Parallel System for Abnormal Cell Growth Prediction Based on Fast Numerical Simulation

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Abstract. The paper focuses on a numerical method for detecting, visualizing and monitoring abnormal cell growth using large-scale mathematical simulations. The discretization of multi-dimensional partial differential equation (PDE) is based on finite difference method. The predictor system depending on users input data via a user interface, generating the initial and boundary condition generated from parabolic or elliptic type of PDE. The processing large sparse matrixes are based on multiprocessor computer systems for abnormal growth visualization. The multi-dimensional abnormal cell has produced the numerical analysis and understanding results at the target area for the potential improvement of detection and monitoring the growth. The development of the prediction system is the combinations of the parallel algorithms, open source software on Linux environment and distributed multiprocessor system. The paper ends with a concluding remark on the parallel performance evaluations and numerical analysis in reducing the execution time, communication cost and computational complexity.

Keywords: parallel system, abnormal cell growth simulation, IADE method, AGE method, distributed memory systems.

1 Introduction

This paper focuses on the development of mathematical modelling and simulation for the human tumour growth involving for simulation up to 10^{-9} computational skill. The implementation of parallel algorithm of the large sparse matrix is visualised and monitor the tumour growth characteristics. The development of the parallel system is suitable as the prediction of the abnormal cell growth such as breast, brain and thyroid tumor. Based on the initial and boundary condition on the properties of tumor cells growth, some mathematical models have been developed to quantify the proliferation and invasion dynamics of tumor within anatomically accurate heterogeneous human cells [7][4]. Andrews et al. [1] and Angelis et al. [2] was presented mathematical

C.H. Hsu and V. Malyshkin (Eds.): MTPP 2010, LNCS 6083, pp. 222–231, 2010.

model of glioblastoma tumor spheroid invasion and an advective-diffusion model for solid tumor evolution in vivo respectively. The contribution of this paper is a fast numerical method to visualize abnormal cell growth in multidimensional space. This research relates to visualizing abnormal cells growth depending on users input data by the implementation of fast numerical methods in providing the high speed and accurate prediction.

1.1 Problem Statement

The mathematical modeling and simulation on one, two and three dimensional of PDE for abnormal tumor growth involving parameter estimation, growth rate, diffusion coefficient, initial and boundary condition. The model of Swanson, Alvord and Murray [10], Bellomo and Preziosi [11], 3D Parabolic Equation (3D) are discretized using finite difference method. Next, numerical schemes called New Iterative Alternating Decomposition Explicit (IADE), [5], New Alternating Group Explicit Methods (AGE) [7], BRIAN [7][4] and Red Black Gauss Seidel methods (RBGS) are used for solving linear system of equation generated by the finite difference method. The flowchart (see Figure 1) below illustrates the machine interface of multi-dimensional mathematical simulation for the abnormal cell growth.

