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Modelling the Position of Pumping Well Located Between Two Parallel Contaminant Sources

Shaymaa Mustafa, Mohamad Darwish

***Abstract*—This article provides an analytical model to determine the suitable location of pumping well located between two polluted parallel sources. The first source is assumed to be a polluted river, while the second one is an arbitrary contaminant source such as landfill leachate site or wastewater. The location of the well is specified so that it gets the highest amount of river water that satisfies water quality requirements. To validate the model results, a numerical simulation is conducted using MODFLOW to calculate the concentration of contaminants at various locations determined by the proposed analytical model. The results confirmed the suitability of the analytical model to determine the location of pumping well. Additionally, it is found that any increase in pumping time, pumping rate or initial contaminant concentration increases the distance between river and well.**

***Index Terms*— Pumping well, Environmental pollution, Analytical modelling, Green's function, Wastewater, Landfill leachate**

I. INTRODUCTION

THE surface water pollution is a significant public health issue that currently affects various countries.

As one of those countries, Malaysia depends on surface water as the main source of water supply, which makes the country prone to experience river pollution problem. River water can be polluted by multiple sources such as wastewater, landfill leachates, agricultural fertilizers, and transportation. This results in different kinds of dangerous diseases that can be fatal for individuals. For example, usage of synthetic fertilizers in agriculture can cause contamination of surface water by nitrates compounds that are dangerous for infants, toddlers, elderly, pregnant and nursing women. Moreover, the high cost of conventional treatment of river water forced tropical countries to use groundwater as a supplementary source of water supply.

Extraction of groundwater from a pumping well near a polluted river stream may end up capturing polluted water from the said river. Contaminants that migrate from the polluted river or the second source towards the well are naturally removed by bacteria. However, the success of this process depends on several factors such as pumping process (rate and time), microbial activity, clogging layer under the riverbed, site hydrogeology, well type and location, and degree of contamination of river water. In complex cases, numerical models and solutions are recommended. For example, Timpitak and Pochai [1] modeled one-dimensional groundwater pollution through heterogeneous soil. They used Saulyev finite difference and forward time center space techniques to approximate the solutions.

Generally, the process of extracting groundwater via drilling a pumping well next to a polluted river needs a variety of pervious critical decisions such as specifying the well location. The fundamental issue in selecting the well location is obtaining high quality underground water. At the same time, the position needs to be as near to the river as possible to achieve high percentage of infiltrated river water. Unfortunately, in real cases, the river is not the only source of contaminants. Usually, other contaminations reach the groundwater from the opposite side which could originated from agriculture or housing area, landfill site or even another polluted river. In this case, the well is located between two parallel contaminant sources which makes it more difficult to decide where the well should be drilled.

Several modeling efforts have been conducted in the literature to simulate the contaminants transport in aquifer [2-8]. However, most of these models did not consider two parallel sources of contaminants and did not focus on calculating the shortest distance between rivers and well that produces high quality water. In fact, the researchers assumed that the wells are already drilled and tend to alter other factors, such as pumping rate, to control the quality of water obtained. Nevertheless, it is more effective to simulate the suitable location of the well before drilling, as the pumping rates, time period and site hydrology should be taken into consideration. It is crucial to ensure that the well is able to produce high quality water for long time period with the highest percentage amount of infiltrated river water. Reference [9] specified the suitable location and pumping rate of the well based on the percentage of river water in the well and whether the aquifer is fully penetrated by the stream, without taking into account the level of contaminant concentration in the well.

One of the classical approaches that can solve such problem is the image well theory and graphical method described by [10]. The method is based on complex analysis

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Shaymaa Mustafa is a senior lecturer at the Mathematical Department, Faculty of Science, Universiti Teknologi Malaysia, Johor, Malaysia (e-mail: mdshaymaa@utm.my)

Mohamad Darwish is a senior lecturer at Faculty of Civil Engineering, Universiti Teknologi Malaysia, Johor, Malaysia (corresponding author; e-mail: sjmohamad@utm.my).

and was applied in various groundwater modelling studies such as [11-13]. However, these previous studies focused on the capture zone delineation, based on pumping rates, well location and groundwater flow. Mustafa et al. [9] used Green's function approach to determine the well location based on two factors; rate of bank filtration share and level of contamination of the pumped water. Another method to solve this problem is through MODFLOW software by using trial-and-error approach. To do this, initially, the well is positioned in a specified location in the model and a simulation is run to calculate the contaminant concentration at this location. If the concentration results are acceptable, then the location is suitable for the well. Otherwise, the location should be changed and the simulation is repeated, which is not a practical method. In this article, the location of pumping well between two parallel sources of contaminants (one of them is a polluted river) will be determined analytically with the aim to obtain high quality water and to high percentage of infiltrated river water for a long time period.

