Neutrosophic Systems with Applications, Vol. 10, 2023
https://doi.org/10.61356/j.nswa.2023.69

Neutrosophic Bicubic B-spline Surface Interpolation Model for Uncertainty Data

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Abstract: Dealing with the uncertainty data problem using neutrosophic data is difficult since certain data are wasted due to noise. To address this issue, this work proposes a neutrosophic set (NS) strategy for interpolating the B-spline surface. The purpose of this study is to visualize the neutrosophic bicubic B-spline surface (NBB-sS) interpolation model. Thus, the principal results of this study introduce the NBB-sS interpolation method for neutrosophic data based on the NS notion. The neutrosophic control net relation (NCNR) is specified first using the NS notion. The B-spline basis function is then coupled to the NCNR to produce the NBB-sS. This surface is then displayed using an interpolation method that comprises surfaces representing truth, indeterminacy, and false membership. There is a numerical example for constructing the NBB-sS using interpolation and will use quantitative data in the form of discrete numerical cases, particularly in neutrosophic numbers. The major conclusion of this study is a mathematical representation of NBB-sS by using the interpolation method was introduced and visualized for a neutrosophic data problem. The scientific value contributed to this study is an acceptance of uncertainty. Therefore, since it incorporates geometric modeling, this work can make a significant contribution to the neutrosophic decision model.

Keywords: Neutrosophic Set Theory; Neutrosophic Control Net Relation; B-spline Surface; Interpolation Method; Uncertainty Data.

1. Introduction

The foundational concept of fuzzy set theory was introduced by Zadeh [1] in order to tackle the complexities associated with uncertainty in complex systems. Several years later, Atanassov [2] introduced the concept of the intuitionistic fuzzy set (IFS), which is an extension of fuzzy set theory that incorporates membership grade, non-membership, and uncertainty. Addressing a complex problem that possesses intuitive and fuzzy characteristics is a challenging task, and it is seldom explored within the realm of spline modeling. Multiple studies have been conducted in the field of datasets and splines, as seen by the references [3-11]. The neutrosophic technique was created by Florentin Smarandache [12] as a mathematical framework for handling ambiguous data by using the concept of neutrality. The concept of neutrosophic set (NS) is characterized by membership degrees, non-membership, and indeterminacy. In the present context, the term "neutrosophic set" pertains to the process of resolving and representing complex problems that encompass numerous domains. Neutrosophic set theory allows for the simultaneous assignment of true, false, and indeterminate membership degrees to an element. This facilitates the representation of complex forms of uncertainty and indeterminacy, such as scenarios when a proposition can possess both truth and falsehood simultaneously. Academics have also utilized geometric modeling to implement neutrosophic set approaches [13, 14,19].

The objective of this research is to provide a geometric framework that can effectively handle uncertain data, specifically emphasizing the neutrosophic bicubic B-spline surface (NBB-sS) interpolation model. Before constructing the NBB-sS, it is necessary to define the neutrosophic control point relation (NCPR) using the properties of the NS. The control points are employed in conjunction with the B-spline basis function to construct the NBB-sS model, which is subsequently visualized by interpolation. The structure of this document is outlined as follows. The introductory component of this investigation presented relevant contextual information. Section 2 provides an overview of the neutrosophic fundamental notion, NCPR and neutrosophic control net relation (NCNR). Section 3 illustrates the application of the NCPR method for the purpose of interpolating the NBB-sS. Section 4 includes both a mathematical illustration and a graphic depicting NBB-sS. The investigation includes a review of the characteristics of the surface, together with the methodology employed for its construction. The inquiry will be concluded in Part 5.

2. Preliminaries

This part describes the NS, including the core concept of NS and the NCPR. The IFS can handle limited data but not paraconsistent data [12]. "There are three memberships: a truth membership function, T, an indeterminacy membership function, I, and a falsity membership function, F, with the parameter 'indeterminacy' added by the NS specification" [12].

