Journal of Quality Measurement and Analysis *JQMA* 20(2) 2024, 35-48
e-ISSN: 2600-8602 *http://www.ukm.my/jqma*
https://doi.org/10.17576/jqma.2002.2024.04

HYPERSTABILITY RESULTS FOR THE GENERAL LINEAR FUNCTIONAL EQUATION IN NON-ARCHIMEDEAN 2-BANACH SPACES

(Hiperstabil bagi Persamaan Fungsi Linear Am di dalam Ruang Bukan Archimedean 2- Banach)

SHUJAUDDIN SHUJA, AHMAD FADILLAH EMBONG* & NOR MUHAINIAH MOHD ALI

ABSTRACT

Let X be a 2-normed space over $\mathbb{R}$, Y be a non-Archimedean 2-Banach space over non-Archimedean field $\mathbb{K}$, $r, s\ \mathbb{R} \setminus \{0\}$, and $R, S\ \in \mathbb{K} \setminus \{0\}$. In this paper, a short preface on non-Archimedean 2-Banach spaces $(Y, \|\cdot,\cdot\|_*)$ is given. Then, we reformulate the Brzdek fixed point theorem in non-Archimedean 2-Banach spaces. Using the Brzdek fixed point method, we prove hyperstability results of the general linear functional equation $h(rx + sy) = Rh(x) + Sh(y)$, $x, y\ \in X,$ in non-Archimedean 2-Banach spaces. In fact, under some natural assumptions on control function $\gamma: X \times X \times Y \to [0, \infty)$, we show that every map satisfying $\|h(rx + sy) - Rh(x) - Sh(y), z\|_* \leq \gamma(x, y, z),\ \ x, y\ \in X,\ \ z\ \in Y$, is hyperstable in the class of functions $h: X \to Y$.

Keywords: non-Archimedean 2-Banach spaces; general linear functional equation; hyperstability; fixed point method

ABSTRAK

Biarkan X menjadi ruang nyata 2-norma di atas $\mathbb{R}$, Y menjadi ruang bukan Archimedean 2-norma di atas $\mathbb{K}$, $r, s\ \mathbb{R} \setminus \{0\}$, dan $R, S\ \in \mathbb{K} \setminus \{0\}$. Dalam penyelidikan ini, ringkasan mengenai ruang bukan Archimedean 2-Banach $(Y, \|\cdot,\cdot\|_*)$diberikan. Kemudian, kami merumuskan semula teorem titik tetap Brzdek di dalam ruang bukan-Archimedean 2- Banach. Menggunakan kaedah titik tetap Brzdek, kami membuktikan ciri-ciri hiperstabil bagi persamaan fungsi linear am $h(rx + sy) = Rh(x) + Sh(y),\ \ x, y\ \in X,$ di dalam ruang bukan Archimedean 2-Banach. Malah, di bawah beberapa andaian pada fungsi kawalan $\gamma: X \times X \times Y \to [0, \infty)$, kami menunjukkan bahawa setiap pemetaan yang memenuhi kondisi $\|h(rx + sy) - Rh(x) - Sh(y), z\|_* \leq \gamma(x, y, z),\ \ x, y\ \in X,\ \ z\ \in Y$, adalah hiperstabil di dalam fungsi kelas $h: X \to Y$.

Kata kunci: ruang bukan Archimedean 2-Banach; persamaan fungsi linear am; hiperstabil; kaedah titik tetap

1. Introduction

In this article, $\mathbb{N}$ indicates the set of positive integers and $\mathbb{R}$ is the set of real numbers. As well as, it is denoted that $\mathbb{N}_0 = \mathbb{N} \cup \{0\},\ X_0 = X \setminus \{0\},\ \mathbb{R}_+ := [0, \infty),\ \mathbb{N}_{m_0}$ the set of integers $\geq m_0$, and E_1, E_2 are normed spaces.

For the first time, a normed space which does not follow Archimedean property was introduced by Hensel (Hensel 1897) and called non-Archimedean or ultrametric spaces. The concept of non-Archimedean spaces has some nice applications which are accessible in (Katsaras & Beloyiannis 1999; Khrennikov 1997; Nyikos 1999). Let us recall some basic definitions on non-Archimedean field and non-Archimedean 2-Banach spaces (Brzdek & Cieplinski 2011; El-Fassi 2018; El-Fassi *et al.* 2020).

Definition 1.1. The field $\mathbb{K}$ which is equipped with a function (valuation) $|\cdot|_*: \mathbb{K} \to \mathbb{R}_+$ is called non-Archimedean field if the following conditions hold for all $\alpha, \beta \in \mathbb{K}$.

- $|\alpha|_* = 0$ if and only if $\alpha = 0$,
- $|\alpha\beta|_* = |\alpha|_*|\beta|_*$,
- $|\alpha + \beta|_* \leq \max\{|\alpha|_*, |\beta|_*\}$.

The pair $(\mathbb{K}, |\cdot|_*)$is called a valued field. From Definition1.1, we have that in any non-Archimedean field, $|1|_* = |-1|_* = 1$ and $|n|_* \leq 1$ for all $n \in \mathbb{N}$. The valuation $|\cdot|_*$ is trivial valuation if $\forall \alpha \in \mathbb{K}$, $|\alpha|_* = 1$, $\alpha \neq 0$, and $|0|_* = 0$. In this article, we work with non-Archimedean non-trivial field and write non-Archimedean field instead non-Archimedean non-trivial field. The p-adic number field is a well-known example of non-Archimedean fields. In this direction we refer to (Diagana & Ramaroson 2016).

Definition 1.2. Let E be a vector space with dimension more than 1 over non-Archimedean field$\mathbb{K}$. A function $\|\cdot,\cdot\|_*: E^2 \to \mathbb{R}_+$ is a non-Archimedean 2-norm if it satisfies the following conditions:

- $\|x, y\|_* = 0$ if and only if x and y are linearly dependent,
- $\|x, y\|_* = \|y, x\|_*$,
- $\|\alpha x, y\|_* = |\alpha|_*\|x, y\|_*$,
- $\|x + y, z\|_* \leq \max\{\|x, z\|_*, \|y, z\|_*\}$,

for all $x, y, z \in E, \alpha \in \mathbb{K}$. The pair $(E, \|\cdot,\cdot\|_*)$ is a non-Archimedean 2-normed space. For a complete non-Archimedean 2-normed space, we call non-Archimedean 2-Banach space.

Example 1.3. (El-Fassi *et al.* 2020) Let $\mathbb{K}$ be a non-Archimedean field, $\lambda \in \mathbb{K}$ and $x = (x_1, x_2)$, $y = (y_1, y_2) \in E = \mathbb{K}^2$ with $\lambda x = (\lambda x_1, \lambda x_2)$ and $x + y = (x_1 + y_2, x_2 + y_2)$ then, the non-Archimedean 2-norm on $E$is defined by $\|x, y\|_* = |x_1 y_2 - x_2 y_1|_*$.