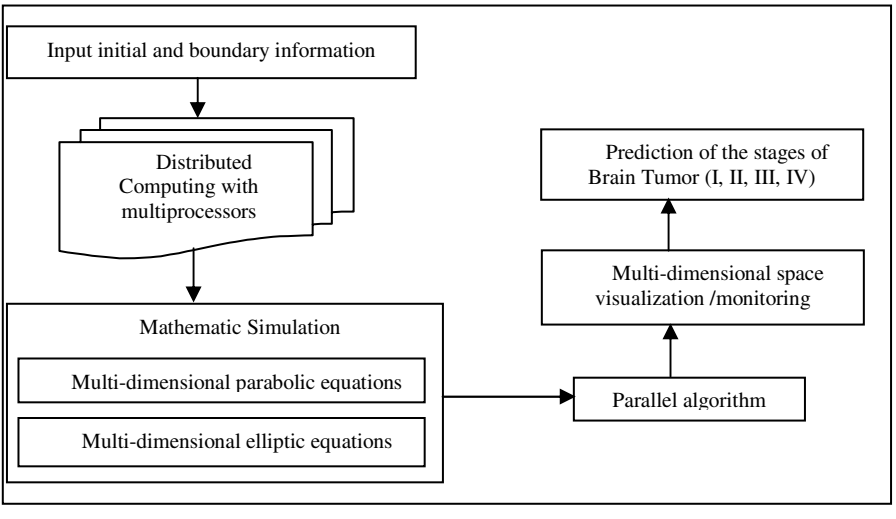

Fig. 1. Flowchart illustrating machine interface of the abnormal cell growth visualization software of the present invention

2 Solving the Cell Growth Model

The model of Swanson, Alvord and Murray [10] is as below:

$$\frac{\partial c}{\partial t} = \nabla \cdot (D(x)\nabla c) + \rho c \tag{1}$$

where is the concentration of cells at any position x and time t, ρ is the units of per day and represents the net rate of growth of abnormal cells, including proliferation,

loss and death, D denotes the units of cm2 per day and represents the diffusion coefficient of cells in brain tissue, = D_g (constant for x in grey matter) and = D_w (constant for x in white matter).

∇ represents the spatial gradient. The diffusion term describes the active migration of the glioma cells using a simple Fickian diffusion [12] where cells move from regions of higher to lower densities. Abnormal cells are assumed to grow exponentially.

The 2D PDE with parabolic type is referred to as the model implemented by Bellomo and Preziosi [11]:

$$\frac{\partial u}{\partial t} = -\nabla \cdot (Wu) + \nabla \cdot (Q \nabla u) + \tau - Lu \tag{2}$$

Where, $\tau = \tau(u)$ is the proliferation coefficient,

$L = L(u)$ is the death coefficient,

W is the diffusion coefficient,

Q is the drift velocity field and in two dimensions, $W = (P, R)$

The equation (equation 3) below is a three Dimensional Parabolic Equation (3D) that is implemented in this study,

$$\frac{\partial u}{\partial t} = -\frac{\partial (Pu)}{\partial x} - \frac{\partial (Ru)}{\partial y} - \frac{\partial (Su)}{\partial z} + Q\frac{\partial^2 u}{\partial x} + \frac{\partial Q \partial u}{\partial x \partial x} + Q\frac{\partial^2 u}{\partial y^2} + \frac{\partial Q \partial u}{\partial y \partial y} + Q\frac{\partial^2 u}{\partial z^2} + \frac{\partial Q \partial u}{\partial z \partial z} + \tau - Lu \tag{3}$$

2.1 Parallel Systems

The abnormal cell mathematical model involves some parameters need to be counted. Obviously the problems need huge repetitive calculations on large amounts of data to produce the valid results.

To achieve implementation in parallel, suitable software and hardware must be provided as a platform that supports the simultaneous execution of heterogeneous distributed memory computer systems. The software services are built on Linux platform which is based on open source technology. The algorithm is programmed in C language while the software tools involve a web browser, Apache web server, Perl-CGI, HTML pages, PHP, XML, UML, PVM, MPI, C programming and MySQL database. A related factor is that multiple computers often have more total main memory than a single computer, enabling problems that require larger amounts of main memory to be tackled. The software is designed and analysed for its efficiency, distributional, robustness, adaptiveness and stability of the algorithms.

3 Fast Numerical Simulation

Like many other numerical methods, the approach begins with the domain discretization of multivariable the parabolic equation. The implement of fast numerical simulation under consideration are some explicit methods called IADE, AGE and Brian [9]. RBGS is chosen as the comparison method among the IADE, AGE and Brian methods. The originality of this research shows that these methods have found a generally applicable high-level expression of parallelism in solving a large system of

heat transfer of multidimensional PDEs. Those numerical methods are straightforward and well suit implemented into the distributed memory parallel computer platform [8]. The numerical analysis and the parallel performance evaluations are analyzed from the aspect accuracy, convergence efficiency, effectiveness and temporal performance.

