Journal of Physics: Conference Series

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To cite this article: Yuat Hoong Cheah and Su Hoe Yeak 2023 *J. Phys.: Conf. Ser.* **2609** 012003

The fitting of roundabout model with gradient-based minimization

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Abstract. The objective of this research is to create an appropriate roundabout model for all countries. To date, the four-arm roundabout macroscopic model has been created. In order to ensure the feasibility and suitability of the model for all countries, the fitting process must be implemented since the speed of vehicles varies in each country. Thus, the parameter estimation on the rate of exiting roundabout is to be determined because the mean speed of vehicles is related to the rate of exiting the roundabout. In the minimization process, we have proposed an efficient and reliable framework as it includes the calculation of gradients used in minimization so called the user supplied-gradient minimization, as compared to non-user supplied-gradient minimization. The including of the calculation of gradient is to produce more accurate results by the built-in MATLAB minimization routine for parameter fitting. In this research, five pseudo experiments with numerous parameters are carried out. The rate of exiting the roundabout is set initially in order to compute the Total Travel Time and Total Waiting Time. The simulation showed a highly converged and accurate solution by the user supplied-gradient minimization. Lastly, this parameter estimation can be implemented that will enable the roundabout model to be applied worldwide if there is actual data for Total Travel Time and Total Waiting Time.

1. Introduction

The roundabout plays a pivotal role in managing road congestions and improving traffic movements as it can replace the T-junction and intersecting roads. Essentially, it provides an alternative to the conventional traffic junctions by reducing the number of traffic flow junctions. Roundabouts can significantly increase the smoothness of traffic flow movements through a multiple-road junction, both in terms of throughput and safety. Hence, this is the reason that the roundabout is gaining much momentum and the number of roundabouts is on the increase in countries around the world.

The earliest model of the roundabout is called the Circus, originally named King's Circus. Completed in 1768, it was designed by architect John Wood, the elder. These circular junctions are widespread in many countries, for instance, France, America, Netherlands, Germany, and so on. Until the year 1960, modern roundabouts were developed by the UK's Transport Research Laboratory engineers who re-calibrated and standardized the circular intersections. Then, this modern roundabout has spread to Europe and North America since 1970 and is still being used today. Malaysia has the largest roundabout in the world, which is located at Putrajaya. In fact, the roundabout is a viable option that ensures safe and efficient traffic flow, and is being used globally.

Many researchers have created the roundabout model for the purpose of studying the performance of traffic flow at the roundabout. In 2000, Robinson and Rodegerdts presented a comprehensive discussion of roundabout performance analysis and capacity curves for urban compact roundabouts, single-lane roundabouts and double-lane roundabouts. In the calculation of these three kinds of roundabouts, the different equations were defined according to the capacity curves developed by different countries as well as the size of roundabouts [1]. Subsequently, Wang and Ruskin proposed a new model to study traffic flow on a single-lane urban roundabout using the multi-state cellular automata (CA) ring. This model included driver behaviour at the roundabout entrance and is considered as the stochastic model which is randomly grouped into four categories, based on the amount of space required to enter the roundabout. Driver behaviour refers to rational, urgent and reckless behaviours [2]. A few years later, they proposed the Multi-stream Minimum Acceptable Space (MMAS) Cellular Automata (CA) model to study unsignalized multi-lane urban roundabouts. This method is also based on the heterogeneity and inconsistency of driver behaviour. However, this model is only applicable in left-hand traffic countries, for instance, Ireland, New Zealand and the UK [3].

The suitability of the roundabout model for a country is crucial and it has to ensure the accuracy of the performance of the roundabout. The new Highway Capacity Manual 2010 (HCM 2010) capacity formulae for single-lane and two-lane roundabouts were developed in 2010 [4]. However, this research only focused on creating models for United Kingdom, with the adaption of HCM 2010, in which the HCM 2010 was based on research covering United State roundabouts, as described in the National Cooperative Highway Research Program (NCHRP) report 572. It has since been adopted in the new Transportation Research Board – Federal Highway Administration (TRB – FHWA) Roundabout Informational Guide [5]. Recently, the HCM 2010 delay model has been modified and applied to the haphazard heterogeneous traffic conditions in India, which has a mix of vehicle categories with different static and dynamic characteristics. The HCM model is derived by multiplying the average control delay in the Indian heterogeneous traffic conditions [6].

