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A Neutrosophic Approach for B-Spline Curve by Using Interpolation Method

Siti Nur Idara Binti Rosli [1] and Mohammad Izat Emir Bin Zulkifly [2,*]

[1,2] Department of Mathematic, Faculty of Sciences, University of Technology Malaysia (UTM) 81310, Johor Bahru, Malaysia.
Emails: idara-1999@graduate.utm.my; izatemir@utm.my.

* Correspondence: izatemir@utm.my.

Abstract: This study introduces a B-spline curve interpolation model based on the neutrosophic set technique. To begin, the neutrosophic notion is used to define the neutrosophic control point relation. After that, the neutrosophic control point is combined with the B-spline basis function. Besides, the neutrosophic B-spline curve interpolation model is illustrated using the interpolation method. Furthermore, an example and the methods are shown for creating the right curve.

Keywords: Neutrosophic Set; Curve of B-Spline; Method of Interpolation; Neutrosophic Control Points.

1. Introduction

The link between a curve created from control polygon vertices and the curve is technically dependent i on some interpolation or approximation approach [1]. Basis function determines this scheme. Bézier curves are generated via the Bernstein basis. Piegl and Tiller [1] also noted that two Bernstein basis features restrict curve flexibility. The curve's polynomial order depends on the number of polygon vertices supplied. A four-vertex, three-span polygon defines a cubic curve. Six-vertex polygons always generate fifth-degree curves. Decrease the number of vertices to decrease the curve degree, and vice versa.

According to Piegl and Tiller [1], the second limiting feature of the Bernstein foundation is globality. For all parameter values along the curve, the blending function is nonzero. Because each point is formed by mixing all control vertices, a change in one control vertex affects the entire Bézier curve. This avoids local alterations to the curve. The end slopes of a Bézier curve are defined by the orientations of the first and last polygon spans, so changing the center vertex of a five-point polygon has no effect. The Bernstein basis modifies the curvature of the curve worldwide. A lack of local control could be problematic. As a result, Bernstein basis is a subset of B-spline basis. This foundation does not operate on a global scale [1]. Since each vertex has a basis function, B-spline curves are non-global. Thus, each vertex only influences curve shape in the parameter range where its basis function is nonzero . The B-spline basis allows user to change the order of the Basis functions and the degree of the curve without changing the control polygon vertices . B-splines were invented by Schoenberg [2]. Cox [3] and De Boor [4] each created their own definition of recursive numerical computing. The B-spline basis was used by Gordon and Riesenfeld [5] to define curves.

Data points, according to Hoschek and Lasser [6] require curves. Data analysis and representation are complicated by noise and ambiguity. This problem is addressed by fuzzy set theory and geometric modelling. Tuohy and Patrikalakis [7] proposed using regionally scattered geophysical data to rebuild ambiguous surfaces. Their technique has been extended to describe volume data with periodic B-spline volume function [8,9]. Based on Tuohy and Patrikalakis [7], they developed enclosing or gap specific B-spline geometry [10] to describe underwater geophysical data and sensor measurement error. Tuohy and Patrikalakis [11] depicted functions with uncertainty defining an observed geophysical parameter using interval B-spline.

Anile et al. extended the techniques presented in [12] to data modelling and data reduction difficulties [13-15]. Anile et al. [16] improved on the modelling of Patrikalakis et al. [10]. They begin by reducing a big data set to fuzzy integers with suitable membership functions . They created fuzzy B-splines to interpret fuzzy data and rapid algorithms to calculate spline alpha values. Anile and Spinella [17] created the fuzzy B-splines methodology and used fuzzy arithmetic concepts to uncertain sparse data caused by measurement mistakes, data reduction issues, and modelling flaws. Using rigorous procedures, fuzzy B-splines that fit uncertain sparse data were generated and examined.

To address uncertainty problems, Wahab et al. [18] employ fuzzy numbers and Zadeh's [19] fuzzy set theory. The ideas of instability in data, fuzzy numbers, implementation of control measures, B-spline, and Bézier were employed. In CAGD, the approaches are used to construct fuzzy Bézier and B-spline curves. Each crisper control point is composed of left and right vague control points, each of which has a different degree of similarity to the initial control points (crisp control points). Its membership function is left and right continuous in a closed interval at each alpha value.