Lemma 1.4. (El-Fassi *et al.* 2020) *Let* $(E, \|\cdot,\cdot\|_*)$ *be non-Archimedean 2-normed space, if* $x \in E$, *and* $\|x, y\|_* = 0$, *then* $x = 0$ *for all* $y \in E$.

The following definition states the concept of hyperstability. The family of all functions mapping a non-empty set B into a non-empty set A is symbolized with A^B.

Definition 1.5. (El-Fassi 2018) Consider A as a non-empty set, (Z, d)a metric space, $\gamma: A^n \to \mathbb{R}_+$, B a non-empty subset of A^n , and $\mathcal{H}_1, \mathcal{H}_2$ mapping a nonempty $\mathcal{D} \subset Z^A$ into Z^{A^n}. The conditional equation

$$\mathcal{H}_1\varphi(x_1, \cdots, x_n) = \mathcal{H}_2\varphi(x_1, \cdots, x_n), \quad (x_1, \cdots, x_n) \in B \tag{1}$$

is γ-hyperstable if every $\varphi_0 \in \mathcal{D}$, satisfying

$$d\big(\mathcal{H}_1\varphi_0(x_1, \cdots, x_n), \mathcal{H}_2\varphi_0(x_1, \cdots, x_n)\big) \leq \gamma(x_1, \cdots, x_n), \quad (x_1, \cdots, x_n) \in B \tag{2}$$

is a solution to Eq. (1). Briefly, every approximately solution in the format of Eq. (2) is the solution of Eq. (1).

The notion results from Definition 1.5 reminds the Ulam stability problem for various equations. Let us recall the Ulam's question.

Ulam's question. Let $(G_1,\diamond)$ be a group and $(G_2,\circledast)$ be a metric group with a meter $\rho: G_2 \times G_2 \to [0,\infty)$. Given $\delta > 0$, does there exist $\epsilon > 0$ such that if a function $h: G_1 \to G_2$ satisfies the inequality $\rho\big(h(x \diamond y), h(x) \circledast h(y)\big) \leq \delta$ for all $x, y \in G_1$ then there exist a homomorphism $l: G_1 \to G_2$ with $\rho\big(h(x), l(x)\big) < \epsilon$ for all $x \in G_1$?

The first answer to this question was given by D. H. Hyers (Hyers 1941) in Banach spaces concerning the stability of additive functional equation. The most classical result which states Ulam's stability is seen in following equation called Cauchy equation

$$f(x+y) = f(x) + f(y), \quad x, y \in E_1. \tag{3}$$

The following theorem describes Ulam's stability in better manner.

Theorem 1.6. *(El-Fassi 2018) Let E_1, E_2 be normed spaces, and $f: E_1 \to E_2$ fulfils the next inequality for all $x, y \in E_1$,*

$$\|f(x+y) - f(x) - f(y)\| \leq \theta(\|x\|^p + \|y\|^p) \tag{4}$$

where θ and p are real constants with $\theta > 0$ and $p \neq 1$. Then, the following statements are valid.

(1) If E_2 is a complete normed space and $p \geq 0$, a unique solution $H: E_1 \to E_2$ of Eq. (3) exists such that

$$\|f(x) - H(x)\| \leq \frac{\theta}{|1-2^{p-1}|}\|x\|^p, \quad x \in E_1 \setminus \{0\}. \tag{5}$$

(2) The f is additive if $p < 0$, means that Eq. (3) hold.

Now, the Theorem 1.6 is stating the first result of stability considered by Hyers (Hyers 1941) if $p = 0$ and Aoki (Aoki 1950) for $0 < p < 1$. Gajda (Gajda 1991) beside obtaining result for $p > 1$, illustrated an example showing that this theorem fails when $p = 1$ to answer a question of Rassias regarding values of p. Furthermore, Rassias (Rassias 1991) observed that a similar result is valid also for $p < 0$ (Rassias & Semrl 1992; Bourgin 1951). Now, it is clear that statement (2) is valid, and it states that f must be additive. This case proved for the first time (Lee 2008) and next in (Brzdek 2013a) on restricted domain. The stability of some functional equations have been obtained by many authors (see, e.g. Cieplinski 2011; Brzdek 2013c; El-Fassi & Kabbaj 2015a; Gordji & Savadkouhi 2010; Khodaei *et al.* 2012; Mirmostafaee 2010; Moslehian & Rassias 2007; Moslehian & Sadeghi 2008) in various spaces.

For the first time, the concept of hyperstability was used probably in (Maksa & Pales 2001); but, the first hyperstability result was published in (Bourgin 1949) concerning ring homomorphism. Hyperstability results for various functional equations can be found in survey paper (Brzdek & Cieplinski 2013) and related articles (Bahyrycz & Piszczek 2014; Brzdek 2013a; 2013b ; 2014; 2015; El-Fassi & Kabbaj 2015b ; El-Fassi *et al.* 2016; El-Fassi 2017; El-Fassi 2018; El-Fassi *et al.* 2020; Gselmann 2009; Almahalebi & Chahbi 2017) in both Banach and non-Archimedean Banach spaces.

Definition 1.7. (Piszczek 2015) Let E_1, E_2 be normed spaces over numbered fields $\mathbb{F}$ and $\mathbb{K}$ respectively. A function $h: E_1 \to E_2$ is (r,s)-linear provided it satisfies the functional equation

$$h(rx + sy) = Rh(x) + Sh(y), \qquad x, y \in E_1 \tag{6}$$

for $r, s\ \mathbb{F} \setminus \{0\}$, and $R, S \in \mathbb{K} \setminus \{0\}$.

Putting $r = s = R = S = 1$, we get Cauchy functional equation while the Jensen functional equation results when $r = s = R = S = \frac{1}{2}$ in Eq. (6). In 2015, the hyperstability of general linear functional Eq. (6) studied by M. Piszczek (Piszczek 2015) in Banach spaces. I. El-Fassi (El-Fassi *et al.* 2018) obtained some hyperstability result for Eq. (6) in β-Banach spaces. In 2024, S. Shuja studied some results on hyperstability of the general linear functional equation was studied in non-Archimedean Banach spaces (Shuja *et al.* 2024).

In this paper, we use an extended version of fixed point method which is resulted from Brzdek and Cieplinski (2011)'s Theorem 1 for proving hyperstability of Eq. (6) in non-Archimedean 2-Banach spaces. This method contains some hypotheses and a theorem. Before start the theorem, we mention the following hypotheses to state the theorem easily.