II. MATHEMATICAL FORMULATION

A. Assumptions and Description

In this model, suitable location of the pumping well between a polluted river and a parallel contaminant source on the opposite side can be adjusted. The model assumed that the pumping well has a constant pumping rate and the aquifer is isotropic, homogeneous, confined, has finite width *d* and initially free of contamination. The river is assumed as a line source of contaminant and positioned at the origin (Fig. 1). The total distance between the two parallel sources equals to L_T. It is supposed that the second source of contaminant has an infinite dimension along *y* axis, otherwise the well can be drilled far from the second source to ignore its effect.

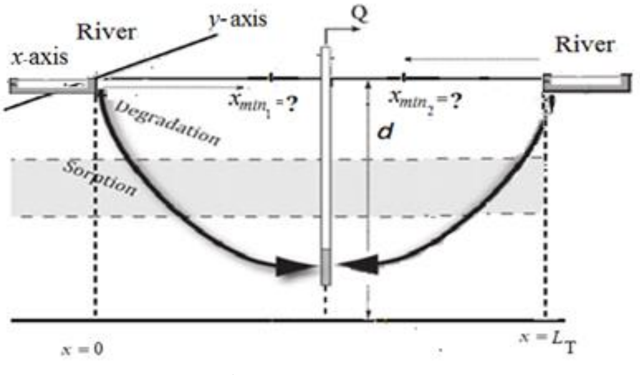

Fig. 1. General geometry of the problem.

The governing equations were derived by assuming only one source of contaminant. Firstly, the model is applied to calculate the distance between the river and the well, which is denoted by $x_{\min_1}$. Secondly, the model is repeated to determine the distance between well and the second contaminant source, which is denoted by $x_{\min_2}$. At these distances, it is supposed that the contaminant reaches its maximum allowable concentration (C_s). Thus, if the well is located within the interval $(x_{\min_1}, x_{\min_2})$, it is expected that the quality of water produced is acceptable. On the other hand, if the well location *x* is outside of this range as follows:

$$x \leq x_{\min_1} \quad \text{or} \quad x \geq x_{\min_2} \Rightarrow C \geq C_s$$

then the level of pollutions in water produced is not acceptable. If the $(x_{\min_1}, x_{\min_2})$ =0, or if

$$L_T > x_{\min_1} + x_{\min_2},$$

then it is not safe to drill the well. In this case, either the pumping rate or pumping time period should be lowered.

B. Calculating the Distance Between Well and Contaminant Sources

In this article, an inverse model for the contaminant transport equation from river to well is developed. The contaminant transport equation was proposed by Mustafa et al. [10]:

$$C(x,t) = \frac{1}{\alpha\sqrt{\pi}R}\frac{q}{Q}C_0 \exp(-\frac{\beta t_1}{R})$$

$$(2\sqrt{\frac{U_x t_2}{d}}\exp(-\frac{R}{4D_x t_2}(x-\frac{U_x t_1}{R})^2)-$$

$$\sqrt{\pi}\sqrt{\frac{U_x R}{dD_x}}(x-\frac{U_x t_1}{R})erfc(\frac{\sqrt{R}}{2\sqrt{D_x t_2}}(x-\frac{U_x t_1}{R}))), \quad (1)$$

$$C(x,t) = 0 \quad \text{for} \quad x \to \infty \quad \text{and} \quad t \geq 0$$

$$C(x,t) = S_o f(t) \quad \text{for} \quad x = 0 \quad \text{and} \quad t \geq 0$$