Fuzzy set theory (FST) only takes into account membership data, but not non-membership data and uncertainty. In 1986, Krasimir Atanassov expanded FST to include truth, falsehood, and uncertainty degrees [20]. It is best to accept ambiguity. As FST only accepts full membership data, Intuitionistic Fuzzy Sets (IFS) can be used when the data for categorization and processing is insufficient [21]. Florentin Smarandache, on the other hand, proposed mathematical theory, and neutrosophy advocates equality [22]. Neutrophil sets might be members, non-members, or undecided. Transdisciplinary challenges are addressed and described using Neutrosophic Set (NS) approaches. A true, incorrect, or ambiguous NS theory element can exist. This allows for more nuanced doubt and ambiguity, for as when two statements contradict each other. Geometric modelling has been employed by certain academics to build neutrosophic set procedures [23,24].

The Neutrosophic B-spline Curve Interpolation (NB-SCI) Model will be the primary focus of this project's creation of a geometric model that can deal with uncertainty data. The Neutrosophic Control Point Relation (NCPR) must be determined using neutrosophic set theories and the qualities it holds before generating the B-spline interpolation. These control points, together with the B-spline basis function, are used to build NB-SCI models, which are subsequently displayed using an interpolation method. The following section shows how to use the format of this paper. The initial part of this paper gave background information on the issue. Section 2 introduces the reader to the basic concept of Neutrosophic Set (NS), followed by Neutrosophic Point Relation (NPR) and Neutrosophic Control Point Relation (NCPR). The third section discusses how to calculate the NB-SCI using NCPR. The fourth section includes a numerical example, a graphical representation of NB-SCA, and the model-creation algorithm. The fifth and final segment will conclude the probe.

2. Preliminaries

The intuitionistic set in fuzzy systems can accommodate imperfect information but not indeterminate or inconsistent information [25]. A NS has three membership functions. With the addition of the "indeterminacy" parameter to the NS specification [25], there are three sorts of membership functions: a membership function (denoted by the letter T), an indeterminacy membership function (denoted by the letter I), and a non-membership function (denoted by the letter F).

Definition 1: [22] Let Z be the main of conversation, with element in Z denoted as z. The NS is an item in the form below and $\hat{N}$ denoted as NS.

$$\hat{N} = \{\langle z : T_{\hat{N}(z)}, I_{\hat{N}(z)}, F_{\hat{N}(z)} \rangle | \ z \in Z\} \quad (1)$$

where, the degrees $T, I, F : Z \to]^{-}0,1^{+}[$ the meaning of accordingly, the degree to which an element is a member of the truth, the degree to which it is indeterminate, and the degree to which it is a member of the false $z \in Z$ to the set Z with the condition;

$$0^{-} \le T_{\hat{N}}(z) + I_{\hat{N}}(z) + F_{\hat{N}}(z) \le 3^{+} \quad (2)$$

There is no restriction to values of $T_{\hat{N}}(z), I_{\hat{N}}(z)$ and $F_{\hat{N}}(z)$

NS will pick a value from either one of the actual standard subsets or one of the non-standard subsets of $]^{-}0,1^{+}[$. The actual value of the interval $[0,1]$, on the other hand, $]^{-}0,1^{+}[$ will be utilized in technical applications since its utilization in real data such as the resolution of scientific challenges, will be physically impossible. As a direct consequence of this, membership value utilization is increased.

$$\begin{gathered} \hat{N} = \{\langle z : T_{\hat{N}(z)}, I_{\hat{N}(z)}, F_{\hat{N}(z)} \rangle | \ z \in Z\} \text{ and} \\ T_{\hat{N}}(z), I_{\hat{N}}(z), F_{\hat{N}}(z) \in [0,1] \end{gathered} \quad (3)$$