Definition 1: [12] Let Y be the main of conversation, with element in Y denoted as y. The Neutrosophic set is an object in the form.

$$\hat{B} = \{\left\langle y : T_{\hat{B}(y)}, I_{\hat{B}(y)}, F_{\hat{B}(y)} \right\rangle | \ y \in Y\} \tag{1}$$

where, the functions $T, I, F : Y \to]^{-}0,1^{+}[$ define, respectively, the degree of truth membership, the degree of indeterminacy, and the degree of false membership of the element $y \in Y$ to the set $\hat{B}$ with the condition;

$$0^{-} \leq T_{\hat{B}}(y) + I_{\hat{B}}(y) + F_{\hat{B}}(y) \leq 3^{+} \tag{2}$$

There is no limit to the amount of $T_{\hat{B}}(y), I_{\hat{B}}(y)$ and $F_{\hat{B}}(y)$

A value is chosen by NS from one of the real standard subsets or one of the non-standard subsets of $]^{-}0,1^{+}[$. The actual value of the interval $[0,1]$, on the other hand, $]^{-}0,1^{+}[$ will be utilized in technical applications since its utilization in real data such as the resolution of scientific challenges, will be physically impossible. As a direct consequence of this, membership value utilization is increased.

$$\hat{B} = \{\left\langle y : T_{\hat{B}(y)}, I_{\hat{B}(y)}, F_{\hat{B}(y)} \right\rangle | \ y \in Y\} \text{ and } T_{\hat{B}}(y), I_{\hat{B}}(y), F_{\hat{B}}(y) \in [0,1] \tag{3}$$

There is no restriction on the sum of $T_{\hat{B}}(y), I_{\hat{B}}(y), F_{\hat{B}}(y)$ .Therefore,

$$0 \leq T_{\hat{B}}(y) + I_{\hat{B}}(y) + F_{\hat{B}}(y) \leq 3 \tag{4}$$

Definition 2: [13, 14] Let $\hat{B} = \{\left\langle y : T_{\hat{B}(y)}, I_{\hat{B}(y)}, F_{\hat{B}(y)} \right\rangle | \ y \in Y\}$ and $\hat{C} = \{\left\langle z : T_{\hat{C}(z)}, I_{\hat{C}(z)}, F_{\hat{C}(z)} \right\rangle | \ z \in Z\}$ be neutrosophic elements. Thus, $NR = \{\left\langle (y,z) : T_{(y,z)}, I_{(y,z)}, F_{(y,z)} \right\rangle | \ y \in \hat{B}, z \in \hat{C}\}$ is a neutrosophic relation on $\hat{B}$ and $\hat{C}$.

Definition 3: [13,14] Neutrosophic set of $\hat{B}$ in space Y is Neutrosophic Point (NP) and $\hat{B}=\{\hat{B}_i\}$ where $i=0,...,n$ is a set of NPs where there exists $T_{\hat{B}}:Y\rightarrow[0,1]$ as truth membership, $I_{\hat{B}}:Y\rightarrow[0,1]$ as indeterminacy membership and $F_{\hat{B}}:\hat{Y}\rightarrow[0,1]$ as false membership with

$$T_{\hat{B}}(\hat{B})=\begin{cases}0 & if\ \hat{B}_i\notin\hat{B}\\ a\in(0,1) & if\ \hat{B}_i\hat{\in}\hat{B}\\ 1 & if\ \hat{B}_i\in\hat{B}\end{cases}$$

$$I_{\hat{B}}(\hat{B})=\begin{cases}0 & if\ \hat{B}_i\notin\hat{B}\\ b\in(0,1) & if\ \hat{B}_i\hat{\in}\hat{B}\\ 1 & if\ \hat{B}_i\in\hat{B}\end{cases} \tag{5}$$

$$F_{\hat{B}}(\hat{B})=\begin{cases}0 & if\ \hat{B}_i\notin\hat{B}\\ c\in(0,1) & if\ \hat{B}_i\hat{\in}\hat{B}\\ 1 & if\ \hat{B}_i\in\hat{B}\end{cases}$$

