

NUMERICAL SOLUTION OF ELASTOHYDRODYNAMIC  
LUBRICATION WITH NON-NEWTONIAN FLUID FLOW

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A project report submitted in partial fulfillment of  
the requirements for the degree of  
Master of Engineering (Mechanical)

Faculty of Mechanical Engineering  
University of Technology Malaysia

APRIL 2010

*To my beloved parent and family especially my wife Fadzlina and my daughters  
Nani and Dira. Thanks for your understanding during the hard time.*

## **ACKNOWLEDGEMENT**

Thanks Allah, finally I was able to complete this project research.

I wish to express my sincere appreciation to my supervisor, Assoc. Prof. Dr. Kahar bin Osman and Mr Zubil bin Bahak and to all my other post graduate lecturers, for the encouragement, guidance, advices, criticism and friendship. Without their guidance and wide ranging knowledge, this thesis would not have been the same as presented here.

I also wish to thank to all my course mates for your support, kind words, encouragement, comments, and for our thought-provoking conversations. My heartfelt thanks also to each and everyone, who have shared their beautiful words, lovely thoughts, and heartfelt blessings, and finally, to all of my dear friends and fellow colleagues, postgraduate's students and others who have provided assistance and support on various occasions.

## ABSTRACT

In this project, a line contact elastohydrodynamic lubrication is modelled through an infinite cylinder on a plane to represent the application of roller bearing. The numerical solution of this problem is presented embedded with free volume model of pressure-viscosity equation using Doolittle-Tait model. Newton-Raphson method is used to solve the Reynolds equation simultaneously with pressure-viscosity equation and elastic deformation in order to find the pressure profile and film thickness. The behavior of non-Newtonian fluid also was investigated using power law fluid model. The bearing performances parameters such as pressure, film thickness and friction coefficient of lubricated contacts are calculated. Non-uniform grid also used to improve the pressure spike stability. The results show that the peak pressure increases as the parameters such as velocity, load, material parameter and power law index are increases and the spike is found to shift to the center of roller. The coefficient of friction is reduced as the power law index and slide to roll ratio are decreased.

## ABSTRAK

Dalam projek ini, pelinciran sentuhan garisan elastohydrodynamic dimodelkan melalui silinder tidak terhingga ke atas suatu satah untuk mewakili penggunaan gelas jenis *roller* . Penyelesaian berangka terhadap permasalahan pelinciran *elastohydrodynamic* jenis sentuhan garisan dipersembahkan dengan memasukan model isipadu bebas terhadap persamaan tekanan-kelikatan menggunakan model *Doolittle-Tait*. Kaedah *Newton-Raphson* digunakan untuk menyelesaikan persamaan Reynolds secara serentak bersama persamaan tekanan-kelikatan dan elastik deformasi untuk mengira tekanan dan ketebalan filem. Kelakuan bendalir bukan *Newtonian* juga diasasat menggunakan model hukum kuasa bendalir. Perkara yang melibatkan prestasi gelas seperti tekanan, ketebalan filem and pekali geseran terhadap permukaan yang mengalami pelinciran dikira. Saiz sela yang tidak sekata telah digunakan untuk memperbaiki kesan ketidakstabilan pepaku tekanan. Hasil kajian menunjukkan puncak tekanan meningkat apabila parameter seperti halaju, beban, parameter bahan dan indeks hukum kuasa ditingkatkan dan puncak tekanan didapati bergerak ke tengah silinder. Pekali geseran berkurang apabila indeks hukum kuasa dan nisbah gelincir-golek dikurangkan.

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## LIST OF SYMBOL

$b$	–	Hertzian contact radius (m)
$C_j$	–	Weigthing factor used in linear equation
$D_{ij}$	–	Influence coefficient
$E$	–	Elastic Modulus (Pa)
$E'$	–	Effective Elastic Modulus
$G$	–	Material Parameter
$H$	–	Dimensionless film thickness
$H_e$	–	Dimensionless film thickness where $dP/dX=0$
$H_0$	–	Dimensionless Central Film Thickness
$h$	–	Film thickness (m)
$i,j$	–	Nodes
$N$	–	Number of nodes used in linear equation
$N_{max}$	–	Maximum Number of nodes
$n$	–	power law index
$p$	–	Pressure (Pa)
$P$	–	Dimensionless Pressure
$p_H$	–	Hertzian Pressure (Pa)
$R$	–	Radius of roller (m)
$u$	–	Average entrainment rolling speed (m/s)
$U$	–	Dimensionless Speed
$W$	–	Dimensionless Load
$w$	–	Load per unit length (N/m)

