memberangsangkan berbanding kaedah setara yang lain.

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LIST OF SYMBOLS

X_i	Position of the *i*th agent
x_i^d	Position of the *i*th agent in *d*th dimension
M_i	Mass of the object *i*
$F_{ij}^d(t)$	Force from mass *j* on mass *i* at time *t* in *d*th dimension
$G(t)$	Gravitational constant at time *t*
R_{ij}	Distance between the two objects *i* and *j*
$a_i^d(t)$	Acceleration of the agent *i* at time *t* in *d*th dimension
$v_i^d(t)$	Velocity of the agent *i* at time *t* in *d*th dimension
$fit_i(t)$	Fitness value of the agent *i* at time *t*
best(t)	Fitness value of the strongest agent at time *t*
worst(t)	Fitness value of the weakest agent at time *t*

LIST OF ABBREVATIONS

TSP	Traveling Salesman Problem
NP	Non-Deterministic Polynomial-time
GSA	Gravitational Search Algorithm
GA	Genetic Algorithm
STSP	Symmetric Traveling Salesman Problem
ATSP	Asymmetric Traveling Salesman Problem
GTSP	Generalized Traveling Salesman Problem
BB	Branch and bound
EA	Evolutionary Algorithm
SGA	Simple Genetic Algorithm
ACO	Ant Colony Optimization
PSO	Particle Swarm Optimization
COA	Chaotic Optimization Algorithm
TSPLIB	Traveling Salesman Problem Library

CHAPTER 1

INTRODUCTION

1.1 Introduction

In the middle 1930s computer science was not yet a well defined academic discipline. In fact, fundamental concepts, such as *algorithm*, or *computational problem*, have not been formalized until just some year before. In these years the Austrian mathematician Karl Menger invited the research community to consider from a mathematical point of view the following problem taken from the everyday life. A traveling salesman has to visit, exactly once, each one of a list of m cities and then return to the home city. He knows the cost of traveling from any city i to any other city j. Thus, which is the tour of the least possible cost the salesman can take?

The Traveling Salesman Problem (for short, TSP) was born. However, many other researchers have come across the same problem in their real lives in the same era of time. Therefore, it could be said that the real origin of the TSP is obscure. There will be a more detailed discussion on the history of the TSP in chapter 2.

The TSP is a representative of a large class of problems known as the combinatorial optimization problems. Among them, TSP is one of the most important, since it is very easy to describe, but very difficult to solve. Actually, TSP belongs to the NP-hard class. Hence, an efficient algorithm for TSP (that is, an algorithm computing, for any TSP instance with m nodes, the tour of least possible cost in polynomial time with respect to m) probably does not exist. More precisely, such an algorithm exists if and only if the two computational classes P and NP coincide, a very improbable hypothesis, according to the last years of research developments.

From a practical point of view, it means that it is quite impossible to find an exact algorithm for any TSP instance with m nodes, for large m, that has a behavior considerably better than the algorithm which computes any of the $(m-1)!$ possible distinct tours, and then returns the least costly one.

There are several applications of the TSP that extend beyond the route planning of a traveling salesman. If we are looking for applications, a different approach can be used. Given a TSP instance with m nodes, any tour passing once through any city is a feasible solution, and its cost leads to an upper bound to the least possible cost. Algorithms that construct in polynomial time with respect to m feasible solutions, and thus upper bounds for the optimum value are called heuristics. In general, these algorithms produce solutions but without any quality guarantee as to how far their cost is from the least possible one. If it can be shown that the cost of the returned solution is always less than k times the least possible cost, for some real number $k>1$, the heuristic is called a k-approximation algorithm. The heuristics will be defined in more details later on.

1.2 Problem Background

The Travelling Salesman Problem (TSP) is a problem in combinatorial optimization, studied in operations research and theoretical computer science. As explained before, given a list of cities and their pair-wise distances, the task is to find the shortest possible tour that visits each city exactly ones. Mathematical problems related to the travelling salesman problem were treated in the 1800s by the Irish mathematician Sir William Rowan Hamilton and the British mathematician Thomas Penyngton Kirkman. The Austrian mathematician, Karl Menger, originated what we know as the TSP.

Eventually, the TSP gained notoriety as the prototype of a hard problem in the combinatorial optimization. Nowadays as the vast field of artificial intelligence has thrived, many algorithms are designed and used to solve this classic problem in the best possible way. However, it should be noted that many of these existing algorithms have serious flaws. Some only work for very small problems, some fail noticeably when it comes down to accuracy, and some consume a long time in spite of managing to solve the problem.