2.1 *Neutrosophic Point Relation*

Neutrosophic point relation (NPR) depends on the NS notion, which was addressed in the previous section. If P,Q is a group of Euclidean universal space points and $P,Q\in\mathbf{R}^2$ then NPR is defined as follows:

Definition 4 [19]
Let X,Y be collection of universal space points with non-empty set and $P,Q,I\in\mathbf{R}\times\mathbf{R}\times\mathbf{R}$, then NPR is defined as

$$\hat{R}=\left\{\left\langle\left(p_i,q_j\right),T_R(p_i,q_j),I_R(p_i,q_j),F_R(p_i,q_j)\right\rangle \mid T_R(p_i,q_j),I_R(p_i,q_j),F_R(p_i,q_j)\in I\right\} \tag{6}$$

where $\left(p_i,q_j\right)$ is an ordered pair of coordinates and $\left(p_i,q_j\right)\in P\times Q$ while $T_R(p_i,q_j),I_R(p_i,q_j),F_R(p_i,q_j)$ are the truth membership, indeterminacy membership and false membership that follows the condition of neutrosophic set which is $0\le T_{\hat{B}}(\hat{y})+I_{\hat{B}}(\hat{y})+F_{\hat{B}}(\hat{y})\le 3$.

2.2 *Neutrosophic Control Net Relation*

A spline surface's geometry can only be specified by all of the points required to construct the surface and this is what the term "control net" refers to [18]. The control net is critical in the development, control, and manufacturing of smooth surfaces [18]. In this part, the NCPR is defined by first using the control point concept from the research reported in [15-17] in the following way:

Definition 5: [19] Let $\hat{R}$ be a NPR, then NCPR is defined as set of point $n+1$ that indicates the positions and coordinates of a location and is used to describe the curve and is denoted by

$$\begin{aligned}\hat{P}_i^T&=\left\{\hat{p}_0^T,\hat{p}_1^T,...,\hat{p}_n^T\right\}\\ \hat{P}_i^I&=\left\{\hat{p}_0^I,\hat{p}_1^I,...,\hat{p}_n^I\right\}\\ \hat{P}_i^F&=\left\{\hat{p}_0^F,\hat{p}_1^F,...,\hat{p}_n^F\right\}\end{aligned} \tag{7}$$

where $\hat{P}_i^T$, $\hat{P}_i^I$ and $\hat{P}_i^F$ are neutrosophic control point for membership truth, indeterminacy and i is one less than n.

This study primarily focuses on surface modelling. Consequently, it is essential to present the concept of neutrosophic control net relation. In the context of surface modelling, the net control relation is formed by combining the relations of each control point [18]. In contrast to the curve model that simply requires control points [18]. Thus, the NCNR can be defined as follows.

Definition 6: Let $\hat{P}$ be an NCPR, and then define an NCNR as points $n+1$ and $m+1$ for $\hat{P}$ in their direction, and it can be denoted by $\hat{P}_{i,j}$ that represents the locations of points used to describe the surface and may be written as

$$\hat{P}_{i,j} = \begin{bmatrix} \hat{P}_{0,0} & \hat{P}_{0,1} & \cdots & \hat{P}_{0,j} \\ \hat{P}_{1,0} & \hat{P}_{1,1} & \cdots & \hat{P}_{1,j} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{P}_{i,0} & \hat{P}_{i,1} & \cdots & \hat{P}_{i,j} \end{bmatrix} \tag{8}$$

where $\hat{P}_{i,j}$ are also the points that make up a polygon's control net.