3.1 AGE Fractional Scheme on Based on the Douglas-Rachford Formula

$$(G_1 + rI)u_{(r)}\left(k + \frac{1}{4}\right) = (rI - G_1 - 2G_2 - 2G_3 - 2G_4)u_{(r)}^{(k)} + 2f \tag{4}$$

$$(G_2 + rI)u_{(r)}^{\left(k+\frac{1}{2}\right)} = G_2 u_{(r)}^{(k)} + ru_{(r)}^{\left(k+\frac{1}{4}\right)} \tag{5}$$

$$(G_3 + rI)u_{(r)}^{\left(k+\frac{3}{4}\right)} = G_3 u_{(r)}^{(k)} + ru_{(r)}^{\left(k+\frac{1}{2}\right)} \tag{6}$$

$$(G_4 + rI)u_{(r)}^{(k+1)} = G_4 u_{(r)}^{(k)} + ru_{(r)}^{\left(k+\frac{3}{4}\right)} \tag{7}$$

It is notable that the Brain Method is dependent on the modification values of the acceleration parameter *r with* the time step up to *(k+1.5).*

3.2 Interactive Alternating Decomposition Explicit Method [10]

At time level$\left(k + \frac{1}{2}\right)$,

$$(I + \alpha G_1)u^{k+\frac{1}{2}} = (I + (\alpha + 2r)G_1)(I + 2rG_2) \tag{8}$$

At time level $(k + 1)$,

$$(I + \alpha G_2)u^{k+1} = u^{k+\frac{1}{2}} + \alpha G_2 u^{(k)} \tag{9}$$

With $\alpha = \frac{1}{12} - \frac{2}{3}r$ and $\beta = \frac{2r(3v-2r)}{3}$

4 Parallel Algorithms

The IADE, AGE, Brian and RBGS iterative using parallel computing converging very fast compares to the its sequential algorithm. The fast numerical methods are allowed to divide arrays among local processors and to minimize the communication as shown in Figure 2. The implementation of domain decomposition technique requires the non-overlapping sub-domain. The parallel algorithm of fully explicit of IADE, AGE, BRIAN and RBGS can therefore be used to maximum advantage on distributed memory system. The flowchart in Figure 2 illustrates the parallel algorithm and the communication activities among the server and client processors. The data structure has to be decomposed where given set of ranges assigned to particular processors must be physically sent to those processors in order for the processing to be done. The result must be sent back to whichever processors responsible for coordinating the global result. As the parallel computing executes the same task on multiple processors simultaneously, it can reduce the time execution of computational complexity.