There are two main types of models for designing the traffic flow which are macroscopic and microscopic models. Theoretically, the macroscopic model considered a huge number of vehicles on a road, thus the flow of vehicles is assumed as flowing in a tube [7]. It describes the most crucial characteristics of traffic flow which are the formation and dissipation of queues and shock waves. Normally, the variables in this model are density, flow and speed. On the contrary, microscopic model describes both the condition of traffic flow and interaction between driver behaviour. It simulates single vehicle-driver characteristics. The dynamic variables of this model are the position and velocity of each vehicle [7-9]. The first microscopic traffic flow model called follow-the-leader model was developed in 1961 [10]. Recently, there are many researchers derived the microscopic model in terms of velocity or delay or both in order to create a realistic model [11-14].

As mentioned, the HCM model above, is used to study the traffic flow in India by multiplying the average control delay. Otherwise, to study the traffic flow in other countries, the average control delay of the country must be multiplied into the HCM model. The multiplication of the average control delay into the HCM model is similar to the requirements of this research. In order to create a model that can be applied across countries, the parameter-fitting process must be carried out to meet the objective of this research. We implemented the minimization process for obtaining parameter estimation [15] with two methods: non-user supplied-gradient minimization and user supplied-gradient minimization. We derived the gradient algorithm [16, 17] and this equation incorporate to the MATLAB program with its built-in function known as fmincon. It is a nonlinear minimization function. This method is so-called user supplied-gradient minimization or partial exact gradient minimization. The aim of we derived the gradient algorithm is to increase the accuracy of the minimization results. On the contrary, the non-user supplied-gradient minimization is not included in our derived gradient equation, it is only relying on the built-in gradient-based of Matlab software. Lastly, the exiting of roundabout rate will be the parameter that has to be fitted, as this rate is corresponding to the mean speed of cars. For instance, the higher the exiting the roundabout rate indicates the faster of the cars and vice versa. As a result, our model will be a convenient model which can be fitted to suit the traffic flow condition of each country if there is actual data for Total Travel Time and Total Waiting Time.

2. Roundabout model

In this research, the model of roundabout is described in the paper as "Modeling of Traffic Flow on Roundabouts" [18]. Furthermore, the model has been modified with the coupling of waiting system in demand as in the paper "Modeling and Simulation of Roundabout with Waiting System" [19]. Thereupon, the model is macroscopic model.

3. Research methodology

3.1. Determination of parameter estimation and pseudo experiment data

First of all, the relationship between the traffic flow on the roundabout, the time consumption and driver behaviour must be understood. In reality, drivers drive at different speeds, and this is related to the Total Travel Time (TTT) on the road network and Total Waiting Time (TWT) at the entrance of the secondary road. Logically, in the event where the drivers drive at a higher speed, the (TTT) and (TWT) will be reduced, and vice versa. Besides, it actually corresponds to the exiting the roundabout rate; if the drivers drive at a faster speed, the exiting the roundabout rate will be higher. The speed taken by the driver is related to the driver's behaviour because it is attributed to one's personality and is often in tandem with the driver's temperament. From this scenario, we can verify that the traffic flow on roundabouts, time consumption and driver behaviour are interrelated [20, 21]. As a result, the exiting the roundabout rate plays a crucial role as the parameter estimation; conversely, the TTT and TWT will be the pseudo experiment data.