3. Neutrosophic Bicubic B-spline Surface Interpolation

Piegl and Tiller [18] introduced the B-spline basis function will be mixed with NCNR as stated in the previous section. The NCNR and Definition 1 of the neutrosophic set are used to build the NBB-sS, which is then used to incorporate the B-spline blending function in a geometric model. The interpolation approach model is mathematically stated as follows:

Definition 7: Let $\hat{P}_i^T = \left\{\hat{p}_0^T, \hat{p}_1^T, ..., \hat{p}_n^T\right\}, \hat{P}_i^I = \left\{\hat{p}_0^I, \hat{p}_1^I, ..., \hat{p}_n^I\right\}, \hat{P}_i^F = \left\{\hat{p}_0^F, \hat{p}_1^F, ..., \hat{p}_n^F\right\}$ where $i = 0,1,...,n$ is NCPR. The Cartesian product B-spline surface determined by is the obvious expansion of the Bezier surface. $BsS(u,w)$ denoted as neutrosophic B-spline surface approximation as follows:

$$BsS(u,w) = \sum_{i=0}^{n}\sum_{j=0}^{m} \hat{P}_{i,j} N_i^k(u) M_j^l(w) \tag{9}$$

where $N_i^k(u)$ and $M_j^l(w)$ are the B-spline basis function in the u and w parametric directions.

$$\begin{aligned} N_i^1(u) &= \begin{cases} 1 & if \quad u_i \le u < u_{i+1} \\ 0 & otherwise \end{cases} \\ N_i^1(u) &= \frac{(u-u_i)}{u_{i+k-1}-u_i} N_i^{k-1}(u) + (7)\frac{(u_{i+k}-u)}{u_{i+k}-u_{i+1}} N_{i+1}^{k-1}(u) \end{aligned} \tag{10}$$

$$\begin{aligned} M_j^1(w) &= \begin{cases} 1 & if \quad w_j \le w < w_{j+1} \\ 0 & otherwise \end{cases} \\ M_j^1(w) &= \frac{(w-w_j)}{w_{j+l-1}-u_i} M_i^{l-1}(w) + (8)\frac{(w_{j+l}-w)}{w_{j+l}-w_{j+1}} M_{j+1}^{l-1}(w) \end{aligned} \tag{11}$$

The parametric function neutrosophic B-spline surface in Eq. (9) is defined as follows and is made up of three surfaces: a member surface, a non-member surface, and an indeterminacy surface.

$$BsS^T(u,w) = \sum_{i=0}^{n}\sum_{j=0}^{m} \hat{P}_{i,j}{}^T N_i^k(u) M_j^l(w) \tag{12}$$

$$BsS^{F}(u,w)=\sum_{i=0}^{n}\sum_{j=0}^{m}\hat{P}_{i,j}{}^{F}N_i^k(u)M_j^l(w) \tag{13}$$

$$BsS^{I}(u,w)=\sum_{i=0}^{n}\sum_{j=0}^{m}\hat{P}_{i,j}{}^{I}N_i^k(u)M_j^l(w) \tag{14}$$

The surface for the neutrosophic B-spline will be lie in the NCPR since NBB-sS using interpolation method. Suppose $\hat{F}_{i,j}$ as data point matrix. Thus, the interpolation procedure is as follows:

$$\begin{bmatrix} \hat{F}_{0,0} & \hat{F}_{0,1} & \cdots & \hat{F}_{0,j} \\ \hat{F}_{1,0} & \hat{F}_{1,1} & \cdots & \hat{F}_{1,j} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{F}_{i,0} & \hat{F}_{i,1} & \cdots & \hat{F}_{i,j} \end{bmatrix} = \begin{bmatrix} BsS(u_0,w_0) & BsS(u_0,w_1) & \cdots & BsS(u_0,w_j) \\ BsS(u_1,w_0) & BsS(u_1,w_1) & \cdots & BsS(u_1,w_j) \\ \vdots & \vdots & \ddots & \vdots \\ BsS(u_i,w_0) & BsS(u_i,w_1) & \cdots & BsS(u_i,w_j) \end{bmatrix} \tag{15}$$