(**H1**) X is a nonempty set, Y is a non-Archimedean 2-Banach space over a non-Archimedean valued field $\mathbb{K}$, $h_1, \cdots, h_k: X \to X$and $G_1, \cdots, G_k: X \times Y \to \mathbb{R}_+$ are given maps.

(**H2**) $\mathcal{T}: Y^X \to Y^X$is an operator satisfying

$$\|\mathcal{T}\xi(x) - \mathcal{T}\mu(x), z\|_* \le \max_{1\le i\le k}\left\{G_i(x,z)\ \left\|\xi\big(h_i(x)\big) - \mu\big(h_i(x)\big),\ z\right\|_*\right\},$$

for all $\xi, \mu \in Y^X$, $(x,z) \in X \times Y$.

(**H3**) $\Lambda: \mathbb{R}_+^{X\times Y} \to \mathbb{R}_+^{X\times Y}$ is a non-decreasing operator defined by

$$\Lambda\delta(x,z) := \max_{1\le i\le k}\{G_i(x,z)\delta\big(h_i(x)\big)\}, \quad \delta \in R_+^{X\times Y}, \quad x \in X, z \in Y.$$

Now, it is easy to offer the following fixed point theorem.

Theorem 1.8. *Suppose that hypothesis* (**H1**) - (**H3**) *are valid and the functions* $\varepsilon: X \times Y \to \mathbb{R}_+$ *and* $\varphi: X \to Y$ *satisfy the next conditions*

$$\|\mathcal{T}\varphi(x) - \varphi(x),\ z\|_* \le \varepsilon(x,z), \quad (x,z) \in X \times Y,$$

$$\lim_{n\to\infty} \Lambda^n \varepsilon(x,z) = 0, \quad (x,z) \in X \times Y,$$

then, a unique $\psi \in Y^X$ *exists with*

$$\psi(x) := \lim_{n\to\infty} \mathcal{T}^n \varphi(x), \quad x \in X,$$

which is the fixed point of $\mathcal{T}$ *such that,*

$$\|\varphi(x) - \psi(x),\ z\|_* \le \sup_{n\in\mathbb{N}_0} \Lambda^n \varepsilon(x,z), \quad (x,z) \in X \times Y.$$

Proof. The proof can be directly driven from Brzdek and Cieplinski (2011)'s Theorem 1. In fact, Brzdek proved this theorem in non-Archimedean complete metric spaces, so in the light of Brzdek work, we can easily replace non-Archimedean metric$d$of Brzdek theorem with non-Archimedean 2-norm $\| \cdot,\cdot \|_*$ in our work. □

2. Hyperstability Results

In this section, we prove hyperstability results of the general functional equation in Eq. (6) in non-Archimedean 2-Banach spaces.

Theorem 2.1. *Let X be a real 2-normed space over* $\mathbb{R}$ *with dimension more than 1, and* Y *be a non-Archimedean 2-Banach space over* $\mathbb{K}$*,* $r, s\ \mathbb{R} \setminus \{0\}$*, and* $R, S \in \mathbb{K} \setminus \{0\}$*,* $l \geq 0$*,* $p, q \in \mathbb{R}$*,* $p + q < 0$ *and* $h: X \to Y$ *satisfies*

$$\|h(rx + sy) - Rh(x) - Sh(y), z\|_* \leq l\ \|x, z\|^p\ \|y, z\|^q, \quad (x, y, z) \in X_0^2 \times Y \tag{7}$$

then, h *is* (r, s)*-linear.*

Proof. When $p + q < 0$, so one of p or q must be negative. Suppose $q < 0$. Take $m_0 \in \mathbb{N}$ such that

$$\alpha_m := \max\left\{\left|\frac{1}{R}\right|_* |r + sm|^{p+q}, \left|\frac{S}{R}\right|_* m^{p+q}\right\} < 1, \quad \text{for } m \in \mathbb{N}_{m_0}0.$$

Fix $m \geq m_0$ and replace y by mx in Eq. (7), then

$$\| \frac{1}{R} h(rx + smx) - \frac{S}{R} h(mx) - h(x), z\|_* \leq \frac{l}{|R|_*}\ m^q\ \|x, z\|^{p+q}, \ (x, z) \in X_0 \times Y. \tag{8}$$

We define operator $\mathcal{T}_m: Y^{X_0} \to Y^{X_0}$ by

$$\mathcal{T}_m \xi(x) := \frac{1}{R} \xi\big((r + sm)x\big) - \frac{S}{R} \xi(mx), \quad \xi \in Y^{X_0}, x \in X_0,$$

and write

$$\varepsilon_m(x, z) := \frac{l}{|R|_*}\ m^q\ \|x, z\|^{p+q}, \quad (x, z) \in X_0 \times Y \tag{9}$$

then, the Eq. (8) changes to

$$\|\mathcal{T}_m h(x) - h(x), z\|_* \leq \varepsilon_m(x, z), \quad (x, z) \in X_0 \times Y.$$

Let also define $\Lambda_m: \mathbb{R}_+^{X_0 \times Y} \to \mathbb{R}_+^{X_0 \times Y}$ by

$$\Lambda_m \delta(x, z) := \max\left\{\left|\frac{1}{R}\right|_* \delta\big((r + sm)x, z\big), \left|\frac{S}{R}\right|_* \delta(mx, z)\right\}, \quad \delta \in \mathbb{R}_+^{X_0}, x \in X_0$$

clearly, Λ_m has the form what was before described in **(H3)** with $k = 2$, $h_1(x) = (r + sm)x$, $h_2(x) = mx$, $G_1(x,z) = \left|\frac{1}{R}\right|_*$ and $G_2(x,z) = \left|\frac{S}{R}\right|_*$ for $x \in X_0$, $z \in Y$. Furthermore, for every $\xi, \mu \in Y^{X_0}$, $(x,z) \in X_0 \times Y$,

$$\|\mathcal{T}_m\xi(x) - \mathcal{T}_m\mu(x), z\|_* = \left\|\frac{1}{R}\xi\big((r+sm)x\big) - \frac{S}{R}\xi(mx) - \frac{1}{R}\mu\big((r+sm)x\big) + \frac{S}{R}\mu(mx), z\right\|_* \leq \max\left\{\left|\frac{1}{R}\right|_* \left\|(\xi-\mu)\big((r+sm)x\big), z\right\|_*, \left|\frac{S}{R}\right|_* \|(\xi-\mu)(mx), z\|_*\right\}$$
$$= \max_{1\leq i\leq 2}\left\{G_i(x,z)\left\|(\xi-\mu)\big(h_i(x)\big), z\right\|_*\right\}.$$