$$C(x,t) = 0 \quad \text{for} \quad x \geq 0 \quad \text{and} \quad t = 0,$$

where C is the concentration of contaminant (M/L^3); C_0 is the initial concentration of contaminant at the river (M/L^3), β is the decay rate of contaminants, q is the stream depletion flow rate (L3/T), d is the aquifer distance, t_1 and t_2 are the travelling time of contaminants from the contaminant area towards the pumping well and the pumping time period, respectively. R is the linear retardation factor and U_x is the Darcy velocity (L/T) which can be calculated as follows [16]:

$$U_x = \frac{3Q}{2\pi\phi d x_{\min_1}}, \quad (2)$$

where Q is the pumping rate and ϕ is the porosity. D_x is the diffusivity of mass transport (L2/T) and its equation is [17]:

$$D_x = a\ U_x\ , \quad (3)$$

where a is the dispersivity [L]. Consequently, from Equation (2), the value of D_x becomes:

$$D_x = a\frac{3Q}{2\pi\phi d x_{\min_1}}, \quad (4)$$

Travelling time of contaminants to reach the well can be calculated based on pumping rate and distance as follows:

$$t_1 = \frac{x_{\min_1}}{U_x} = \frac{2\pi\phi d\,(x_{\min_1})^2}{3Q} \quad (5)$$

Since the aquifer is assumed to be fully penetrated by the stream, the percentage of the infiltrated river water $\frac{q}{Q}$ in the pumping well can be measured as follows [11]:

$$\frac{q}{Q} = erfc(\sqrt{\frac{S_x(x_{\min_1})^2}{4Tt}}), \qquad (6)$$

where T is the transmissivity (L^2/T) and S_x is the storage coefficient. By substituting Equations (3, 4, 5 and 6) in Equation (1) and replacing x by $x_{\min_1}$, we get:

$$\begin{aligned} C(x_{\min_1},t) = {} & \frac{1}{\alpha\sqrt{\pi}R} erfc(\sqrt{\frac{S_x(x_{\min_1})^2}{4Tt}})C_0 \\ & \exp(-\frac{2\beta\pi\phi d(x_{\min_1})^2}{3QR})(2\sqrt{\frac{3t_2Q}{2\pi\phi d^2(x_{\min_1})^2}} \\ & \exp(-\frac{\pi\phi d\, x_{\min_1} R}{6at_2Q}(x_{\min_1} - \frac{x_{\min_1}}{R})^2) - \\ & \sqrt{\pi}\sqrt{\frac{R}{ad}}(x_{\min_1} - \frac{x_{\min_1}}{R}) \qquad (7) \\ & erfc(\frac{\sqrt{R}}{2\sqrt{\frac{3aQ}{2\pi\phi d\, x_{\min_1}}t_2}}(x_{\min_1} - \frac{x_{\min_1}}{R}))), \end{aligned}$$

Since it is required to calculate the position where the contaminant decreases until it reaches the value of C_s, $C(x_{\min_1},t) = C_s$ is substituted in Equation (7). Thus, Equation (7) is in one variable ($x_{\min_1}$), and the solution can be approximated to the least integer greater than or equal to the value of $x_{\min_1}$. Since the contaminant concentration decreases along the path from river to well, so for any distance x after $x_{\min_1}$ we get $C(x,t) \le C_s$. The same calculation is repeated for $x_{\min_2}$ by assuming the other contaminant source.

III. RESULTS AND DISCUSSIONS

A. Comparison with the Model Developed by [19]

In order to validate the proposed model, researcher needs to check the location of the two existing pumping wells in a river bank filtration site in Langat Basin, Selangor, Malaysia [12]. The first well (W1) and second well (W2) are located 40 m and 18 m away from the stream, respectively. The proposed model is used to compute the locations of the wells from river edge, in which the results obtained is compared with the real locations of the two wells on site. During simulation, a pumping rate of 3,075 m^3/d was implemented with two pumping time periods: 9.7 days and 4.6 days. The initial contaminant concentration considered during the simulation was 16 mg/L. Fig. 2 shows the results of estimated concentration obtained using the proposed model with different expected concentrations around the well.