3.2. Mathematical derivation

With reference to our paper [18], the Total Travel Time (TTT) and Total Waiting Time (TWT) are considered as the pseudo experiment data whereas the exiting the roundabout rate β is the parameter estimation. Therefore, in the minimization process, the parameter fitting is to minimize the following objective function is derived as

$$E(\beta) = \sum_{Fin=F_1}^{F_N} \left(TTT(F_{\text{in}}, \beta_{Ip}) - TTT(F_{\text{in}}, \beta)\right)^2 + \left(TWT(F_{\text{in}}, \beta_{Ip}) - TWT(F_{\text{in}}, \beta)\right)^2, N = 1,2,3,4. \tag{1}$$

in which

$$TTT = \sum_{n=1}^{N} \int_0^T \int_{I_n}^{N} \rho(x,t;\beta)\,dx\,dt + \sum_{n=1}^{N} \int_0^T l_n(t;\beta)\,dt + T \cdot \sum_{n=1}^{N} \int_{I_n}^{N} \rho(x,T;\beta)\,dx + T \cdot \sum_{n=1}^{N} l_n(T;\beta) \tag{2}$$

and

$$TWT = \sum_{n=1}^{N} \int_0^T l_n(t;\beta)\,dt + T \cdot \sum_{n=1}^{N} l_n(T;\beta) \tag{3}$$

where N is the number of arms of the roundabout, I is the interval between the arm junctions, Ip is the initial guess of parameter estimation, F_{in} is the flux entering the secondary lane, and l is the queue length. In equation (1), the terms $TTT(F_{\text{in}}, \beta)$ and $TWT(F_{\text{in}}, \beta)$ represent pseudo experiment data. Note that, there is no unit for TTT and TWT for equations (2) and (3).

The next derivation is for the gradient calculation which means differentiating the Equation (1) with respect to β, yield

$$\begin{aligned} \frac{d}{d\beta} E(\beta) &= \sum_{Fin=F_1}^{F_N} \frac{d}{d\beta}\left(TTT(F_{\text{in}}, \beta_{Ip}) - TTT(F_{\text{in}}, \beta)\right)^2 + \frac{d}{d\beta}\left(TWT(F_{\text{in}}, \beta_{Ip}) - TWT(F_{\text{in}}, \beta)\right)^2 \\ &= \sum_{Fin=F_1}^{F_N} -2\left(TTT(F_{\text{in}}, \beta_{Ip}) - TTT(F_{\text{in}}, \beta)\right) \frac{d}{d\beta} TTT(F_{\text{in}}, \beta) + \\ &\quad \sum_{Fin=F_1}^{F_N} -2\left(TWT(F_{\text{in}}, \beta_{Ip}) - TWT(F_{\text{in}}, \beta)\right) \frac{d}{d\beta} TWT(F_{\text{in}}, \beta) \end{aligned} \tag{4}$$

where

$$\frac{d}{d\beta}TTT(F_{\text{in}},\beta) = \sum_{n=1}^{N}\int_0^T\int_{I_n}^{N}\frac{d}{d\beta}\rho(x,t;\beta)\,dx\,dt + \sum_{n=1}^{N}\int_0^T\frac{d}{d\beta}l_n(t;\beta)\,dt + T\cdot \sum_{n=1}^{N}\int_{I_n}^{N}\frac{d}{d\beta}\rho(x,T;\beta)\,dx + T\cdot\sum_{n=1}^{N}\frac{d}{d\beta}l_n(T;\beta) \tag{5}$$

and

$$\frac{d}{d\beta}TWT(F_{\text{in}},\beta) = \sum_{n=1}^{N}\int_0^T\frac{d}{d\beta}l_n(t;\beta)\,dt + T\cdot\sum_{n=1}^{N}\frac{d}{d\beta}l_n(T;\beta) \tag{6}$$

From equations (5) and (6), the central difference method is applied yields

$$\frac{d}{d\beta}\rho(x,t;\beta) \approx \frac{\rho(x,t;\beta+\Delta\beta)-\rho(x,t;\beta-\Delta\beta)}{2\Delta\beta} \tag{7}$$

$$\frac{d}{d\beta}l_n(t;\beta) \approx \frac{l_n(t;\beta+\Delta\beta)-l_n(t;\beta-\Delta\beta)}{2\Delta\beta} \tag{8}$$

Note that equations (7) and (8) are only valid if the range for $\beta+\Delta\beta$ and $\beta-\Delta\beta$ is the same as the range of β which is $[0,1]$.