Therefore, **(H2)** is also valid. Let us show that for each $(x,z) \in X_0 \times Y$,

$$\Lambda_m^n\, \varepsilon_m(x,z) = \frac{l}{|R|_*} m^q\, \|x,z\|^{p+q}\, \alpha_m^n \tag{10}$$

where $\alpha_m := \max\left\{\left|\frac{1}{R}\right|_* |r+sm|^{p+q}, \left|\frac{S}{R}\right|_* m^{p+q}\right\}$. For $n = 0$ the result is trivial and Eq. (10) satisfies Eq. (9). Next, we assume that Eq. (10) holds for $n = k$, for $k \in \mathbb{N}$. Then, we have

$$\Lambda_m^{k+1}\varepsilon_m(x,z) = \Lambda_m\left(\Lambda_m^k\varepsilon_m(x,z)\right) = \max\left\{\left|\frac{1}{R}\right|_* \Lambda_m^k\varepsilon_m\big((r+sm)\,x, z\big), \left|\frac{S}{R}\right|_* \Lambda_m^k\varepsilon_m(mx,z)\right\}$$
$$= \max\left\{\frac{l}{|R|_*}m^q\, \|x,z\|^{p+q}\, \alpha_m^k \left|\frac{1}{R}\right|_* |r+sm|^{p+q},\ \frac{l}{|R|_*}\, m^q\, \|x,z\|^{p+q}\, \alpha_m^k \left|\frac{S}{R}\right|_* m^{p+q}\right\}$$
$$= \frac{l}{|R|_*}\, m^q\, \|x,z\|^{p+q}\, \alpha_m^k \max\left\{\left|\frac{1}{R}\right|_* |r+sm|^{p+q},\ \left|\frac{S}{R}\right|_* m^{p+q}\right\}$$
$$= \frac{l}{|R|_*}\, m^q\, \|x,z\|^{p+q}\, \alpha_m^{k+1}, \quad (x,z) \in X_0 \times Y$$

It means that Eq. (10) holds for $n = k + 1$, so Eq. (10) holds for all $n \in \mathbb{N}_0$. It can be obtained from Eq. (10) that for all $x \in X_0$

$$\lim_{n\to\infty} \Lambda^n\varepsilon_m(x,z) = 0$$

Therefore, by Theorem 1.8, a unique solution $F_m : X_0 \to Y$of the equation

$$F_m(x) = \frac{1}{R}F_m\big((r+sm)x\big) - \frac{S}{R}F_m(mx), \quad x \in X_0$$

exists such that

$$\|h(x) - F_m(x), z\|_* \leq \sup_{n\in\mathbb{N}_0}\left\{\frac{l}{|R|_*}m^q\|x,z\|^{p+q}\alpha_m^n\right\}, \quad (x,z) \in X_0 \times Y.$$

Moreover, for all$x \in X_0$,

$$F_m(x) := \lim_{n\to\infty}\mathcal{T}_m^n h(x).$$

Now we show that for every $(x,y,z) \in X_0^2 \times Y$ and $n \in \mathbb{N}_0$,

$$\|\mathcal{T}_m^n h(rx+sy) - R\mathcal{T}_m^n h(x) - S\mathcal{T}_m^n h(y), z\|_* \le l\, \alpha_m^n\, \|x,z\|^p\, \|y,z\|^q. \tag{11}$$

Therefore, if $n = 0$, then Eq. (11) is simply Eq. (7). Take $k \in \mathbb{N}_0$ and suppose that Eq. (11) holds for $n = k$. Then for $n = k + 1$,

$$\begin{aligned}
&\|\mathcal{T}_m^{k+1} h(rx+sy) - R\mathcal{T}_m^{k+1} h(x) - S\mathcal{T}_m^{k+1} h(y), z\|_* = \Big\|\tfrac{1}{R}\mathcal{T}_m^k h\big((r+sm)(rx+sy)\big) - \\
&\tfrac{S}{R}\mathcal{T}_m^k h\big(m(rx+sy)\big) - R\tfrac{1}{R}\mathcal{T}_m^k h\big((r+sm)x\big) + R\tfrac{S}{R}\mathcal{T}_m^k h(mx) - S\tfrac{1}{R}\mathcal{T}_m^k h\big((r+sm)y\big) + \\
&S\tfrac{S}{R}\mathcal{T}_m^k h(my), z\Big\|_* \\
&\le \max\Big\{\Big|\tfrac{1}{R}\Big|_* \big\|\mathcal{T}_m^k h\big((r+sm)(rx+sy)\big) - R\mathcal{T}_m^k h\big((r+sm)x\big) - D\mathcal{T}_m^k h\big((r+ \\
&sm)y\big), z\big\|_*, \Big|\tfrac{S}{R}\Big|_* \big\|\mathcal{T}_m^k h\big(m(rx+sy)\big) - R\mathcal{T}_m^k h(mx) - S\mathcal{T}_m^k h(my), z\big\|_*\Big\} \\
&\le \max\Big\{\Big|\tfrac{1}{R}\Big|_*\, l\, \alpha_m^k\, \|x,z\|^p\, \|y,z\|^q\, |r+sm|^{p+q}\ \Big|\tfrac{S}{R}\Big|_*\, l\, \alpha_m^k\, \|x,z\|^p\, \|y,z\|^q\, m^{p+q}\Big\} \\
&= l\, \alpha_m^{k+1}\, \|x,z\|^p\, \|y,z\|^q, \quad (x,y,z) \in X_0^2 \times Y,
\end{aligned}$$

so, by induction it is clear that Eq. (11) holds for every $n \in \mathbb{N}_0$. Letting n tends to ∞ in Eq. (11), then

$$F_m(rx+sy) = RF_m(x) + SF_m(x), \quad x \in X_0.$$

From Theorem 1.8, there exist a sequence $\{F_m\}_{m \ge m_0}$ of linear functions on X_0 such that

$$\|h(x) - F_m(x), z\|_* \le \sup_{n \in \mathbb{N}_0} \Big\{\frac{l}{|R|_*}\, m^q\, \|x,z\|^{p+q}\, \alpha_m^n\Big\}, \quad (x,z) \in X_0 \times Y$$

which implies that

$$\|h(x) - F_m(x), z\|_* \le \frac{l}{|R|_*} m^q\, \|x,z\|^{p+q}, \quad (x,z) \in X_0 \times Y$$

and let m tends to ∞ in last inequality, it results that h is (r,s)-linear. □

In the next theorem, we consider the previous theorem when condition $p + q > 0$.