The analytical result for W1 was 22 m when the concentration around the well was expected to be less than 0.5 mg/l. To reduce the concentration to less than 0.1mg/l, the distance needs to be set at 32 m. These results confirmed that the real location of W1 well at 40 m from the river is acceptable. However, according to the analytical results, in order to get more proportion of river water from the well, the well can be located 30 m away and the water quality is still acceptable.

For W2, the analytical result was 15 m away from the stream for expected concentration of 0.5 mg/l. These numbers agree with the real location of the well at 18 m away from stream.

However, based on the proposed analytical results, to get more quality water pumped with concentration less than 0.1 mg/l, the well should be drilled 25 m away from the stream.

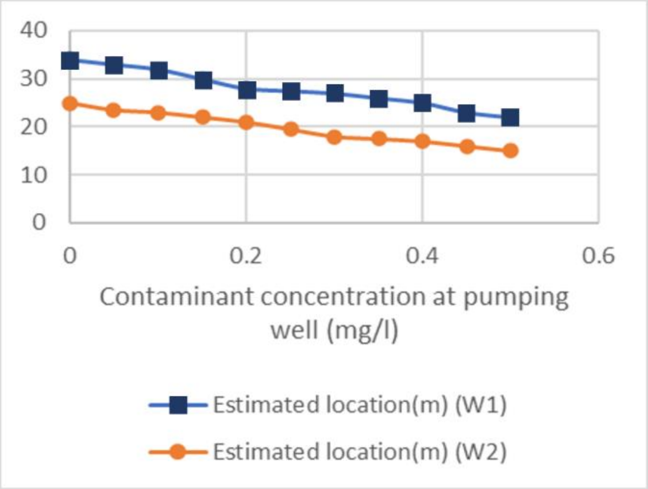

Fig. 2. Locations of W1 and W2 wells with different values of contamination at pumping well.

B. Comparison with the Model Developed by reference [9]

Reference [9] determined the location of well from the shore based on the percentage of river water ($\frac{q}{Q}$) required for the well. The model is performed for 7 days pumping period and 3,075 m^3/d pumping rate. Additionally, two values of $\frac{q}{Q}$ were considered: 33% and 38%. Also, in this comparison, the initial contaminant concentration considered during the simulation was 16 mg/L, while the concentration around the well was expected to be within the range from 0.3 to 0 mg/L. The results are summarized in Fig. 3.

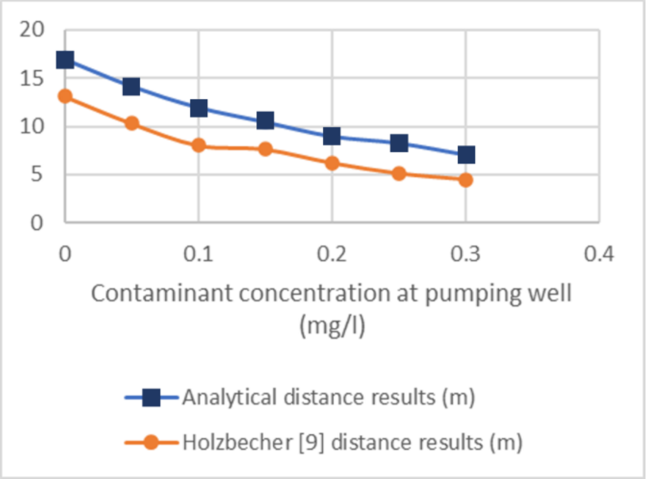

Fig. 3. Comparison between analytical results and results obtained by using Holzbecher [13] model.

From Fig. 3, it is noticed that the proposed analytical results are higher than [9] results by 2 to 4 meter. For example, when the concentration at the well is 0.3 mg/l, the distances are 5 m using [9] model and around 7 m using the analytical model. Next, for concentration of 0.1 mg/l, the distances are approximately 8 m for [9] and around 12 m for the analytical model. Unlike Holzbecher [13], the location is not only based on the participation of river water in the well, but also based on the contamination level in the pumped water.

C. Simulation of the Effect of Pumping Rates

In this section, the effects of pumping rates on the well location is simulated for the case of two polluted rivers. A numerical simulation is performed using MODFLOW to measure the concentration values at the well locations obtained from the proposed analytical model. The values of all input parameters for the numerical simulation are reported in Table 1. The values of $x_{\min_1}$ and $x_{\min_2}$ are calculated using Equation (7) by considering that the concentration of pollutants around the well is expected to be less than 0.5 mg/L. The initial values of concentration at the river and the second source are represented by C_{0_1} and C_{0_2}, respectively.