Each $BsS(u,w)$ can be expressed as a matrix product as follows:

$$BsS(u_i,w_j)=\begin{bmatrix} N_0^k(u_i) & N_1^k(u_i) & \cdots & N_i^k(u_i) \end{bmatrix} \times \begin{bmatrix} \hat{P}_{0,0} & \hat{P}_{0,1} & \cdots & \hat{P}_{0,j} \\ \hat{P}_{1,0} & \hat{P}_{1,1} & \cdots & \hat{P}_{1,j} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{P}_{i,0} & \hat{P}_{i,1} & \cdots & \hat{P}_{i,j} \end{bmatrix} \times \begin{bmatrix} M_0^l(w_j) \\ M_1^l(w_j) \\ \vdots \\ M_i^l(w_j) \end{bmatrix} \tag{16}$$

All the separate equations can be combined into a single matrix equation:

$$\hat{F}=N^T\hat{P}M \tag{17}$$

where $\hat{F}$ denotes the data points from Eq. (15), and $\hat{P}$ denotes the matrix containing the unknown control points $\hat{P}_{i,j}$. The values of the B-spline polynomials at the given parameters are contained in the matrix M^T and N:

$$N^T=\begin{bmatrix} N_0^k(u_0) & N_1^k(u_0) & \cdots & N_i^k(u_0) \\ N_0^k(u_1) & N_1^k(u_1) & \cdots & N_i^k(u_1) \\ \vdots & \vdots & \ddots & \vdots \\ N_0^k(u_i) & N_1^k(u_i) & \cdots & N_i^k(u_i) \end{bmatrix} \tag{18}$$

$$M=\begin{bmatrix} M_0^l(w_0) & M_0^l(w_0) & \cdots & M_0^l(w_j) \\ M_1^l(w_1) & M_1^l(w_1) & \cdots & M_1^l(w_j) \\ \vdots & \vdots & \ddots & \vdots \\ M_j^l(w_j) & C_j^l(w_j) & \cdots & C_j^l(w_j) \end{bmatrix} \tag{19}$$

To find the control point $\hat{P}_{i,j}$, Eq. (17) can be easily simplified as follows since this study is an interpolation method. Therefore, the matrices should be inverse to find the interpolate data.

$$\hat{P}=\left(N^T\right)^{-1}\hat{F}\left(M\right)^{-1} \tag{17}$$

To demonstrate the neutrosophic bicubic B-spline surface, consider the following $\hat{P}_{i,j}$ to find NBB-sS with degrees of truth membership, false membership, and indeterminacy with $i=3, j=3$ for bicubic case.

$$\begin{bmatrix} \hat{P}_{0,0} & \hat{P}_{0,1} & \hat{P}_{0,2} & \hat{P}_{0,3} \\ \hat{P}_{1,0} & \hat{P}_{1,1} & \hat{P}_{1,2} & \hat{P}_{1,3} \\ \hat{P}_{2,0} & \hat{P}_{2,1} & \hat{P}_{2,2} & \hat{P}_{2,3} \\ \hat{P}_{3,0} & \hat{P}_{3,1} & \hat{P}_{3,2} & \hat{P}_{3,3} \end{bmatrix} \tag{18}$$

4. Applications of Neutrosophic Bicubic B-spline Surface Interpolation

To illustrate the neutrosophic B-spline surface interpolation, let considered a neutrosophic B-spline surface for 4×4 NCNR with the degree of truth membership, indeterminacy membership and false membership as follow in Table 1. The example given below shows that each NCPR follow the condition $0^{-} \le T_{\hat{B}}(y)+I_{\hat{B}}(y)+F_{\hat{B}}(y) \le 3^{+}$. Therefore, it is satisfying the neutrosophic set problem.