There is no restriction on the sum of $T_{\hat{N}}(z), I_{\hat{N}}(z), F_{\hat{N}}(z)$ .Therefore,

$$0 \le T_{\hat{N}}(z) + I_{\hat{N}}(z) + F_{\hat{N}}(z) \le 3 \quad (4)$$

Definition 2: [23, 24] Let $\hat{N} = \{\langle z : T_{\hat{N}(z)}, I_{\hat{N}(z)}, F_{\hat{N}(z)} \rangle | \ z \in Z\}$ and $\hat{M} = \{\langle y : T_{\hat{M}(y)}, I_{\hat{M}(y)}, F_{\hat{M}(y)} \rangle | \ y \in Y\}$ be neutrosophic elements. Thus, $NR = \{\langle (z,y) : T_{(z,y)}, I_{(z,y)}, F_{(z,y)} \rangle | \ z \in \hat{N}, y \in \hat{M}\}$ is a Neutrosophic Relation (NR) on $\hat{N}$ and $\hat{M}$.

Definition 3: [23,24] NS of $\hat{N}$ in space Z is Neutrosophic Point (NP) and $\hat{N} = \{\hat{N}_i\}$ where $i = 0, \dots, n$ is a collection of NPs where the existences $T_{\hat{N}} : Z \to [0,1]$ as truth degree, $I_{\hat{N}} : Z \to [0,1]$ as indeterminacy degree and $F_{\hat{N}} : Z \to [0,1]$ as false degree with

$$\begin{gathered} T_{\hat{N}}(\hat{N}) = \begin{cases} 0 & \text{if } \hat{N}_i \notin \hat{N} \\ a \in (0,1) & \text{if } \hat{N}_i \hat{\in} \hat{N} \\ 1 & \text{if } \hat{N}_i \in \hat{N} \end{cases} \\ I_{\hat{N}}(\hat{N}) = \begin{cases} 0 & \text{if } \hat{N}_i \notin \hat{N} \\ b \in (0,1) & \text{if } \hat{N}_i \hat{\in} \hat{N} \\ 1 & \text{if } \hat{N}_i \in \hat{N} \end{cases} \\ F_{\hat{N}}(\hat{N}) = \begin{cases} 0 & \text{if } \hat{N}_i \notin \hat{N} \\ c \in (0,1) & \text{if } \hat{N}_i \hat{\in} \hat{N} \\ 1 & \text{if } \hat{N}_i \in \hat{N} \end{cases} \end{gathered} \quad (5)$$

2.1 Neutrosophic Point Relation (NPR)

The concept of the NS, which was discussed in the previous section, serves as the cornerstone for NPR. If is a group of Euclid eternal space points and then, the following is how NPR is described:

Definition 4: Let N, M be a grouping of elements in global area that are part of a set that is not null and $N, M, O \in \mathbf{R}\times\mathbf{R}\times\mathbf{R}$, then the term "NPR" refers to

$$\hat{R} = \left\{ \begin{array}{l} \left\langle \left(n_i, m_j\right), T_R(n_i, m_j), I_R(n_i, m_j), F_R(n_i, m_j) \right\rangle \\ \mid T_R(n_i, m_j), I_R(n_i, m_j), F_R(n_i, m_j) \in I \end{array} \right\} \tag{6}$$

Where $\left(n_i, m_j\right)$ is a set of ordered positions and $\left(n_i, m_j\right) \in N \times M$ while $T_R(n_i, m_j), I_R(n_i, m_j), F_R(n_i, m_j)$ are the truth membership, the indeterminacy membership, and the false membership that follows the condition of the neutrosophic set which is respectively, $0 \le T_{\hat{N}}(z) + I_{\hat{N}}(z) + F_{\hat{N}}(z) \le 3$.