$x$	–	Distance along rolling direction (m)
$X$	–	Dimensionless Distance
$\alpha$	–	Piezoviscous coefficient ( $\text{m}^2/\text{N}$ )
$\eta$	–	Viscosity of lubricant ( $\text{Ns}/\text{m}^2$ )
$\eta_0$	–	Viscosity at ambient pressure ( $\text{Ns}/\text{m}^2$ )
$\bar{\eta}$	–	Dimensionless Viscosity
$\rho$	–	Density of lubricant ( $\text{kg}/\text{m}^3$ )
$\rho_0$	–	Density at ambient pressure ( $\text{kg}/\text{m}^3$ )
$\bar{\rho}$	–	Dimensionless Density
$\nu$	–	Poisson Ratio
$\tau$	–	Shear Stress (Pa)

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## **CHAPTER 1**

### **INTRODUCTION**

#### **1.1 Background**

The whole idea of this study is to write a programming code that can provide a solution for fluid film lubrication problem related to elastohydrodynamic lubrication. This can further be applied to investigate the effect of parameters on bearing design and performance such as pressure, film thickness and friction.

Once completed, the programming code will also be highly suitable for investigating the behavior of Non-Newtonian lubricant in elastohydrodynamic lubrication problem.

##### **1.1.1 Tribology and Fluid Film Lubrication**

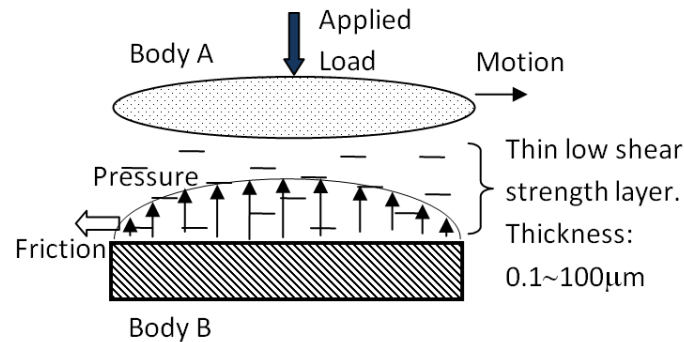
During the past few decades, there has been an increasing research interest in the area of friction and wear characteristics of various bearing designs, lubricants,

and materials for bearings. This discipline, named Tribology, is concerned with the friction, lubrication, and wear of interacting surfaces in relative motion. Tribology, which derived from the Greek word 'tribos' meaning rubbing or sliding. It is a new field of science defined in 1967 by a committee of the Organization for Economic Cooperation and Development. The word tribology has gained gradual acceptance from time to time [ 1 ].

Wear is the major cause of material wastage and loss of mechanical efficiency and performance. Any improvement to reduce wear can result in considerable cost savings. Friction is a principle cause of wear and energy dissipation. Friction is a phenomenon that cannot be eliminated but can be reduced. There are basically three methods in reducing friction: use of bearings, air cushioning and lubrication. Considerable cost savings can be made by controlling and improving the friction. Among others, lubrication is an effective means of controlling wear, agent of a cooling system and reducing friction.

In fluid film lubrication, thin low shear strength layers of gas, liquid and solid (Figure 1.1) are interposed between two surfaces in order to improve the smoothness of movement of one surface over another and to prevent damage. These layer of material separate contacting solid bodies and are usually very thin and very difficult to observe by naked eyes. In general, the thickness of these films ranging from 1~100  $\mu\text{m}$ .