Theorem 2.2. *Let X be a real 2-normed space over* $\mathbb{R}$ *with dimension more than 1, and Y be a non-Archimedean 2-Banach space over non-Archimedean field* $\mathbb{K}$, $r, s\ \mathbb{R} \setminus \{0\}$, *and* $R, S \in \mathbb{K} \setminus \{0\}$, $l \ge 0$, $p, q \in \mathbb{R}$, $p + q > 0$ *and* $h: X \to Y$ *satisfies Eq. (7), then h is* (r,s)*-linear.*

Proof. Take $m_0 \in \mathbb{N}$, such that,

$$\alpha_m := \max\Big\{\Big|\frac{1}{R}\Big|_* \Big|r - \frac{r}{m}\Big|^{p+q}, \Big|\frac{S}{R}\Big|_* \Big|\frac{r}{sm}\Big|^{p+q}\Big\} < 1, \quad \text{for } m \in \mathbb{N}_{m_0}$$

Since $p + q > 0$, one of p, q must be positive. Let $q > 0$ and replace y by $-\frac{r}{sm}x$ in Eq. (7), then

$$\left\|\frac{1}{R}h\left(\left(r-\frac{r}{m}\right)x\right)-\frac{S}{R}h\left(-\frac{r}{sm}x\right)-h(x),z\right\|_* \le \frac{l}{|R|_*}\left|\frac{r}{sm}\right|^q \|x,z\|^{p+q} \tag{12}$$

for $(x,z)\in X_0\times Y$. Define $\mathcal{T}_m: Y^{X_0}\to Y^{X_0}$ by

$$\mathcal{T}_m\xi(x) := \frac{1}{R}\xi\left(\left(r-\frac{r}{m}\right)x\right)-\frac{S}{R}\xi\left(-\frac{r}{sm}x\right),\quad \xi\in Y^{X_0}, x\in X_0$$

and write

$$\varepsilon_m(x,z) := \frac{l}{|R|_*}\left|\frac{r}{sm}\right|^q \|x,z\|^{p+q},\quad (x,z)\in X_0\times Y \tag{13}$$

then, the Eq. (12) takes the form

$$\|\mathcal{T}_m h(x)-h(x),z\|_*\le\varepsilon_m(x,z),\quad (x,z)\in X_0\times Y.$$

We also define $\Lambda_m:\mathbb{R}_+^{X_0\times Y}\to\mathbb{R}_+^{X_0\times Y}$ by

$$\Lambda_m\delta(x,z) := \max\left\{\left|\frac{S}{R}\right|_*\delta\left(\left(r-\frac{r}{m}\right)x,z\right),\left|\frac{S}{R}\right|_*\delta\left(-\frac{r}{sm}x,z\right)\right\},\quad \delta\in\mathbb{R}_+^{X_0}, x\in X_0$$

and see that Λ_m has the form described in **(H3)** with $k=2$, $h_1(x)=\left(r-\frac{r}{m}\right)x$, $h_2(x)=-\frac{r}{sm}x$, $G_1(x,z)=\left|\frac{1}{R}\right|_*$ and $G_2(x,z)=\left|\frac{S}{R}\right|_*$.

Furthermore, for every $\xi,\mu\in Y^{X_0}$, $(x,z)\in X_0\times Y$,

$$\|\mathcal{T}_m\xi(x)-\mathcal{T}_m\mu(x),z\|_* = \left\|\frac{1}{R}\xi\left(\left(r-\frac{r}{m}\right)x\right)-\frac{S}{R}\xi\left(-\frac{r}{sm}x\right)-\frac{1}{R}\mu\left(\left(r-\frac{r}{m}\right)x\right)+\right.$$
$$\left.\frac{S}{R}\mu\left(-\frac{r}{sm}x\right),z\right\|_* \le \max\left\{\left|\frac{1}{R}\right|_*\left\|(\xi-\mu)\left(\left(r-\frac{r}{m}\right)x\right),z\right\|_*,\left|\frac{S}{R}\right|_*\left\|(\xi-\mu)\left(-\frac{r}{sm}x\right),z\right\|_*\right\}$$
$$=\max_{1\le i\le 2}\left\{G_i(x,z)\|(\xi-\mu)\left(h_i(x)\right),z\|_*\right\}$$

so, **(H2)** is valid.

Using mathematical induction, we show that

$$\Lambda_m^n\,\varepsilon_m(x,z) = \frac{l}{|R|_*}\left|\frac{r}{sm}\right|^q \|x,z\|^{p+q}\ \alpha_m^n,\quad (x,z)\in X_0\times Y \tag{14}$$

where $\alpha_m=\max\left\{\left|\frac{S}{R}\right|_*\left|r-\frac{r}{m}\right|^{p+q},\left|\frac{S}{R}\right|_*\left|\frac{r}{sm}\right|^{p+q}\right\}$. For $n=0$, Eq. (14) satisfies Eq. (13). Next, we suppose that Eq. (14) holds for $n=k$, $k\in\mathbb{N}$. Then, we have

$$\Lambda_m^{k+1}\varepsilon_m(x,z)=\Lambda_m\left(\Lambda_m^k\varepsilon_m(x,z)\right)$$
$$=\max\left\{\left|\frac{1}{R}\right|_*\Lambda_m^k\varepsilon_m\left(\left(r-\frac{r}{m}\right)x,z\right),\ \left|\frac{S}{R}\right|_*\Lambda_m^k\varepsilon_m\left(-\frac{r}{sm}x,z\right)\right\}$$
$$=\max\left\{\frac{l}{|R|_*}\left|\frac{r}{sm}\right|^q\ \|x,z\|^{p+q}\,\alpha_m^k\left|\frac{1}{R}\right|_*\left|r-\frac{r}{m}\right|^{p+q},\ \frac{l}{|R|_*}m^q\,\|x,z\|^{p+q}\,\alpha_m^k\left|\frac{S}{R}\right|_*\left|\frac{r}{sm}\right|^{p+q}\right\}$$

$$= \frac{l}{|R|_*}\left|\frac{r}{sm}\right|^q \|x,z\|^{p+q}\alpha_m^k \max\left\{\left|\frac{1}{R}\right|_*\left|r-\frac{r}{m}\right|^{p+q}, \left|\frac{S}{R}\right|_*\left|\frac{r}{sm}\right|^{p+q}\right\}$$
$$= \frac{l}{|R|_*}\left|\frac{r}{sm}\right|^q \|x,z\|^{p+q}\ \alpha_m^{k+1}, \quad (x,z) \in X_0 \times Y.$$

We observe that Eq. (14) holds for $n = k+1$. So, Eq. (14) holds for all $n \in \mathbb{N}$. Clearly, from Eq. (14), we obtain that for all $x \in X_0$,

$$\lim_{n\to\infty} \Lambda^n \varepsilon_m(x,z) = 0.$$

So, Theorem 1.8 states that there exist a unique solution $F_m: X_0 \to Y$ of the equation

$$F_m(x) = \frac{1}{R}F_m\left(\left(r-\frac{r}{m}\right)x\right) - \frac{S}{R}F_m\left(-\frac{r}{sm}x\right), \quad x \in X_0,$$

such that

$$\|h(x) - F_m(x), z\|_* \le \sup_{n\in\mathbb{N}_0}\left\{\frac{l}{|R|_*}\left|\frac{r}{sm}\right|^q \|x,z\|^{p+q}\alpha_m^n\right\}, \quad (x,z) \in X_0 \times Y.$$