2.2 *Neutrosophic Control Point Relation (NCPR)*

The geometry of a spline only be determined by all of the data required to form the curve. The word "control point" relates to this. The control point is essential in the design, control, and production of smooth curves. In this section, the neutrosophic control point relationship (NCPR) is defined by first employing the concept of fuzzy control point from the research published in Wahab et al. [26] in the following:

Definition 5: Let $\hat{R}$ be a NPR, then NCPR is viewed as a group of points $n+1$ that denotes a locations and coordinates and is used to describe the curve and is indicated by

$$\begin{aligned} \hat{P}_i^T &= \left\{ \hat{p}_0^T, \hat{p}_1^T, ..., \hat{p}_n^T \right\} \\ \hat{P}_i^I &= \left\{ \hat{p}_0^I, \hat{p}_1^I, ..., \hat{p}_n^I \right\} \\ \hat{P}_i^F &= \left\{ \hat{p}_0^F, \hat{p}_1^F, ..., \hat{p}_n^F \right\} \end{aligned} \tag{7}$$

Where $\hat{P}_i^T$, $\hat{P}_i^I$ and $\hat{P}_i^F$ are NCP for membership truth, indeterminacy and i is one less than n.

3. Neutrosophic B-Spline Curve Interpolation (NB-SCI)

This section may be divided by subheadings. It should provide a concise and precise description of the experimental results, their interpretation as well as the experimental conclusions that can be drawn.

The NB-SCI is defined as follows after combining NCPR with a B-spline basis function:

Definition 6: Let $\hat{P}_i^T = \left\{ \hat{p}_0^T, \hat{p}_1^T, ..., \hat{p}_n^T \right\}, \hat{P}_i^I = \left\{ \hat{p}_0^I, \hat{p}_1^I, ..., \hat{p}_n^I \right\}, \hat{P}_i^F = \left\{ \hat{p}_0^F, \hat{p}_1^F, ..., \hat{p}_n^F \right\}$ where $i = 1, 2, ..., n+1$ is NCPR and NB-SCI denoted by BSC with the vector along curve as parameter t. As a result of combining, it with the blending function, NB-SCI is described as

$$BSC(t) = \sum_{i=1}^{n+1} \hat{P}_i N_i^k(t) \tag{8}$$

Where $t_{\min} \le t \le t_{\max}$ and $2 \le k \le n+1$ when $\hat{P}_i$ are the position vector of $n+1$ as control polygon vertices and N_i^k as the B-spline basis function. The $N_i^k(t)$ is describe as

$$N_i^1(t) = \begin{cases} 1 & if \quad t_i \le t < t_{i+1} \\ 0 & otherwise \end{cases} \tag{9}$$

and

$$N_i^k(t) = \frac{(t - t_i)}{t_{i+k-1} - t_i} N_i^{k-1}(t) + \frac{(t_{i+k} - t)}{t_{i+k} - t_{i+1}} N_{i+1}^{k-1}(t) \tag{10}$$

The parametric function NB-SCI in (8) is defined as follows and is made up of three curves: a member curve, a non-member curve, and an indeterminacy curve.

$$BSC(t)^T = \sum_{i=1}^{n+1} \hat{P}_i^T N_i^k(t) \tag{11}$$

$$BSC(t)^F = \sum_{i=1}^{n+1} \hat{P}_i^F N_i^k(t) \tag{12}$$

$$BSC(t)^I = \sum_{i=1}^{n+1} \hat{P}_i^I N_i^k(t) \tag{13}$$

Assuming the data points are in the NB-SCI range, then the data point should be (8). For each data point indicated by M_j, equation (8) has been modified as follows:

$$\begin{aligned} \hat{Q}_1(t_1) &= M_1^k(t_1)\hat{S}_1 + M_2^k(t_1)\hat{S}_2 + \ldots + M_{n+1}^k(t_1)\hat{S}_{n+1} \\ \hat{Q}_2(t_2) &= M_1^k(t_2)\hat{S}_1 + M_2^k(t_2)\hat{S}_2 + \ldots + M_{n+1}^k(t_2)\hat{S}_{n+1} \\ &\vdots \\ \hat{Q}_j(t_j) &= M_1^k(t_j)\hat{S}_1 + M_2^k(t_j)\hat{S}_2 + \ldots + M_{n+1}^k(t_j)\hat{S}_{n+1} \end{aligned} \tag{14}$$