Otherwise, the forward difference method is applied for the case of $\beta-\Delta\beta<0$,

$$\frac{d}{d\beta}\rho(x,t;\beta) \approx \frac{\rho(x,t;\beta+\Delta\beta)-\rho(x,t;\beta)}{\Delta\beta} \tag{9}$$

$$\frac{d}{d\beta}l_n(t;\beta) \approx \frac{l_n(t;\beta+\Delta\beta)-l_n(t;\beta)}{\Delta\beta} \tag{10}$$

and for the case of $\beta+\Delta\beta>1$, the backward difference method is applied,

$$\frac{d}{d\beta}\rho(x,t;\beta) \approx \frac{\rho(x,t;\beta)-\rho(x,t;\beta-\Delta\beta)}{\Delta\beta} \tag{11}$$

$$\frac{d}{d\beta}l_n(t;\beta) \approx \frac{l_n(t;\beta)-l_n(t;\beta-\Delta\beta)}{\Delta\beta} \tag{12}$$

The error calculation is

$$\varepsilon = \beta - \beta_{pe} \tag{13}$$

where β is exiting the roundabout rate (exact) and β_{pe} is the estimated exiting the roundabout rate (parameter estimation).

3.3. Framework topology

Generally, the unknown parameters of dynamic models are estimated from the pseudo experimental data. In practice, it is necessary to verify the obtained parameter estimation. The procedures consist of the following steps:

1. Set a value of exiting the roundabout rate, β which is referred to as the exact value, and several values of flux entering the secondary lane, F_{in} in order to compute the pseudo experiment data, which are Total Travel Time, $TTT(F_{\text{in}},\beta)$ and Total Waiting Time, $TWT(F_{\text{in}},\beta)$.
2. Use the computed results of Total Travel Time, $TTT(F_{\text{in}},\beta)$ and Total Waiting Time, $TWT(F_{\text{in}},\beta)$ to estimate the exiting the roundabout rate (parameter estimation), β_{pe} and to define the initial guess of parameter estimation, Ip.
3. Firstly, execute the minimization with non-user supplied-gradient.
4. Secondly, execute the minimization with user supplied-gradient.
5. Compare the exiting the roundabout rate (parameter estimation), β_{pe} obtained by step 3 and step 4 with the exact values of exiting the roundabout rate, β.

Note that, if the estimated exiting the roundabout rate, β_{pe} in the third and fourth steps are very close or similar to the exiting the roundabout rate, β which was set in the first step, the produced parameter estimation is accurate and converging. The roundabout model can be used for fitting purposes.

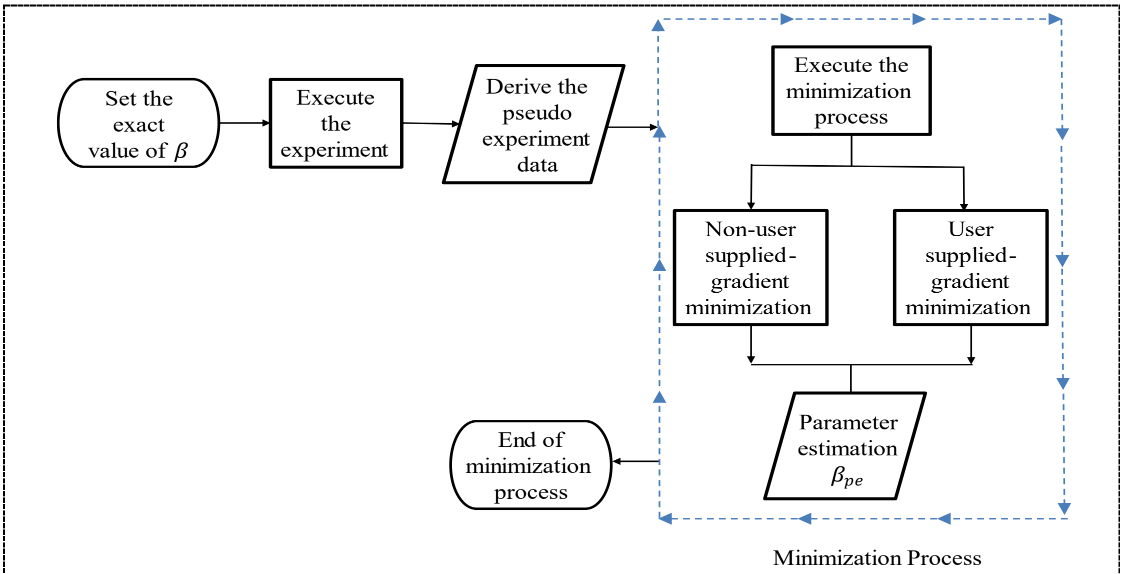


Figure 1. Parameter estimation verification flow chart.