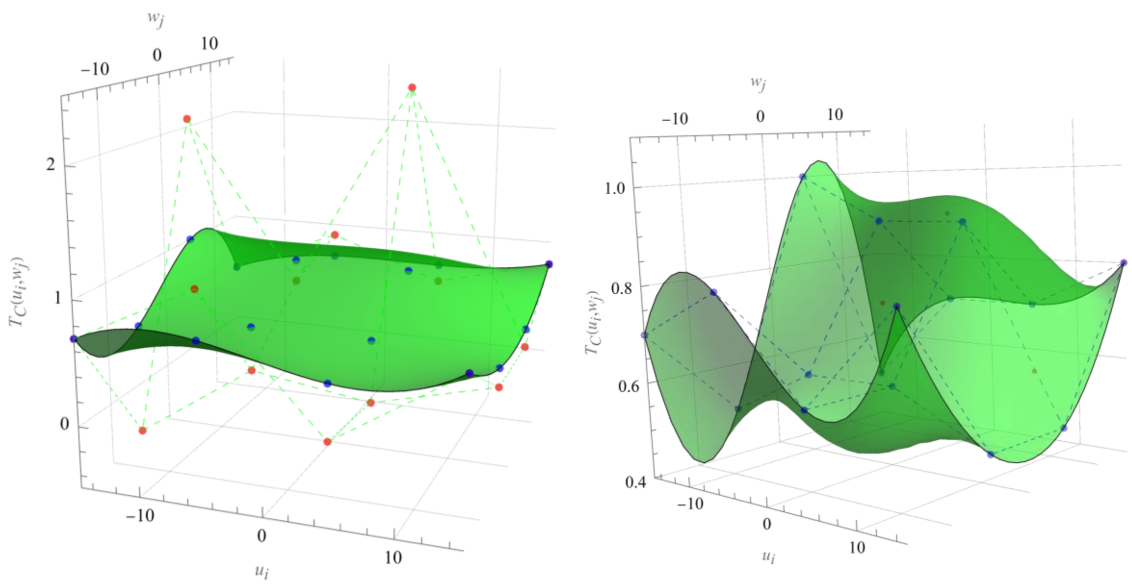
Table 1. NCPR with its respective degrees.

NCPR $\hat{P}_{i,j}$	**Truth Membership** $\hat{P}^{T}_{i,j}$	**False Membership** $\hat{P}^{F}_{i,j}$	**Indeterminacy Membership** $\hat{P}^{I}_{i,j}$
$\hat{P}_{0,0}=(-15,15)$	0.5	0.8	0.3
$\hat{P}_{0,1}=(-15,5)$	1.0	0.4	0.2
$\hat{P}_{0,2}=(-15,-5)$	0.5	0.5	0.6
$\hat{P}_{0,3}=(-15,-15)$	0.7	0.6	0.3
$\hat{P}_{1,0}=(-5,15)$	0.7	0.5	0.4
$\hat{P}_{1,1}=(-5,5)$	0.9	0.3	0.4
$\hat{P}_{1,2}=(-5,-5)$	0.6	0.6	0.4
$\hat{P}_{1,3}=(-5,-15)$	0.8	0.5	0.3
$\hat{P}_{2,0}=(5,15)$	0.7	0.3	0.6
$\hat{P}_{2,1}=(5,5)$	0.9	0.5	0.2
$\hat{P}_{2,2}=(5,-5)$	0.6	0.8	0.2
$\hat{P}_{2,3}=(5,-15)$	0.6	0.4	0.6
$\hat{P}_{3,0}=(15,15)$	0.8	0.4	0.5
$\hat{P}_{3,1}=(15,5)$	0.5	0.7	0.4
$\hat{P}_{3,2}=(15,-5)$	0.5	0.7	0.4
$\hat{P}_{3,3}=(15,-15)$	0.8	0.5	0.3

Table 1 is also can represented in matrix form. $\langle t, f, i\rangle$ denoted as truth, false and indeterminacy membership.