When $2 \le k \le n+1 \le j$. Equation (14) is expressed as a matrix as

$$\left[\hat{Q}\right] = \left[M\right]\left[\hat{S}\right] \tag{15}$$

where

$$\begin{aligned} \left[\hat{Q}\right]^T &= \left[\hat{Q}_1(t_1) \quad \hat{Q}_2(t_2) \quad \cdots \quad \hat{Q}_j(t_j)\right] \\ \left[M\right] &= \begin{bmatrix} M_1^k(t_1) & \cdots & \cdots & M_{n+1}^k(t_1) \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ M_1^k(t_j) & \cdots & \cdots & M_{n+1}^k(t_j) \end{bmatrix} \\ \left[\hat{S}\right]^T &= \left[\hat{S}_1 \quad \hat{S}_2 \quad \cdots \quad \hat{S}_j\right] \end{aligned} \tag{16}$$

The measurement of data points along NB-SCI is the metric value t_j for each output. The parametric value on data point to l for data point is j as follows.

$$t_1 = 0$$
$$\frac{t_l}{t_{\max}} = \frac{\sum_{r=2}^{l}\left|\hat{Q}_r - \hat{Q}_{r-1}\right|}{\sum_{r=2}^{j}\left|\hat{Q}_r - \hat{Q}_{r-1}\right|}; l \geq 2 \tag{17}$$

The greatest parameter is indicated by $t_{\max}$, which is usually considered as the greatest value for the knot vector. If $2 \leq k \leq n+1 = j$, then $[M]$ is a square matrix, and the control polygon is derived immediately using an inverse matrix, such as

$$\left[\hat{Q}\right] = \left[M\right]^{-1}\left[\hat{S}\right] \qquad 2 \leq k \leq n+1 = j \tag{18}$$

As a result, NB-SCI can be acquired using (18).

3.1. Properties of Neutrosophic B-Spline Curve Interpolation (NB-SCI)

Since a B-spline basis is utilized to define a B-spline curve, numerous features, in addition to those already described, are easily understood:

- For any parameter value t, the sum of the B-spline basis functions is [4, 5]

$$\sum_{i=1}^{n+1} N_i^k(t) \equiv 1 \tag{19}$$

- For all values of parameters, each basis functional is either positive or zero. Thus, $N_i^k \geq 0$
- Each basis function, $k=1$ with the exception of first-order basis functions with, has a single highest value.
- The highest order of the curve matches the number of control polygon vertices. The highest value is one degree less.
- The curve demonstrates the variation-diminishing characteristic. As a result, the curve does not oscillate more frequently around any straight line than its control polygon.
- In general, the curve follows the shape of the control polygon.
- Any affine transformation is applied to the curve by transforming the control polygon vertices, which transforms the curve.
- The control polygon's convex hull contains the curve.

4. Numerical Example and Visualization

This section will go over the application of NB-SCAI and visualization. The examples will only use a numerical example at random and will employ an interpolation method. A NB-SCI will be shown that consists of NCPR with a degree of polynomial of four $n=4$.

4.1. Application of Neutrosophic B-Spline Curve Interpolation (NB-SCI)

To illustrate NB-SCA, let's consider NB-SCA with five neutrosophic control point relation as in Table 1.

Table 1. The NCPRs

NCPR $\hat{P}_i$	Truth Membershi $\hat{P}_i^T$	False Membership $\hat{P}_i^F$	Indeterminacy Membership $\hat{P}_i^I$
$\hat{P}_0 = (2,2)$	0.7	0.4	0.2
$\hat{P}_2 = (7,8)$	0.5	0.5	0.3
$\hat{P}_3 = (11,13)$	0.8	0.3	0.2
$\hat{P}_4 = (17,18)$	0.6	0.2	0.5
$\hat{P}_5 = (25,23)$	0.3	0.4	0.6

From Figure 1 through Figure 3, the planned interpolation curve is presented on its own with its matching data points (black dots) and NCP (red dots) utilizing (18). A neutrosophic control polygon connects the control points and is made up of truth degree, false degree, and indeterminacy control polygons. Figures 1-3 are also known as "truth membership," "false membership," and "indeterminacy B-spline curve interpolation." The NCP and controlling polygon governed the curve and ensured that the data points were interpolated.