3.4. Numerical setting

In the simulation, we conducted five pseudo experiments with $F_{\text{in}} = 0, 0.2, 0.4, 0.6, 0.8, 1$ which $F_{\text{in}} = [0,1]$ and several values of $\Delta\beta$ such as 0.4, 0.1, 0.01 and 0.001. These several values of F_{in} and $\Delta\beta$ are randomly chosen for examining the accuracy and convergence of the minimization process. The distinct values of P are also supplied in each simulation where P refers to the entering the outgoing main lane rate from incoming main lane and secondary lane. For each pseudo experiment, both results will be generated by non-user supplied-gradient minimization (NUS) and user supplied-gradient minimization (US) for studying the accuracy and convergence of the estimated exiting the roundabout rate (parameter estimation), β_{pe}. The specifications of the roundabout model are four-arm roundabout with a 4-unit circumference and three-arm roundabout with a 3-unit circumference. In addition, the other settings of parameters are space grid size $\Delta x = 0.1$, total time $T = 1$ and initial guess of parameter estimation, $Ip = 0$.

4. Results and discussion

In this research, we have computed the Total Travel Time (TTT) and Total Waiting Time (TWT) for pseudo experiments 1 to 4 by distinct parameters which are $\beta = 0.45$ with $P = 0.4$, $\beta = 0.6$ with $P = 0.8$, $\beta = 0.5$ with $P = 1$ and $\beta = 1$ with $P = 0.4$. These four pseudo experiments are four-arm roundabout with a 4-unit circumference. The computed results are presented in Table 1. Furthermore, the parameter of pseudo experiment 5 is same as pseudo experiment 1which are $\beta = 0.45$ with $P = 0.4$, however the specification of roundabout model is three-arm roundabout with a 3-unit circumference. The result of Total Travel Time (TTT) and Total Waiting Time (TWT) for pseudo experiment 5 is shown in Table 6.

Table 1. Computed Total Travel Time (TTT) and Total Waiting Time (TWT) from pseudo experiments 1 to 4.

F_{in}	Pseudo Experiment 1		Pseudo Experiment 2		Pseudo Experiment 3		Pseudo Experiment 4	
	TTT	TWT	TTT	TWT	TTT	TWT	TTT	TWT
0	0	0	0	0	0	0	0	0
0.2	1.1269	0	1.1156	0	1.1232	0	1.0853	0
0.4	2.2539	0	2.2312	0	2.2463	0	2.1706	0
0.6	3.3802	0.0485	3.3466	0.0230	3.3691	0.0396	3.2559	0
0.8	4.3336	0.7880	4.2981	0.7532	4.3217	0.7764	4.2022	0.6750
1.0	5.2336	1.6880	5.1981	1.6532	5.2217	1.6764	5.1022	1.5750

As mentioned in the numerical setting section, during the minimization process, we conducted two simulations which are non-user supplied-gradient (NUS) minimization and user supplied-gradient (US) minimization to generate the parameter estimation β_{pe}. Tables 2 to 5 show the comparison between them of four-arm roundabout with a 4-unit circumference. The Table 7 presents the comparison of two minimization methods of three-arm roundabout with a 3-unit circumference.

Table 2. Comparison of Parameter Estimation β_{pe} in pseudo experiment 1.

$\Delta\beta$	Non-user Supplied-gradient (NUS)		User Supplied-gradient (US)	
	$\beta_{pe,NUS}$	Error, ε_{NUS}	$\beta_{pe,US}$	Error, ε_{US}
0.4	0.4962	0.0462	0.4499	0.0001
0.1	0.4962	0.0462	0.4499	0.0001
0.01	0.4962	0.0462	0.4499	0.0001
0.001	0.4962	0.0462	0.4499	0.0001

Table 3. Comparison of Parameter Estimation β_{pe} in pseudo experiment 2.