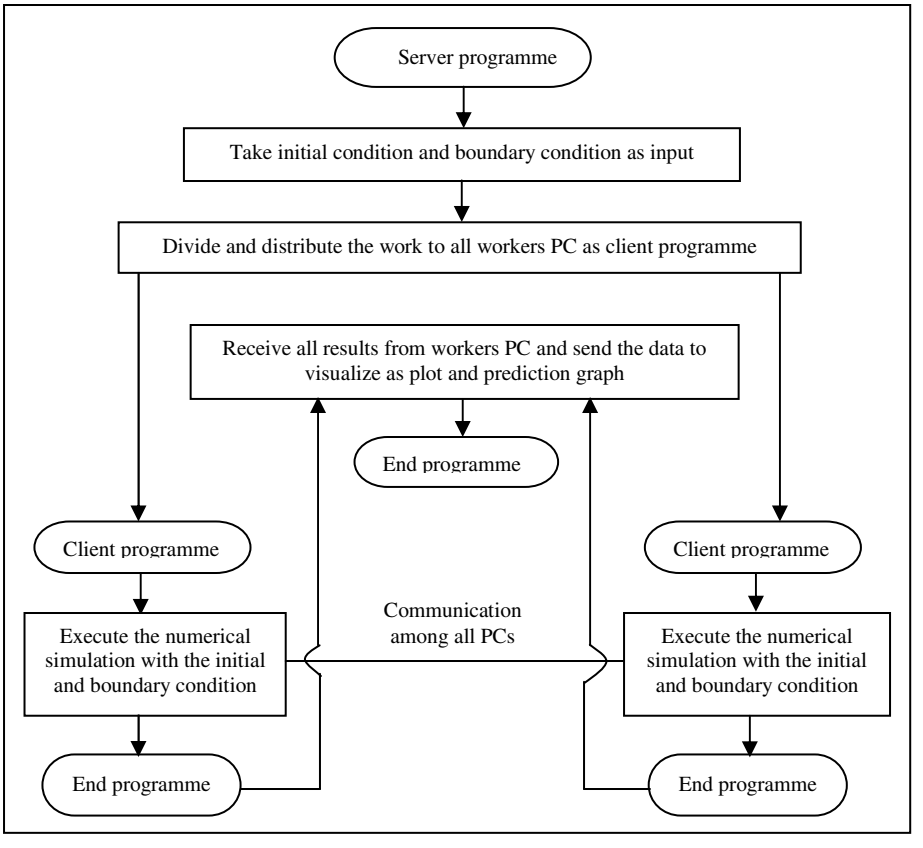

Fig. 2. Parallel algorithms of server and client procedures for the mathematical simulation appeared on the workflow of the high performance computing system

5 Results and Discussion

Based on the initial and boundary condition on the properties of abnormal cells growth, some mathematical models have been modified to quantify the proliferation and invasion dynamics of tumor within anatomically accurate heterogeneous human cells [7][4]. The results obtained reflect the numerical analysis and the parallel performance evaluations are analyzed of accuracy, convergence, stability, errors, residual, speedup, efficiency, effectiveness and temporal performance. The parallel system development are presented the abnormal cell growth graphically and in highly accurate prediction based on the IADE, AGE, Brian and RBGS methods.

The growth rates increase in the first 24 days consistently. After 24 days, the abnormal cells become highly active in evolution. The abnormal cell will grows more than 1000 cells after 30 days.

By data experiment, the values used in solving the mathematical problem are:

- Drift coefficients of P, R and S are 10-5, 10-7 and 10-9 respectively, while the diffusion coefficient, Q is 10-3.
- For each of the proliferation coefficient, and death coefficient, L the values which have been taken are 10^{-2} and 10^{-8}.

A heat capacity for 3000 J/KgºC and a density of 920 Kg/m3 were used for both normal and cancerous tissue. The metabolic heat generation was 450 W/m3 and blood perfusion rate for normal tissue was considered to be 0.00018 ml/s/ml, and for cancerous tissue values of 29,000 W/m3 and 0.009 ml/s/ml were used to account for the higher blood perfusion rates and metabolic heat generation respectively. An effective thermal conductivity of 0.42 W/m, as estimated by Chato J. C., [3], was used for both normal and cancerous tissue. 3D visualization in Figure 3 shows that heat transfer coefficient of 5 W/m2 K was used as a convective boundary condition at the skin surface to account for natural convection.

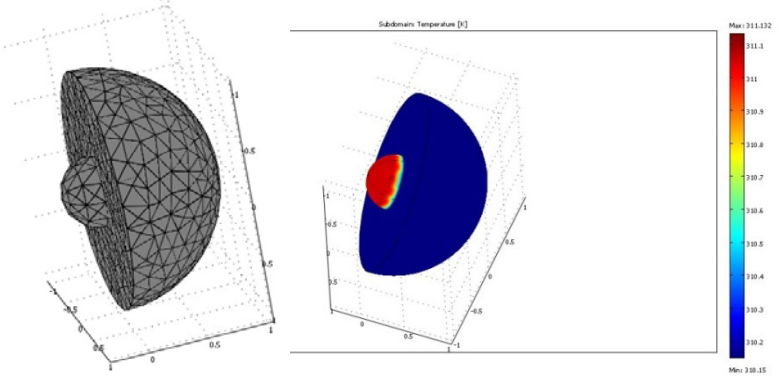

Fig. 3. 3D models of the Mesh of abnormal cell (left) and 3D sub-domain model of abnormal cell visualization by heat detection (right)