In this study, The Gravitational Search Algorithm and the Genetic Algorithm are used to solve the TSP. fuzzy matrices will be used to show the position, velocity, acceleration, and the force between the masses, which will be described in details in chapter 2. The operators used in the original algorithm are replaced by the ones of the fuzzy matrices. Hopefully, a method relatively faster and more accurate than many other existing methods will be achieved.

1.3 Problem Statement

The TSP has several applications even in its purest form, and is always in the center of attention, in order for it to be solved in the best possible way.

Many of the current methods by which the traveling salesman problem is solved, suffer from some considerable but overlooked problems. As the TSP has several practical applications, the accurate and fast results are usually vital. Various heuristics and approximation algorithms, which yield solutions in a reasonable time, have been devised, although most methods can find solutions for large problems with the unsatisfactory probability of 10-20% away from the optimal solution.

The method that will be proposed in this study will hopefully overcome the problems in accuracy and time. The two main algorithms used in this study, which will be explained more thoroughly later on, are the Gravitational Search Algorithm (GSA) and the Genetic Algorithm (GA). Each of these has a certain advantage and a certain disadvantage. By creating a hybrid method, these two algorithms complete each other and a reasonably high-performance TSP solver will be obtained.

Hence, the hypothesis of the study can be stated as:

The Gravitational Search Algorithm and the Genetic Algorithm could yield better accuracy in a reasonable time for the TSP.

1.4 Aim of the Study

The aim of this study is to present a new approach to solving the Traveling Salesman Problem, which can perform better than many of the previous methods in terms of accuracy and speed. The results obtained from the approach will be compared with the ones of the other related algorithms to support the claims made in the study.

1.5 Objectives of the Study

The main goal of this study is to present a new hybrid method by which the TSP can be solved relatively accurately and fast, without being trapped in the local optimum during the process of search.

The Travelling Salesman Problem, or TSP for short, is easy to state:

> "Given a finite number of Cities along with the cost of travel between each pair of them, find the cheapest way of visiting all the cities and returning to your standing point."

In this study, a new solution is suggested that uses three different methods: Fuzzy, the Gravitational Search Algorithm, and the Genetic Algorithm .In most solutions to this much-talked-about problem, several significant flaws can be noticed.

Some of the most important ones include the long duration of time it takes to solve the problem and the local optima which might be found as the accurate answer, while trying to achieve the best one. Hence, this study is carried out with the following objectives:

1. To develop fuzzy matrices for indicating the TSP.
2. To optimize the solution to the point where the process of searching slows down, with the help of the GSA.
3. To reach the precise answer using GA in a short period of time.
4. To evaluate the proposed method by comparing the results with previous methods.

1.6 Scopes of the Study

In this study, the scopes are as follows:

1. Proper fuzzy matrices will be used to show the details of TSP required for GSA.
2. Gravitational Search Algorithm will be applied to solve the problem almost precisely.
3. Genetic Algorithm will find the final answer in a short amount of time.

As mentioned earlier, in this study, a new optimization algorithm based on the law of gravity, namely Gravitational Search Algorithm (GSA) is used to solve the TSP. This algorithm is based on the Newtonian gravity law [70]: Every particle in the universe attracts other particles with a force that is directly proportional to the product of the masses and inversely proportional to the distance between them.

The GSA is basically established to improve the results of algorithms like PSO, while keeping the same complexity. The high point of this algorithm, which this study has used, is that it avoids finding a local optimum as the real answer. However, when the objects, defined in the algorithm, get closer to the global optimum, the speed decreases, and therefore achieving the final result might take a bit longer. Hence, the Genetic Algorithm is used in the end, in order to speed up the process and find the accurate answer fast.

1.7 Significance of the Study

This study is aimed to propose a new solution to a classical combinatorial optimization problem that has been the center of the attention of many researchers for the past few decades. The method presented here, takes advantage of a recently-proposed soft-computing technique, called the Gravitational Search Algorithm. The study can contribute both to the level of performance of the Traveling Salesman problem solving methods and an evaluation of the mentioned soft-computing method.

1.8 Organization of Report

The report for this study consists of four chapters. Chapter 1 presents an introduction to the study, problem background, objectives, scope and significance of this study. Chapter 2 reviews the combinatorial optimization problems, the Traveling Salesman Problem Variations and Applications, classic approaches to solving the TSP, population-based algorithms for solving the TSP, and the Gravitational Search Algorithm, which plays an important role in the proposed method. Chapter 3 discusses the methodology used in this study. It also explains details of the datasets being used. Chapter 4 presents the results of the proposed method and their comparison with other methods presented in this study. Finally, chapter 5 concludes the study.