$\Delta\beta$	Non-user Supplied-gradient (NUS)		User Supplied-gradient (US)	
	$\beta_{pe,NUS}$	Error, ε_{NUS}	$\beta_{pe,US}$	Error, ε_{US}
0.4	0.6194	0.0194	0.5999	0.0001
0.1	0.6194	0.0194	0.5999	0.0001
0.01	0.6194	0.0194	0.5999	0.0001
0.001	0.6194	0.0194	0.5999	0.0001

Table 4. Comparison of Parameter Estimation β_{pe} in pseudo experiment 3.

$\Delta\beta$	Non-user Supplied-gradient (NUS)		User Supplied-gradient (US)	
	$\beta_{pe,NUS}$	Error, ε_{NUS}	$\beta_{pe,US}$	Error, ε_{US}
0.4	0.5339	0.0339	0.5000	0.0000
0.1	0.5339	0.0339	0.5000	0.0000
0.01	0.5339	0.0339	0.5000	0.0000
0.001	0.5339	0.0339	0.5000	0.0000

Table 5. Comparison of Parameter Estimation β_{pe} in pseudo experiment 4.

$\Delta\beta$	Non-user Supplied-gradient (NUS)		User Supplied-gradient (US)	
	$\beta_{pe,NUS}$	Error, ε_{NUS}	$\beta_{pe,US}$	Error, ε_{US}
0.4	1.0000	0.0000	1.0000	0.0000
0.1	1.0000	0.0000	1.0000	0.0000
0.01	1.0000	0.0000	1.0000	0.0000
0.001	1.0000	0.0000	1.0000	0.0000

Table 6. Computed Total Travel Time (TTT) and Total Waiting Time (TWT) of pseudo experiments 5.

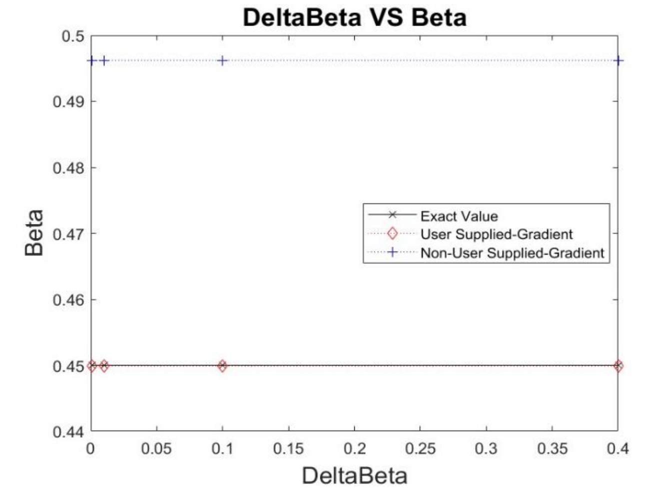
F_{in}	TTT	TWT
0	0	0
0.2	0.8452	0
0.4	1.6904	0
0.6	2.5354	0.0365
0.8	3.2696	0.6105
1.0	3.9696	1.3105

Table 7. Comparison of Parameter Estimation β_{pe} in pseudo experiment 5.

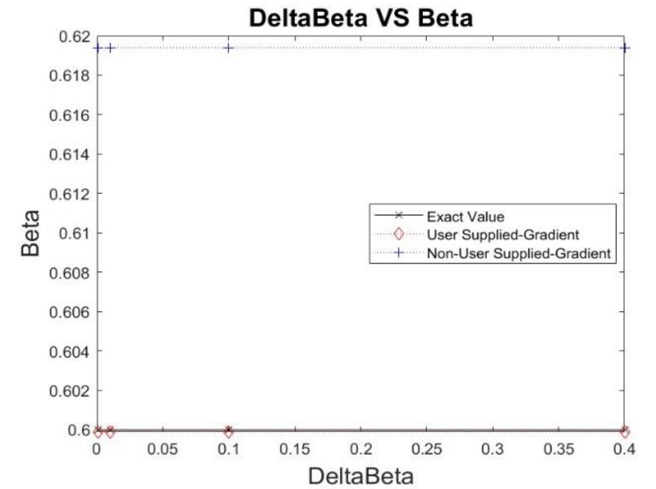
$\Delta\beta$	Non-user Supplied-gradient (NUS)		User Supplied-gradient (US)	
	$\beta_{pe,NUS}$	Error, ε_{NUS}	$\beta_{pe,US}$	Error, ε_{US}
0.4	0.4500	0.0000	0.4500	0.0000
0.1	0.4500	0.0000	0.4500	0.0000
0.01	0.4500	0.0000	0.4500	0.0000
0.001	0.4500	0.0000	0.4500	0.0000