TABLE I

INPUT PARAMETERS USED IN THE MODFLOW NUMERICAL SIMULATION

Parameter	**Description**
ϕ = 0.25	Porosity
a = 10 (m)	Dispersivity along *x* axis (m)
d =20	Saturated thickness of the aquifer (m)
S_x =0.0004	Storage coefficient
T = 1000	Transmissivity (m^2/d)
β =1.2E-3	Degradation factor (1/d)
R=1	Linear retardation factor
Q =3075	Pumping rate (m^3/day)

Fig. 4 represents the relationship between the pumping rates and total distance between the two polluted rivers (L_T). The value of L_T was calculated by using the following equation:

$$L_T = x_{\min_1} + x_{\min_2} + E\,; \tag{8}$$

where E is the length of the interval ($x_{\min_1}$, $x_{\min_2}$). The value of E was set at 5 m. It was found that L_T value increased by increasing the pumping rates. For example, when the pumping rate Q = 1,500 m^3/d, the L_T equals approximately 270 m, whereas when Q is doubled to 3,000 m^3/d, the L_T value is around 350 m. Using a pumping rate of 4,000 m^3/d leads to an increase of the total distance between the stream and the second contaminant to around 400 m. This means that if it is expected that demand for the well is high, it should be drilled in the wide area between the two rivers that is larger than L_T value to make sure that there is enough space to get high quality water.

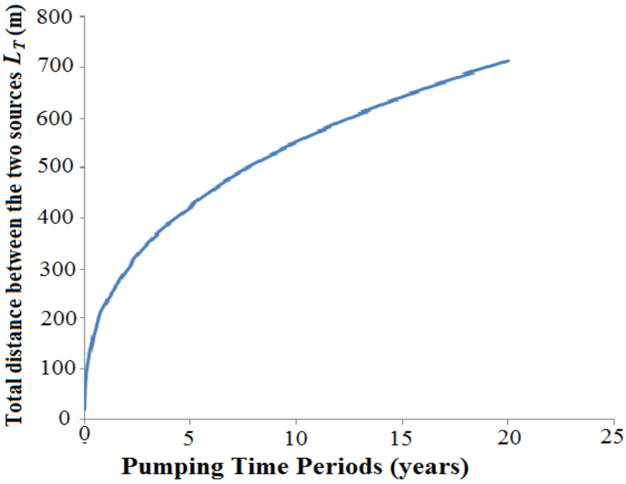

Fig. 4. Relation between pumping rates and total distance between the two polluted rivers.

Table 2 shows the numerical results for the concentration values at pumping wells (C_w) for three different pumping rates. The table demonstrates that the concentration values for the pumped water are very small, which does not exceed 0.0894 mg\L in all observed cases. These numerical results validate the position of the well that was specified based on the analytical results for $x_{\min_1}$ and $x_{\min_2}$. Generally, increasing the pumping rates raises the values of $x_{\min_1}$ and $x_{\min_2}$.

Fig. 5 shows the values of $x_{\min_1}$ and $x_{\min_2}$ at different pumping rates. Similar to the previous case, $x_{\min_1}$ is calculated using C_{0_1} =16 mg/L at the first polluted river, while $x_{\min_2}$ is calculated using C_{0_2} =50 mg/L at the second polluted river. Generally, increasing the pumping rate may increase the distance between pumping well and the second river. Additionally, there is a slight increase in the difference between $x_{\min_1}$ and $x_{\min_2}$ when the pumping rates increase.