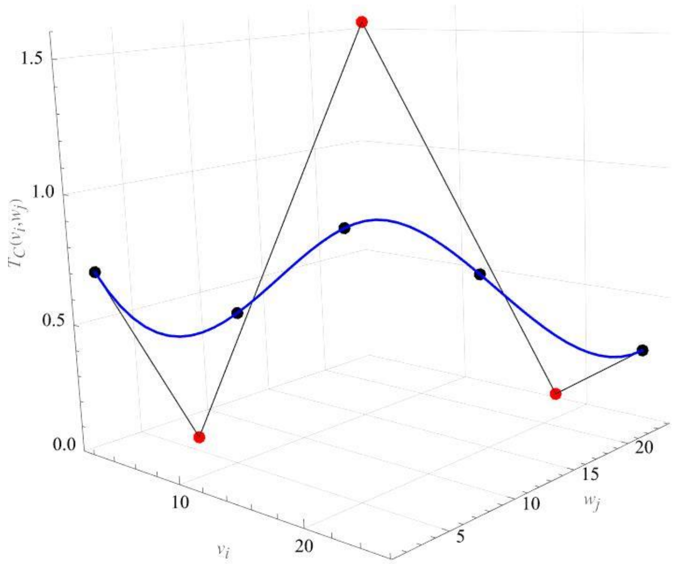

Figure 1. NB-SCI for Truth Membership with its Data Points, NCPs and Control Polygon.

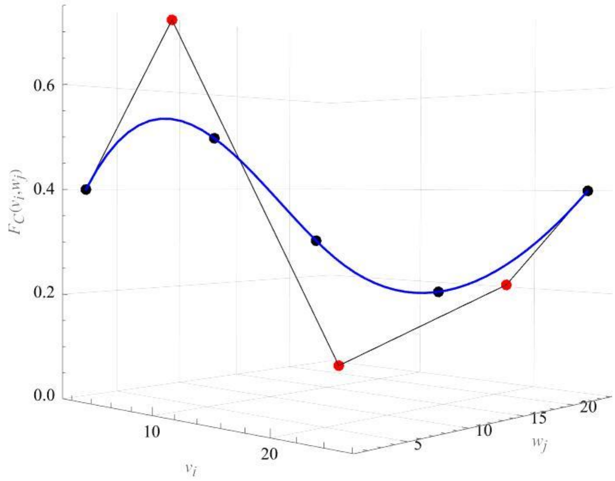

Figure 2. NB-SCI for False Membership with its Data Points, NCPs and Control Polygon

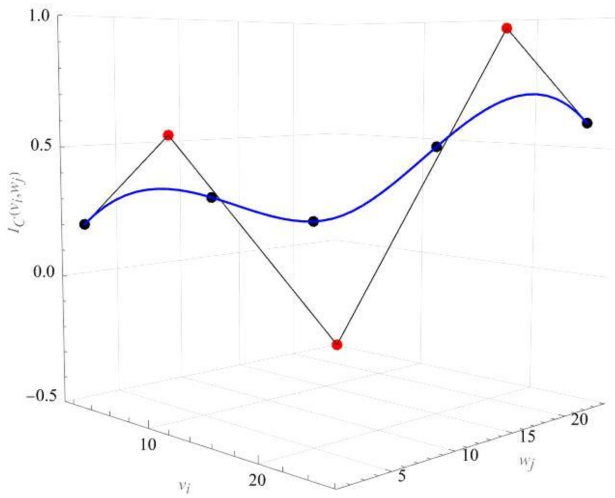

Figure 3. NB-SCI for Indeterminacy Membership with Data Points, NCPs and its Control Polygon

Figures 4 through 6 depict NB-SCI as true membership, false membership, and indeterminacy curves with data points and connected data points, separately.