According to the results in Tables 2 to 5 and Table 7, the distinct values of $\Delta\beta$ do not affect the convergence of each simulation. Meanwhile, it shows that the obtained parameters estimation, β_{pe} are converged for both simulations. Moreover, in these five pseudo experiments, the obtained values of $\beta_{pe,NUS}$ for non-user supplied-gradient simulation and $\beta_{pe,US}$ for user supplied-gradient simulation are very close to the value of β, reflecting a very high degree of accuracy, especially the last two pseudo experiment that indicates zero error. The accuracy and convergence are increasing from pseudo experiment 1 to pseudo experiment 4. The values of β and P will affect the accuracy and convergence of the minimization process. For example, the high accuracy took place in the pseudo experiment 4 because the value of $\beta = 1$, which the exiting the roundabout rate is maximal and low value of $P = 0.4$, which is the crossing the arm junction rate from the secondary lane and incoming main lane to the outgoing main lane of roundabouts. Based on this case, the density on roundabout is low, thus yielded the very high accuracy parameter estimation. In other words, the algorithm of minimization with the small values. Besides that, the pseudo experiment 5 is implemented to compare with the pseudo experiment 1 with the same parameters of β and P. The distinct in these two pseudo experiments is in pseudo experiment 1 and pseudo experiment 5 are the four-arm roundabout with a 4-unit circumference and three-arm roundabout with a 3-unit circumference respectively. It can be described as the algorithm of minimization is not complex. On the other hand, the setting of $\beta = 0.45$ in pseudo experiment 1 is low, it made the high density on the roundabout, thus yielded a lesser accuracy with highest error among four pseudo experiments. Apart from this, it also can be seen that, the user supplied-gradient simulation can perform better than the non-user supplied-gradient simulation in the high density on four-arm roundabout with a 4-unit circumference with the error is only 0.0001.