TABLE II

VALUES OF $x_{\min_1}$, $x_{\min_2}$, C_{0_1}, C_{0_2}, AND L_T AT DIFFERENT PUMPING RATES

Q	C_{0_1}	C_{0_2}	$x_{\min_1}$	$x_{\min_2}$	L_T	C_w
1500	16	16	120	120	250	0.00132
	50	16	144	120	300	0.00894
	50	50	144	144	350	0.0011
3072	16	16	158	158	350	0.00129
	50	16	188	158	400	0.0012279
	50	50	188	188	450	0.00145
4072	16	16	177	177	450	0.00114
	50	16	208	177	400	00.002233
	50	50	208	208	450	0.00438

Using Q= 1,000 m^3/d, the difference between $x_{\min_1}$ and $x_{\min_2}$ is around 30 m, and this value decreases to 26 m for Q= 2,000 m^3/d. At Q= 4,072 m^3/d, this difference increases to 31 m. This shows that, although the initial concentration at the second river is 3 times compared to the first river and the pumping rate is enlarged from 1,000 m^3/d to 4,072 m^3/d, the increase in the difference between $x_{\min_1}$ and $x_{\min_2}$ is minute.

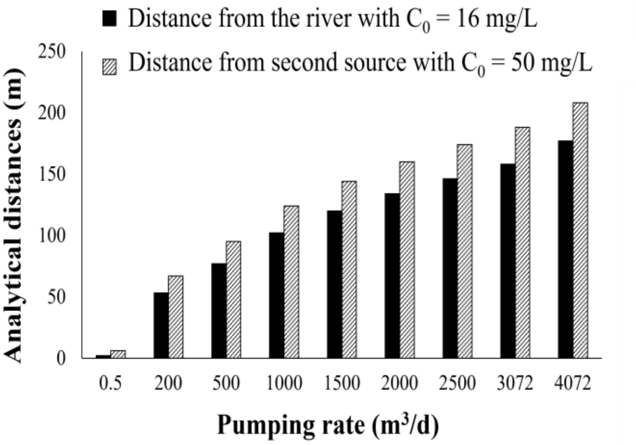

Fig. 5. Analytical distance values between the pumping well and the two polluted rivers at different pumping rates.

D. Simulation of the Effect of Initial Contaminants Concentration

To investigate the relation between contaminant concentration in the river and pumping well location, analytical calculations are conducted continuously for 3 years with different values of initial concentration C_{0_1} (Fig. 6). The same pumping rate (3,075 m^3/d) is used and the values of contaminant concentrations at the second contaminant source are chosen as follow: 0 mg/L, 1 mg/L, 16 mg/L and 50 mg/L. The value of L_T is calculated using Equation (8), and the value of E is set at 5 m. Generally, there is a proportional relationship between river pollution and total distance (L_T) between the river and pumping well area. At 16 mg/L, at second polluted river, the L_T value increases from 160 m to nearly 340 m when the concentration at the river (C_{0_1}) is increased from 0 to 100 mg/L. By applying 50 mg/L at the second source, L_T value increases from 200 m to around 360 m when the C_{0_1} is increased from 0 to 100 mg/L. The same results are obtained when the concentration at the second river is 1 mg/L, while L_T values increases from 60 m to 220 m. Supposed that the second river does not exist, L_T is enlarged up to 150 m. Additionally, at all values assumed for the second river, significant increases are found in L_T values due to the increase of the concentrations at the stream from 0 mg/L to 20 mg/L. The increase of L_T when C_{0_1} > 20 mg/L is negligible and can be attributed to the high consumption of the concentration in the first few meters from the source.

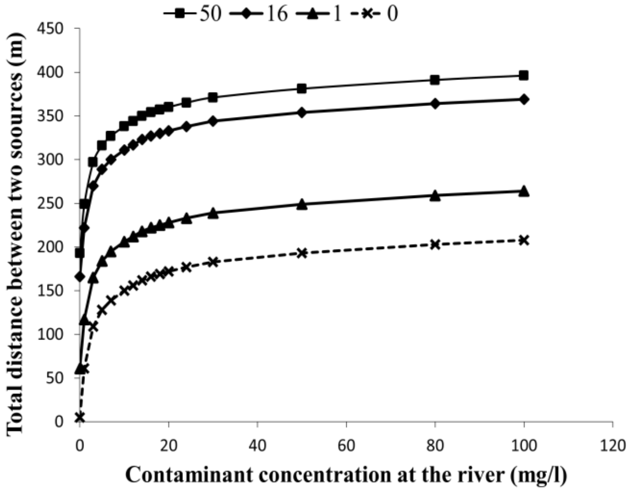

Fig. 6. Relation between initial contaminant concentrations at the river and total distance between the two polluted rivers.