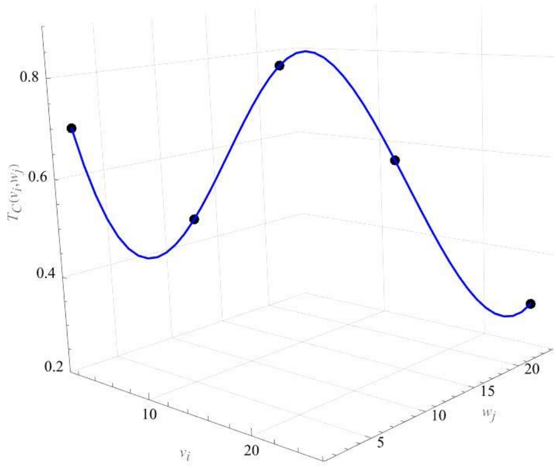

Figure 4. NB-SCI for Truth Membership with its Data Points

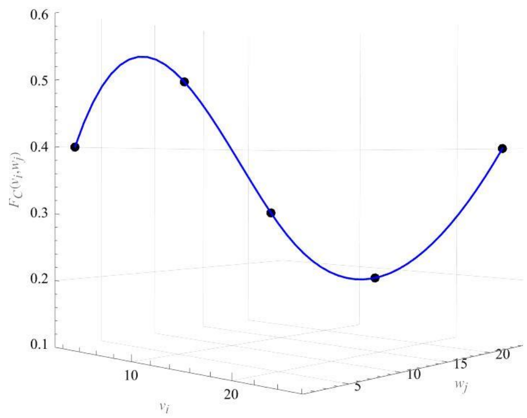

Figure 5. NB-SCI for False Membership with its Data Points

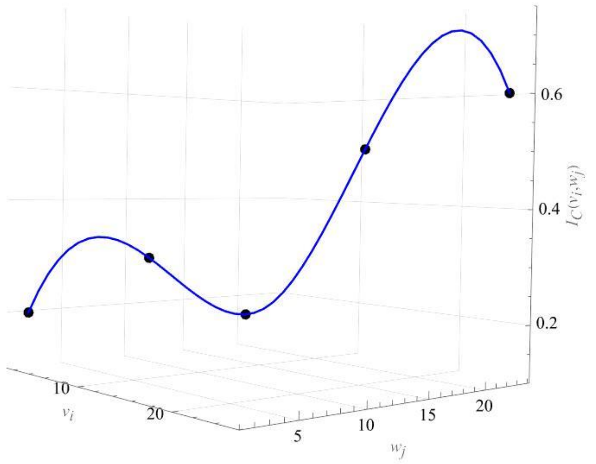

Figure 6. NB-SCI for Indeterminacy Membership with its Data Points

Figures 7 and 8 depicted NB-SCI from various points of view. Figure 7 depicted NB-SCI using data points, NCPs, and control polygons. Finally, Figure 8 depicts NB-SCI with data points. The NB-SCI for blue curve represents truth membership, green curve represents false membership and pink curve represents indeterminacy membership. All the memberships are demonstrated in an axis as Figures 7 and 8 shown.

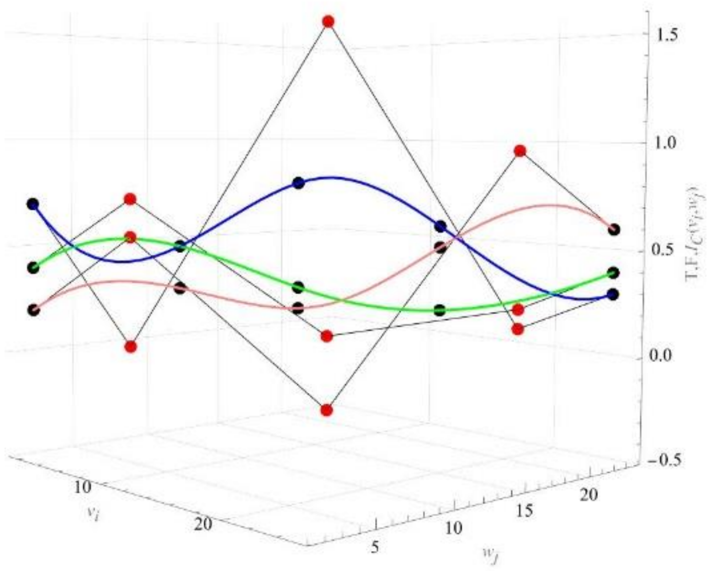

Figure 7. NB-SCI for All Membership with its Data Points, NCPs and Control Polygons.