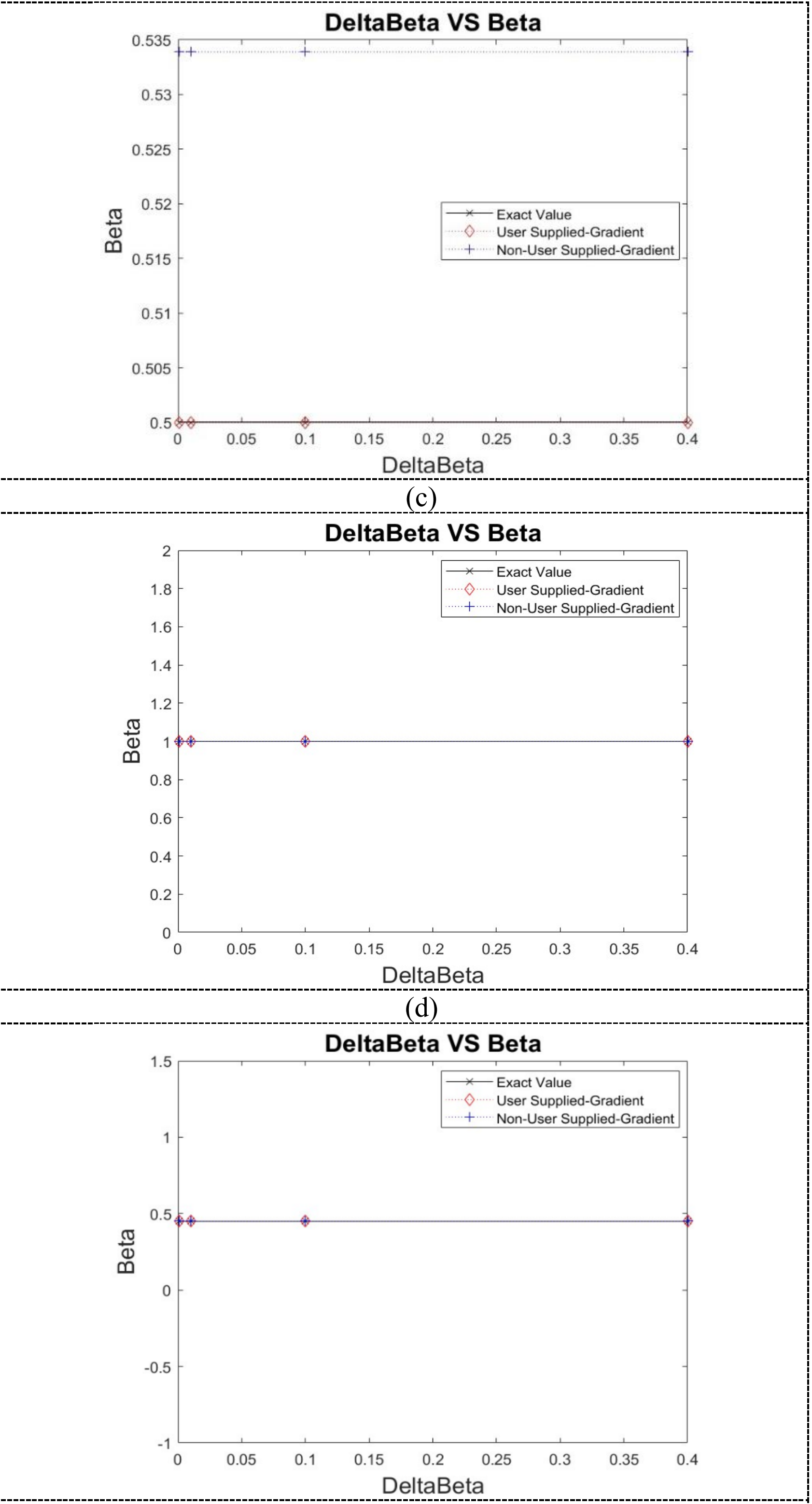
In addition, Figure 2 shows the accuracy and convergence of these five pseudo experiments. From each pseudo experiment, the high accuracy and convergence of the user supplied-gradient is clearly presented as its results almost overlap with the exact value.



(a)



(b)



(c)

(d)

(e)

Figure 2. The accuracy and convergence of pseudo experiments. (a) Pseudo Experiment 1 (b) Pseudo Experiment 2 (c) Pseudo Experiment 3 (d) Pseudo Experiment 4 (e) Pseudo Experiment 5

Once we have analyzed the high accuracy and convergence by the user supplied-gradient minimization, this model is considered can be operated in worldwide. Furthermore, with regards to the fitting process, if there is actual data for Total Travel Time and Total Waiting Time, these values will be used to estimate the parameters estimation, $\boldsymbol{\beta_{pe}}$. Once this parameter estimation is generated, we will set this calibrated parameter estimation in our roundabout model. Hence, it can be implemented to study the traffic flow on roundabouts in other countries.

5. Conclusion

The results show that the obtained parameter estimation for exiting the roundabout rate with user supplied-gradient minimization $\beta_{pe,US}$ is very close and similar to the exact value β, as compared with the parameter estimation for exiting the roundabout rate with non-user supplied-gradient $\beta_{pe,NUS}$. Even though in the high density of traffic flow on roundabout for example the pseudo experiment 1 as it has the lowest rate of β. It is evident that the computation with the user supplied-gradient minimization has converged and it is accurate. Thus, the fitting roundabout model studying the operational performance of traffic flow on roundabouts in other countries can be conducted with the availability of exact Total Travel Time and Total Waiting Time data.

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Acknowledgments

The authors would like to acknowledge the financial support from Universiti Teknologi Malaysia for the funding under UTM Fundamental Research (UTMFR) (QJ130000.2554.20H72).