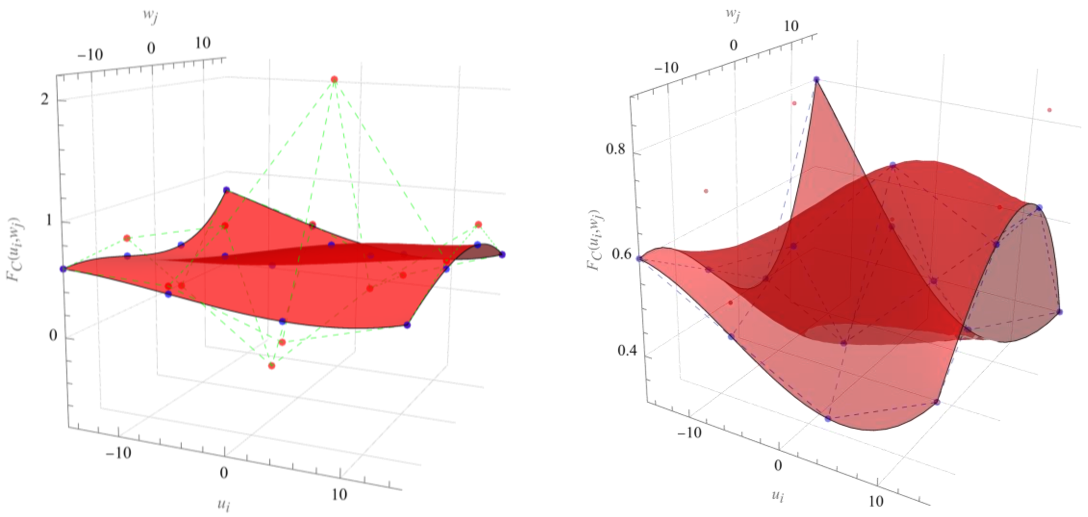
$$\hat{P}_{3,3}=\begin{bmatrix}\langle(-15,15);0.5,0.8,0.3\rangle & \langle(-15,5);1.0,0.4,0.2\rangle & \langle(-15,-5);0.5,0.5,0.6\rangle & \langle(-15,-15);0.7,0.6,0.3\rangle\\ \langle(-5,15);0.7,0.5,0.4\rangle & \langle(-5,5);0.9,0.3,0.4\rangle & \langle(-5,-5);0.6,0.6,0.4\rangle & \langle(-5,-15);0.8,0.5,0.3\rangle\\ \langle(5,15);0.7,0.3,0.6\rangle & \langle(5,5);0.9,0.5,0.2\rangle & \langle(5,-5);0.6,0.8,0.2\rangle & \langle(5,-15);0.6,0.4,0.6\rangle\\ \langle(15,15);0.8,0.4,0.4\rangle & \langle(15,5);0.5,0.7,0.4\rangle & \langle(15,-5);0.5,0.7,0.4\rangle & \langle(15,-15);0.8,0.5,0.3\rangle\end{bmatrix}$$

Thus, by using Definition 7, the respective surface is visualized in Figure 1 until Figure 3 for truth, false and indeterminacy degrees. Since this study focuses on interpolation, each surface will interpolate its respective NCNR. The NCNR has been shown in Figure 1(b), 2(b) and 3(b), the dashed line represents as control net while the dot point denoted as control points. For Figure 1(a), 2(a) and 3(a), the dashed line represents as control net for data point, while the data point denoted as red dot that obtained by utilizing Equation (17). The surface capacity for Figures (a) and (b) has been reduced from 0.7 to 0.5 in order to observe the NCNR phenomenon across the spline in Figure (b). Additionally, the graphics in Figure (b) also appear larger due to adjustments in the range of the graphic.