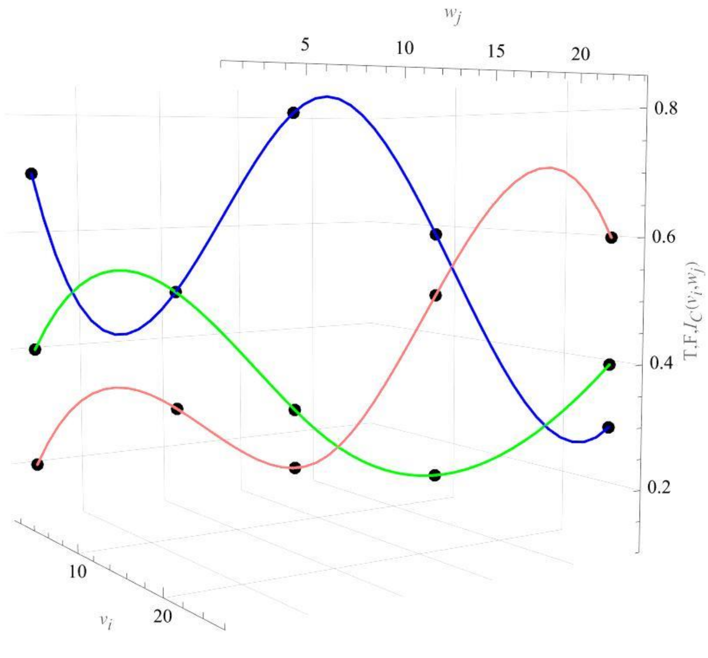

Figure 7. NB-SCI for All Membership with its Data Points only.

Following that, the algorithm for obtaining NB-SCI is summarized below:

Step 1: The knot vector and the neutrosophic data point relation are computed using $\hat{Q} = \left\{\hat{Q}_j\right\}_{j=1}^{n+1}$ and $k = \left\{k_j\right\}_{j=1}^{n+1}$.

Step 2: Determine the parametric value along the neutrosophic B-spline curves that corresponds to each NCPR by using (17).

Step 3:

1. Calculate the chord lengths between each point.

$$\left|\hat{Q}_2-\hat{Q}_1\right|,\left|\hat{Q}_3-\hat{Q}_2\right|,\cdots,\left|\hat{Q}_r-\hat{Q}_{r-1}\right|$$

2. The parameter is computed as.

$$\sum_{r=2}^{r}\left(\hat{Q}_r-\hat{Q}_{r-1}\right) \quad \text{and} \quad t_1,\frac{t_2}{t_{\max}},\ldots,\frac{t_l}{t_{\max}}$$

Step 3: Determine the B-spline basis function based on the knot vector in Step 1 by creating the $[M]$ matrix using (15) and (16).

Step 4: Following that, NCPR can be obtained by using (18).

Step 5: Lastly, the NCPR is combined with the B-spline basis function as shown in (8) - (13) to produce NB-SCI.

5. Conclusions

This paper provides an introduction to NB-SCI as well as some of its characteristics. NB-SCI is an extremely useful methodology that has the potential to be implemented in a broad variety of business sectors, such as real civil engineering concepts, shipbuilding, designs for architecture, aerospace, manufacture and a great deal more besides. Due to the availability of truth degree, false degree, and indeterminacy degree, the neutrosophic approach may solve a greater variety of challenges. This neutrosophic set approach, when combined with tools based on the B-spline, can construct a continuously differentiable smooth curve that is capable of providing a comprehensive description of any explored subject. This technique can be made more effective by utilizing the surface of interpolation or approximation for B-spline and NURBS.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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