Moreover, for all $x \in X_0$,

$$F_m(x) := \lim_{n\to\infty} \mathcal{T}_m^n h(x).$$

Now we show that for every $(x,y,z) \in X_0^2 \times Y$ and $n \in \mathbb{N}_0$,

$$\|\mathcal{T}_m^n h(rx+sy) - R\mathcal{T}_m^n h(x) - S\mathcal{T}_m^n h(y), z\|_* \le l\alpha_m^n\ \|x,z\|^p\ \|y,z\|^q. \tag{15}$$

Put $n = 0$, then Eq. (15) is simply Eq. (7). Take $k \in \mathbb{N}_0$ and suppose that Eq. (15) holds for $n = k$. We see for $n = k+1$,

$$\left\|\mathcal{T}_m^{k+1}h(rx+sy) - R\mathcal{T}_m^{k+1}h(x) - S\mathcal{T}_m^{k+1}h(y), z\right\|_* = \left\|\frac{1}{R}\mathcal{T}_m^k h\left(\left(r-\frac{r}{m}\right)(rx+sy)\right) - \frac{S}{R}\mathcal{T}_m^k h\left(-\frac{r}{dm}(rx+sy)\right) - R\frac{1}{R}\mathcal{T}_m^k h\left(\left(r-\frac{r}{m}\right)x\right) + R\frac{S}{R}\mathcal{T}_m^k h\left(-\frac{r}{sm}x\right) - S\frac{1}{R}\mathcal{T}_m^k h\left(\left(r-\frac{r}{m}\right)y\right) + S\frac{S}{R}\mathcal{T}_m^k h\left(-\frac{r}{sm}y\right), z\right\|_*$$
$$\le \max\left\{\left|\frac{1}{R}\right|_*\left\|\mathcal{T}_m^k h\left(\left(r-\frac{r}{m}\right)(rx+sy)\right) - R\mathcal{T}_m^k h\left(\left(r-\frac{r}{m}\right)x\right) - S\mathcal{T}_m^k h\left(\left(r-\frac{r}{m}\right)y\right), z\right\|_*, \left|\frac{S}{R}\right|_*\left\|\mathcal{T}_m^k h\left(-\frac{r}{sm}(rx+sy)\right) - R\mathcal{T}_m^k h\left(-\frac{r}{sm}x\right) - S\mathcal{T}_m^k h\left(-\frac{r}{sm}y\right), z\right\|_*\right\}$$
$$\le \max\left\{\left|\frac{1}{R}\right|_* l\,\alpha_m^k\,\|x,z\|^p\,\|y,z\|^q\,\left|r-\frac{r}{m}\right|^{p+q}, \left|\frac{S}{R}\right|_* l\,\alpha_m^k\,\|x,z\|^p\,\|y,z\|^q\,\left|\frac{r}{sm}\right|^{p+q}\right\}$$
$$= l\,\alpha_m^k\,\|x,z\|^p\,\|y,z\|^q \max\left\{\left|\frac{1}{R}\right|_*\left|r-\frac{r}{m}\right|^{p+q}, \left|\frac{S}{R}\right|_*\left|\frac{r}{sm}\right|^{p+q}\right\}$$
$$= l\,\alpha_m^{k+1}\,\|x,z\|^p\,\|y,z\|^q, \quad (x,y,z) \in X_0^2 \times Y$$

thus, we obtained that Eq. (15) holds for every $n \in \mathbb{N}_0$. Letting $n \to \infty$ in Eq. (15), we see that,

$$F_m(rx+sy) = RF_m(x) + SF_m(x), \quad x \in X_0.$$

Applying Theorem 1.8, a sequence $\{F_m\}_{m\geq m_0}$of linear functions on X_0 exists such that

$$\|h(x) - F_m(x), z\|_* \leq \sup_{n\in\mathbb{N}_0} \left\{\frac{l}{|R|_*} \left|\frac{r}{sm}\right|^q \ \|x, z\|^{p+q} \ \alpha_m^n\right\}, \quad (x, z) \in X_0 \times Y$$

$$\|h(x) - F_m(x), z\|_* \leq \frac{l}{|R|_*} \left|\frac{r}{sm}\right|^q \ \|x, z\|^{p+q}, \quad (x, z) \in X_0 \times Y$$

and when $m \to \infty$, it follows that h is (r, s)-linear. □

In the next theorem, we prove hyperstability of the general linear functional equation on whole X, when $p, q > 0$.

Theorem 2.3. *Let X be a real 2-normed space over* $\mathbb{R}$ *with dimension more than 1, and Y be a non-Archimedean 2 -Banach space over non-Archimedean field* $\mathbb{K}$, $r, s\ \mathbb{F} \setminus \{0\}$, $R, S \in \mathbb{K} \setminus \{0\}$, $l \geq 0$, $p, q \in \mathbb{R}$, $p, q > 0$ *and* $h: X \to Y$ *satisfies*

$$\|h(rx + sy) - Rh(x) - Sh(y), z\|_* \leq l\ \|x, z\|^p\ \|y, z\|^q, \quad (x, y, z) \in X^2 \times Y \tag{16}$$

if $|r|^{p+q} \neq |R|_*$ *or* $|s|^{r+s} \neq |S|_*$, *then h is* (r, s)*-linear.*

Proof. When p and q are positive, we can set 0 in Eq. (16), and get some auxiliary equalities. We prove in the case $|r|^{p+q} \neq |R|_*$. The same proof can be also applied when $|s|^{p+q} \neq |S|_*$. Assume that $|r|^{p+q} < |R|_*$. Setting $x = y = 0$ in Eq. (16), we get

$$(1 - R - S)h(0) = 0 \quad . \tag{17}$$

Put $y = 0$ in Eq. (16) we have

$$h(rx) = Rh(x) + Sh(0), \quad x \in X$$

and can be written as

$$h(x) = Rh\left(\frac{x}{r}\right) + Sh(0), \quad x \in X.$$

Using Eq. (16) and Eq. (17), we obtain

$$\left\|Rh\left(\frac{rx+sy}{r}\right) - RRh\left(\frac{x}{r}\right) - SRh\left(\frac{y}{r}\right), z\right\|_* \leq l\ \|x, z\|^p\ \|y, z\|^q, \quad (x, y, z) \in X^2 \times Y$$

replacing x by rx and y by ry in the last inequality, then

$$\|h(rx + sy) - Rh(x) - Sh(y), z\|_* \leq l\frac{|r|^{p+q}}{|R|_*}\ \|x, z\|^p\ \ \|y, z\|^q, \quad (x, y, z) \in X^2 \times Y.$$