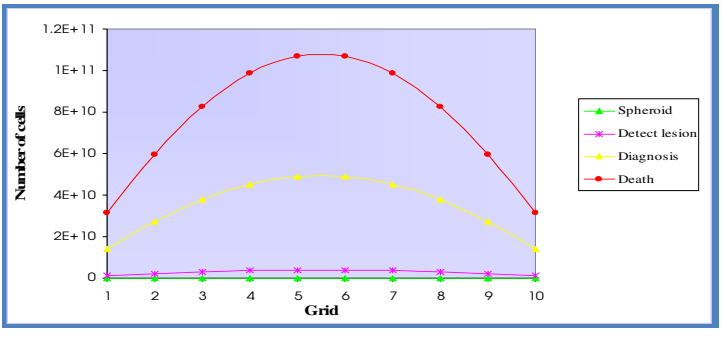

Fig. 4. The number of cell abnormal cell growth in 100 days and visualization for cell growth of untreated patient

In Figure 4, the concentration gradient of abnormal cell growth cells is represented by the curves, the spheroid and macroscopic levels involving the detected lesion, diagnosis and the death. The curve for spheroid level shows that the abnormal cells form only a small dense lesion. However, the cells are highly diffuse after have been detected and when the patients are untreated, the diffusion of these abnormal cells will caused death in only a short term.

5.1 Parallel Performance Evaluation

The speedup, efficiency and effectiveness for the parallel algorithm of fast numerical method are obtained in Table 1. The parallel performance measurements of the parallel algorithm are improved by the increasing of the number of processors. The speedup is increasing as the number of processors, p increasing. Comparable speedups are obtained for all applications with 20 processors. This phenomenon as stated in Amdahl's law shows that the communication cost will eventually become dominant over local computation cost after a certain stage. The efficiency of the parallel algorithm decreases versus number of processors. This could be explained by the fact that several factors lead to the increasing of the communication cost, delay, idle time and load balancing of computational complexity. The effectiveness is increasing up to the optimum number of processors. Then it decreases as number of processors goes beyond 12 processors due to message latency within the processors.

Table 1. Time execution, speedup, efficiency, and effectiveness in respect of the quantity of processors

Num of proc	execution	Speedup	Efficiency	Effective
1	0.0119	1	37.45315	83.4376
4	0.0025	4.739	20.771075	468.4684
8	0.0012	50.649	11.83382	1008.369
12	0.00017	70.005	9.206275	3076.425
16	0.00015	76.337	8.184757	2369.456
20	0.00001	83.062	7.025689	2298.36

In order to assess the communication and computational ratio of the proposed parallel algorithm, some formulations of communication cost is utilized. The computational complexity and communication cost of fast numerical method is given as the following,

Computation complexity: $(15(m\text{-}1)^3+14m^3) T + (13(m\text{-}1)^3+12m^3) D$ (10)

Communication cost: $$12L\left(\frac{(m\times m\times m)}{2}\right)t_{data} + 6L\left(t_{starup} + t_{idle}\right) \quad (11)$$

Where m is the size of matrix, T and D are arithmetic operations, L is a number of iterations.

Table 2 shows the communications cost depends on number of processors. When the numbers of processors increase, consequently the communication time will also increase and the computational time will decrease. The communication cost is

decrease when the numbers of data-sending m and number of iterations to the convergence criteria are increases. The ratio of computational and communication times show that the computational time is much greater that communication time. The domain decomposition strategy is efficiently utilized, straightforward run on parallel computer systems. As the results, sending a larger value of messages and the behavior of communication cost are reflected the communication cost. Figure 5 shows the parallel performance evaluations for IADE, AGE and Brian methods in terms of time execution, speedup, efficiency, effectiveness and temporal performance.