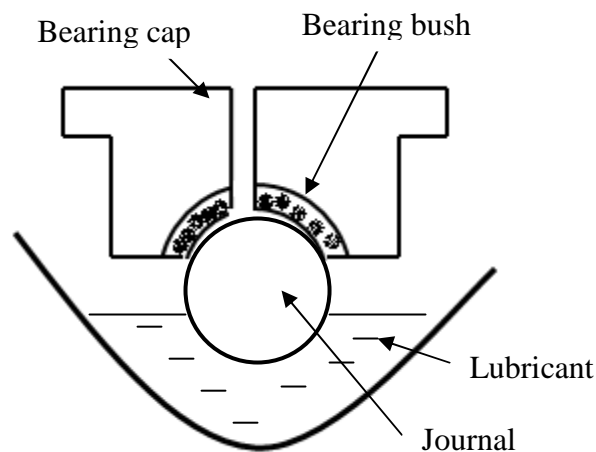


**Figure 1.1** Lubricant film

Detailed analysis of gaseous or liquid films is usually termed **hydrodynamics lubrication (HD)**. While lubrication by solid is termed **solid lubrication**. A specialized form of hydrodynamics lubrication involving physical interaction between the contacting bodies and the liquid lubricant is termed **elastohydrodynamics (EHD) lubrication (EHL)** and is considerable practical significance. Another form of lubrication involves the chemical interactions between contacting bodies and the liquid lubricant is termed **boundary and extreme pressure lubrication**. A form of lubrication that operates involving the external force is termed **hydrostatic lubrication** where liquid or gaseous lubricant is forced into the space between contacting bodies.

The foundations of fluids-film lubrication theory were established by Osborne Reynolds in 1886, following earlier experimental work on railroad journal bearing by Beauchamp Tower [2]. On Tower's test rig (Figure 1.2), he was discovered that the frictional resistance is nearly constant regardless of the bearing load but the frictional resistance increases with sliding speed. Tower also reported that beautiful pressure distribution was observed as was expected. Reynolds then considered this apparent phenomenon of Tower's experiments and suggested that film lubrication was a hydrodynamic action and depended on the viscosity of the lubricant. The lubricant adhere to both the stationary and moving surfaces of the bearing and is dragged into a wedge-shaped gap, converging in the direction of

motion, where it develops a fluid pressure sufficient to carry the load. He developed a governing differential fluid flow equation for a wedge shape film, known as Reynolds equation which is simplified form of Navier-Stokes equation. This theory is the basis of hydrodynamics (HD) and elastohydrodynamics (EHD) lubrication (EHL) that is applied by all researchers.



**Figure 1.2** Tower's test rig

There were two other important fundamental equations derived around that time. In 1881, Hertz [3] published his study of the contact between two spherical bodies, to show how the surfaces deform due to high, local pressure. In 1892, Barus determined how the viscosity of oils increases as a function of pressure.

Bearing technology developed rapidly in subsequent years, but attempts to explain the effective lubrication of highly stressed non-conformal conjunction, such as those in gears, on the basis of hydrodynamic principles alone remained ineffective throughout most of the first half of the twentieth century. It was recognized that the very high pressures associated with such non-conformal conjunctions would enhance the lubricant viscosity and also causes substantial local elastic deformation and that both effects might contribute to satisfactory film deformation.

In the elastohydrodynamics (EHD) lubrication (EHL) regime, the elastic deformation of the bounding solids is large and affects the hydrodynamics lubrication process. EHL is important in non-conforming, heavily loaded contacts such as point contacts of ball bearing, line contacts of roller bearing and gear teeth. The EHL phenomena also occur in some low elastic modulus contacts of high geometry conformity such as lip seals, conventional journal bearing with soft liner, wiper blade and nip coating. In the heavily loaded contacts, high pressure can lead both to change in the viscosity of the lubricant and the elastic deformation of the bodies in contact.

## **1.2 Objective and scope of study**

The objective of this study is to determine the solution for Elastohydrodynamic lubrication for bearing using non-Newtonian Fluid using power law fluids.

Numerical method will be used in the prediction of the parameters that need to be considered in bearing design analysis such as pressure, film thickness and coefficient of friction. As a comparison, this project would also investigate the behavior of non-Newtonian fluids in evaluating the performance of bearing and its lubricant.

The scopes for this project include:

- (i) Solution is limited to two-dimensional line contact problem only.
- (ii) Bearing speed is to be assumed in steady state.
- (iii) Temperature is to be assumed constant.
- (iv) Non-Newtonian Fluid is limited to power law behavior only.

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