Now by induction, we show that

$$\|h(rx + sy) - Rh(x) - Sh(y), z\|_* \leq l\left(\frac{|r|^{p+q}}{|R|_*}\right)^n\ \|x, z\|^p\ \ \|y, z\|^q \tag{18}$$

for all $(x, y, z) \in X^2 \times Y$. Clearly, Eq. (18) satisfies Eq. (16) for $n = 0$. Assume that Eq. (18) holds for $n = k$, $k \in \mathbb{N}$, then

$$\left\| Rh\left(\frac{rx+sy}{r}\right) - RRh\left(\frac{x}{r}\right) - SRh\left(\frac{y}{r}\right), z\right\|_* \leq l\left(\frac{|r|^{p+q}}{|R|_*}\right)^k \|x, z\|^p \|y, z\|^q, \ (x, y, z) \in X^2 \times Y$$

Apply replacements x by rx and y by ry in the last inequality, we get that

$$\|h(rx + sy) - Rh(x) - Sh(y), z\|_* \leq l\left(\frac{|r|^{p+q}}{|R|_*}\right)^{k+1} \|x, z\|^p \|y, z\|^q, \ (x, y, z) \in X^2 \times Y.$$

So, Eq. (18) holds for all $n \in \mathbb{N}_0$ and with $n \to \infty$,

$$h(rx + sy) = Rh(x) + Sh(y), \quad x, y \in X.$$

In the case $|R|_* < |r|^{p+q}$, we use the equation $h(x) = \frac{1}{R}h(rx) - \frac{D}{R}h(0)$ with Eq. (16) and Eq. (17)

$$\left\| \frac{1}{R}h\big(r(rx + sy)\big) - R\frac{1}{R}h(rx) - S\frac{1}{R}h(Ry), z\right\|_* \leq l \|x, z\|^p \|y, z\|^q, \ (x, y, z) \in X^2 \times Y.$$

Replacing x by $\frac{x}{r}$ and y by $\frac{y}{r}$ in the last inequality, we obtain

$$\|h(rx + sy) - Rh(x) - Sh(y), z\|_* \leq l\frac{|R|_*}{|r|^{p+q}} \|x, z\|^p \|y, z\|^q, \quad (x, y, z) \in X^2 \times Y$$

and using induction and same to the first case, we can get

$$\|h(rx + sy) - Rh(x) - Sh(y), z\|_* \leq l\left(\frac{|R|_*}{|r|^{p+q}}\right)^n \|x, z\|^p \|y, z\|^q, \quad (x, y, z) \in X^2 \times Y$$

with $n \to \infty$, results that h is (r, s)-linear. □

Some applications of results are stated in the following corollaries.

Corollary 2.4. *Let X be a real 2-normed space over $\mathbb{R}$ with dimension more than 1, and Y be a non-Archimedean 2-Banach space over non-Archimedean field $\mathbb{K}$, $l \geq 0$, $p, q \in \mathbb{R}$, the generalized Jensen functional equation*

$$f\left(\frac{x+y}{\alpha}\right) = \frac{1}{\alpha}f(x) + \frac{1}{\alpha}f(y), \quad x, y \in X, \alpha \in \mathbb{N}, \ \alpha > 2,$$

is hyperstable in the class of functions $f: X \to Y$ with control function $\varphi(x, y, z) = l \|x, z\|^p \|y, z\|^q$ if the following conditions hold:

- $p + q < 0, \ 0 \notin X,$
- $p + q > 0, \ 0 \notin X,$
- $p, q > 0, \ \ 0 \in X.$

Proof. With a simple comparison, it is clear that the generalized Jensen functional equation satisfies the general linear functional equation in Eq. (6) with $r = s = R = S = \frac{1}{\alpha}$, so when $p + q < 0$ and $0 \notin X$, then by Theorem 2.1, the generalized Jensen functional equation is hyperstable in class of functions $f: X \to Y$. In similar manner, Theorem 2.2 states hyperstability result for $p + q > 0$ and Theorem 2.3, for $p, q > 0$. In this way, the proof becomes complete. □

Corollary 2.5. *Let* X *be a real 2-normed space over* $\mathbb{R}$ *with dimension more than 1 and* Y *be a non-Archimedean 2-Banach space over non-Archimedean field* $\mathbb{K}$, $r, s\ \mathbb{F} \setminus \{0\}$, $R, S \in \mathbb{K} \setminus \{0\}$, $l \geq 0$, $p, q \in \mathbb{R}$, $H: X^2 \to Y$, $H(v, w) \neq 0$ *for some* $v, w \in X$ *and*

$$\|H(x, y), z\|_* \leq l\ \|x, z\|^p\ \|y, z\|^q, \quad (x, y, z) \in X^2 \times Y$$

with existence following conditions

- $p + q < 0,\ 0 \notin X,$
- $p + q > 0,\ 0 \notin X,$
- $p, q > 0,\ 0 \in X,\ |r|^{r+s} \neq |R|_*$ *or* $|s|^{r+s} \neq |S|_*,$

the functional equation

$$h(rx + sy) = Rh(x) + Sh(y) + H(x, y), \quad x, y \in X, \tag{19}$$

has no solution in the class of functions $h: X \to Y$.

Proof. Suppose that $h: X \to Y$ be a solution to Eq. (19). Then Eq. (7) holds. From Theorem 2.1, Theorem 2.2 and Theorem 2.3, it comes that h is (r, s)-linear, so $H(v, w) = 0$ which contradicts the assumption. □

3. Conclusion

Regarding Hyers-Ulam stability, an equation is stable when it has an approximately solution near to the main solution of the equation, and the difference of approximately and the exact solutions is controlling by a function called control function. With some assumptions on control function, somewhere the approximately solution changes to the exact solution. In this case the notion is called hyperstability, and the equation is called hyperstable. In this research the hyperstability of general linear functional equation is considered in non-Archimedean 2-Banach spaces using fixed pint method. In fact, under assumption existing in proved theorems, the existing and uniqueness of a linear map from a 2-normed space to non-Archimedean 2-Banach space is proved.

References

Almahalebi M. & Chahbi A. 2017. Hyperstability of the Jensen functional equation in ultrametric spaces. *Aequationes Mathematicae* **91**(4): 647-661.

Aoki T. 1950. On the stability of the linear transformation in Banach spaces. *Journal of the Mathematical Society of Japan* **2**: 64-66.