Table 2. Parallel execution, communication, idle times, the ratio of computational and communication time for (100 X 100 X 100) and (140 X 140 X 140) size of matrices

M	(100 X 100 X 100)					
p	**parallel**	**comp**	**ratio**	**comm**	**comm1**	**idle**
5	99.9	58.59	1.42	41.31	29.43	11.87
%		58.7		41.4	29.5	11.9
10	73.01	29.29	0.67	43.71	29.43	14.27
%		40.1		59.9	40.3	19.6
15	69.11	19.73	0.4	49.27	29.43	19.84
%		28.6		71.4	42.7	28.8
20	63.99	15.15	0.31	48.85	29.43	19.42
%		23.7		76.3	46.0	30.3

M	(140 X 140 X 140)					
p	**parallel**	**comp**	**ratio**	**comm**	**comm1**	**idle**
5	193	111.8	1.38	81.25	49.14	32.11
%		57.9		42.1	25.5	16.6
10	136.2	55.89	0.7	80.34	49.14	31.21
%		41.0		59.0	36.1	22.9
15	121	36.66	0.43	84.29	49.14	35.15
%		30.3		69.7	40.6	29.1
20	110.7	27.15	0.32	83.55	49.14	34.41
%		24.5		75.5	44.4	31.1

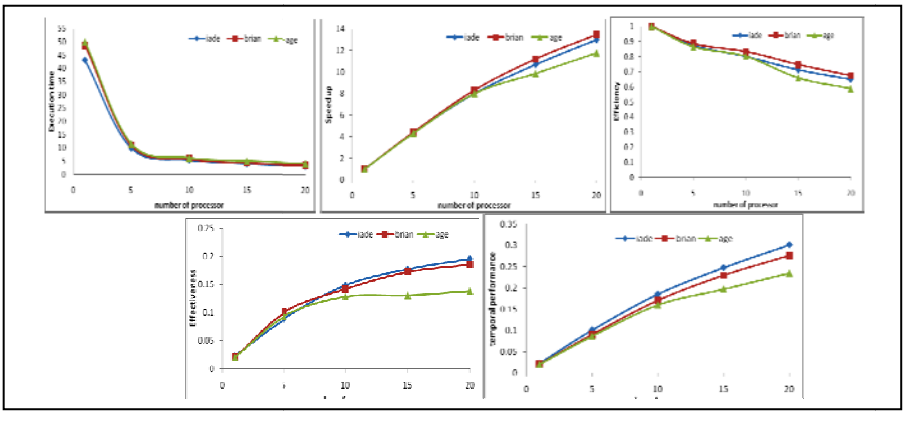

Fig. 5. Parallel performance evaluations for IADE, AGE and Brian in terms of time execution, speedup, efficiency, effectiveness and temporal performance

6 Conclusion

In this paper, the parallel system of this study is being able to visualize the abnormal cell growth in multi-dimensional space using distributed computer systems. . The finite difference method and a weighted approximation of parabolic equation have been formed accurately to obtain the volume of the abnormal cell and rate of cell growth in time and space. The alternative parallel algorithms; IADE, AGE and Brian methods are a well suite and relatively efficient for parallel system development on the distributed parallel computer systems. These schemes are extremely effective in reducing data storage accesses, computational and communication cost on a distributed computer systems. As a conclusion, the parallel system of fast numerical methods can be concluded as a well suite performance tools in solving the grand challenge application of multi-dimensional PDE problems for abnormal cell growth prediction. The future research will be the implementation the higher order of parabolic-elliptic type for the parallel system optimization in terms of numerical analysis and parallel performance evaluations.

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