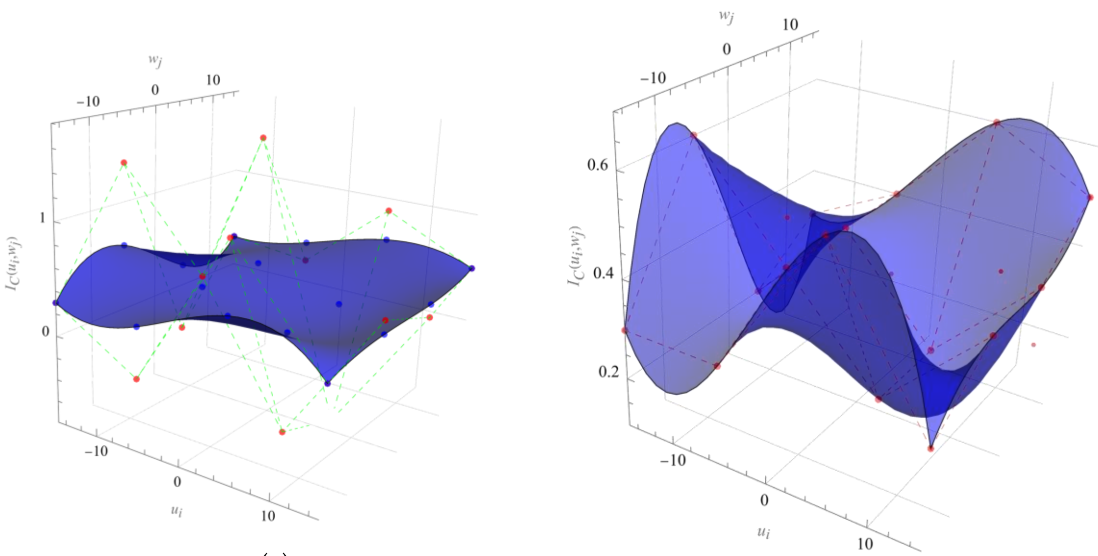
(a) (b)

Figure 1. NBB-sS interpolation for truth membership: **(a)** NBB-sS and its control net for data points with 0.7 capacity of surface; **(b)** NBB-sS and its control net for control points with 0.5 capacity of surface.



(**a**) (**b**)

Figure 2. NBB-sS interpolation for false membership: **(a)** NBB-sS and its control net for data points with 0.7 capacity of surface; **(b)** NBB-sS and its control net for control points with 0.5 capacity of surface.



(**a**) (**b**)

Figure 3. NBB-sS interpolation for indeterminacy membership: **(a)** NBB-sS and its control net for data points with 0.7 capacity of surface; **(b)** NBB-sS and its control net for control points with 0.5 capacity of surface. Since the surface is blue in this case, the control point and control net change to red, which is different from the other surface.

As a results, the novelty of this study as follows:

- The definition of NCNR for NBB-sS based on previous work for NCPR, NPR, NCP, NR, NP, and fundamental notion of NS.
- The mathematical representation of NBB-sS was found after some simplification from Eq. (9) to Eq. (18).

- The visualization of NBB-sS for truth, false and indeterminacy membership degrees.

5. Conclusions

This paper introduced the NCNR-based for creating NBB-sS model. The Bezier and non-uniform rational B-splines (NURBS) functions for surfaces and curves can be used to improve the findings in future study. Aside from that, by utilizing type-2 NS theory as a case of study. Surface data visualization can be utilized in a variety of applications, including geographic information system (GIS) modeling of spatial regions with uncertain borders, remote sensing, object reconstruction from an aerial laser scanner, bathymetric data visualization, and many more. The NBB-sS model can be used to address and solve difficulties characterized by uncertainty. Therefore, the applicability of this study is this study can treat and visualizing the uncertainty data as indeterminacy degree by using geometric modelling for B-spline surface and NS features while the scientific valued is there will be no data wasted during data collection process since all data will be examined and processes. As a result, the NCNR and NBB-sS models may provide a comprehensive analysis and description of a modeling issue that includes for any bicubic surface.

Acknowledgments
The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Data availability
The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest
The authors declare that there is no conflict of interest in the research.

Ethical approval
This article does not contain any studies with human participants or animals performed by any of the authors.

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Received: 26 Apr 2023, **Revised:** 06 Sep 2023,
Accepted: 24 Sep 2023, **Available online:** 01 Oct 2023.