Bahyrycz A. & Piszczek M. 2014. Hyperstability of the Jensen functional equation. *Acta Mathematica Hungarica* **142**(2): 353-365.

Bourgin D.G. 1949. Approximately isometric and multiplicative transformations on continuous function rings. *Duke Mathematical Journal* **16**(2): 385-397.
Bourgin D.G. 1951. Classes of transformations and bordering transformations. *Bulletin of the American Mathematical Society* **57**(4): 223-237.
Brzdek J. 2013a. Hyperstability of the Cauchy equation on restricted domains. *Acta Mathematica Hungarica* **141**(1-2): 58-67.
Brzdek J. 2013b. Remarks on hyperstability of the Cauchy functional equation. *Aequationes Mathematicae* **86**(3): 255-267.
Brzdek J. 2013c. Stability of additivity and fixed point methods. *Fixed Point Theory and Applications* **2013**: 285.
Brzdek J. 2014. A hyperstability result for the Cauchy equation. *Bulletin of the Australian Mathematical Society* **89**(1): 33-40.
Brzdek J. 2015. Remarks on stability of some inhomogeneous functional equations. *Aequationes Mathematicae* **89**: 83-96.
Brzdek J. & Cieplinski K. 2011. A fixed point approach to the stability of functional equations in non-Archimedean metric spaces. *Nonlinear Analysis: Theory, Methods & Applications* **74**(18): 6861-6867.
Brzdek J. & Cieplinski K. 2013. Hyperstability and superstability. *Abstract and Applied Analysis* **2013**(1): 401756.
Cieplinski K. 2011. Stability of multi-additive mappings in non-Archimedean normed spaces. *Journal of Mathematical Analysis and Applications* **373**(2): 376-383.
Diagana T. & Ramaroson F. 2016. *Non-Archimedean Operator Theory.* 1st Ed. New York: Springer.
El-Fassi I. 2017. On a new type of hyperstability for radical cubic functional equation in non-Archimedean metric spaces. *Results in Mathematics* **72**(2): 990-1005.
El-Fassi I. 2018. A new type of approximation for the radical quintic functional equation in non-Archimedean (2,β) -Banach spaces. *Journal of Mathematical Analysis and Applications* **457**(1): 322-335.
El-Fassi I. Elqorachi E. & Khodaei H. 2020. A fixed point approach to stability of k-th radical functional equation in non-Archimedean (n, β)-Banach spaces. *Bulletin of the Iranian Mathematical Society* 47: 487-504.
El-Fassi I. & Kabbaj S. 2015a. Non-Archimedean random stability of σ- quadratic functional equation. *Thai Journal of Mathematics* **14**(1): 151-165.
El-Fassi I. & Kabbaj S. 2015b. On the hyperstability of a Cauchy-Jensen type functional equation in Banach spaces. *Proyecciones (Antofagasta)* **34**(4): 359-375.
El-Fassi I., Kabbaj S. & Chahbi A. 2018. A fixed point approach to the hyperstability of the general linear equation in β-Banach spaces. *Analysis* **38**(3): 115-126.
El-Fassi I., Kabbaj S. & Charifi A. 2016. Hyperstability of Cauchy-Jensen functional equations. *Indagationes Mathematicae* **27**(3): 855-867.
Gajda Z. 1991. On stability of additive mappings. *International Journal of Mathematics and Mathematical Sciences* **14**(3): 431-434.
Gordji M. & Savadkouhi M.B. 2010. Stability of a mixed type cubic--quartic functional equation in non-Archimedean spaces. *Applied Mathematics Letters* **23**(10): 1198-1202.
Gselmann G. 2009. Hyperstability of a functional equation. *Acta Mathematica Hungarica* **124**: 179-188.
Hensel K. 1897. Uber eine neue begr undung der theorie der algebraischen zahlen. *Jahresbericht der Deutschen Mathematiker-Vereinigung* **6**: 83-88.
Hyers D.H. 1941. On the stability of the linear functional equation. *Proceedings of the National Academy of Sciences of the United States of America* **27**(4): 222-224.
Katsaras A.K. & Beloyiannis A. 1999. Tensor products of non-Archimedean weighted spaces of continuous functions. *Georgian Mathematical Journal* **6**: 33-44.
Khodaei H., Gordji M.E., Kim S.S. & Cho Y.J. 2012. Approximation of radical functional equations related to quadratic and quartic mappings. *Journal of mathematical Analysis and Applications* **395**(1): 284-297.
Khrennikov A. 1997. *Non-Archimedean Analysis: Quantum Paradoxes, Dynamical Systems and Biological Models.* Dordrecht: Kluwer Academic Publishers.
Lee Y.-H. 2008. On the stability of the monomial functional equation. *Bulletin of the Korean Mathematical Society* **45**(2): 397-403.
Maksa G. & Pales Z. 2001. Hyperstability of a class of linear functional equations. *Acta Mathematica Academiae Paedagogicae Nyi' regyha'ziensis* **17**: 107-112.
Mirmostafaee A.K. 2010. Hyers-Ulam stability of cubic mappings in non-Archimedean normed spaces. *Kyungpook Mathematical Journal* **50**(2): 315-327.
Moslehian M.S. & Rassias M.T. 2007. Stability of functional equations in non-Archimedean spaces. *Applicable Analysis and Discrete Mathematics* **1**: 325-334.
Moslehian M.S. & Sadeghi G. 2008. Stability of two types of cubic functional equations in non-Archimedean spaces. *Real Analysis Exchange* **33**(2): 375-384.
Nyikos P.J. 1999. On some non-Archimedean spaces of Alexandroff and Urysohn. *Topology and its Applications* **91**(1): 1-23.

Piszczek M. 2015. Hyperstability of the general linear functional equation. *Bulletin of the Korean Mathematical Society* **52**(6): 1827-1238.
Rassias T.M. 1991. On a modified Hyers-Ulam sequence. *Journal of mathematical analysis and applications* **158**(1): 106-113.
Rassias T.M. & Semrl P. 1992. On the behavior of mappings which do not satisfy Hyers-Ulam stability. *Proceedings of the American Mathematical Society* **114**(4): 989-993.
Shuja S., Embong A.F. & Ali N.M.M. 2024. Hyperstability of the general linear functional equation in non-Archimedean Banach spaces. *p-Adic Numbers, Ultrametric Analysis and Applications* **16**: 70-81.
Ulam S.M. 2004. *Problems in Modern Mathematics.* New York: Dover Publications.

Department of Mathematical Sciences
Faculty of Science
Universiti Teknologi Malaysia
81310 UTM Johor Bahru, Malaysia
Email: shuja.ghory@yahoo.com , ahmadfadillah.90@gmail.com * *, normuhainiah@utm.my*

Received: 15 August 2023
Accepted: 27 May 2024

*Corresponding author