Table 3 shows the numerical concentration results at the pumping well computed by MODFLOW using initial concentrations of 16 mg\L and 50 mg\L for 3 years pumping period.

TABLE I

VALUES OF $x_{\min_1}$, $x_{\min_2}$, C_{0_1}, C_{0_2}, L_1, L_2, L_T AND C_w FOR 3 YEARS AND 10 YEARS PUMPING TIME PERIODS

t₂	C_{0_1}	C_{0_2}	$x_{\min_1}$ (m)	$x_{\min_2}$ (m)	L_1 (m)	L_2 (m)	L_T	C_w (mg/l)
3	16	16	158	158	175	175	350	0.00129
	50	16	188	158	225	175	400	0.0012279
	50	50	188	188	225	225	450	0.00145

Based on above table, the concentration ranges from 0.0012 mg\L to 0.0014 mg\L only, which means that the well at this location produces high quality water. Additionally, the increase of $x_{\min_1}$ and $x_{\min_2}$ values due to change of C_{0_1} and C_{0_2} is around 30 m to 40 m. Thus, higher concentration of the river leads to larger distance between its edge and the well.

Fig. 7 shows the L_T values at different contaminant concentration for both polluted rivers. The simulation is performed for 3 years pumping period with pumping rate of

3,075 m^3/d. Values of contaminant concentrations at the second river are chosen as follows: 0 mg/L, 1 mg/L, 16 mg/L and 50 mg/L, while the concentrations at the stream are 1 mg/L, 16 mg/L, 30 mg/L, 50mg/L, 80 mg/L and 100 mg/L. At all values of contaminant concentrations for the river, an increase of L_T value by 200 m is noticed when the concentration of the second river is increased from 0 mg/L to 50 mg/L. For instance, when C_{0_1} = 16 mg/L in the stream, L_T value increases from 150 m to approximately 350 m when C_{0_2} ranges from 0 mg/L to 50 mg/L, which means that the increment is 20 m. The same increment value is noticed for concentrations of 1 mg/L, 30 mg/L, 50 mg/L, 80 mg/L and 100 mg/L in the stream. Moreover, when the concentration at second river (C_{0_2}) is increased from 0 mg/L to 1 mg/L, L_T value increased by nearly 60 m at all values of C_{0_1}. However, when C_{0_2} is increased from 1 mg/L to 16 mg/L, L_T value is raised by nearly 100 m at all values of C_{0_1}. Similarly, when C_{0_2} is increased from 16 mg/L to 50 mg/L, L_T value is raised by nearly 30 m at all values of C_{0_1}. In general, the high pollution in any of the two rivers results in enlargement of the total distance between the two sources.

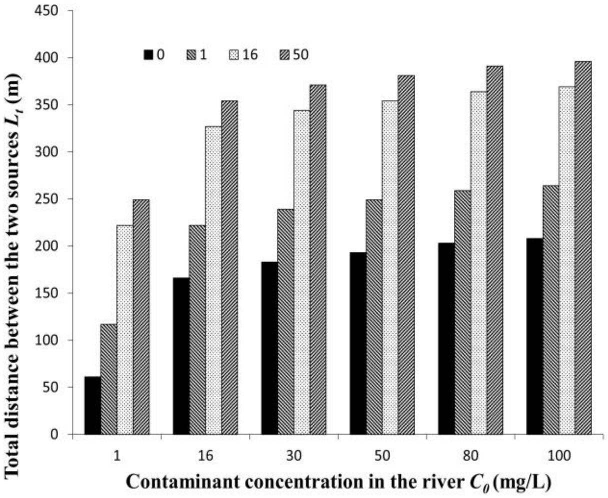

Fig. 7. Total distance between the two rivers at different values of initial contaminant concentration.

E. Simulation of the Effect of Pumping Time Periods

Fig. 8 shows a proportional relationship between the total distance between the two rivers (L_T) and the pumping time periods. For example, taking L_T equals 400 m at 5 years pumping time, this value changes to around 500 m in 10 years pumping period. The total distance between the stream and the second contaminant source continues to extend by increasing the pumping time period until it reaches 700 m in 20 years.

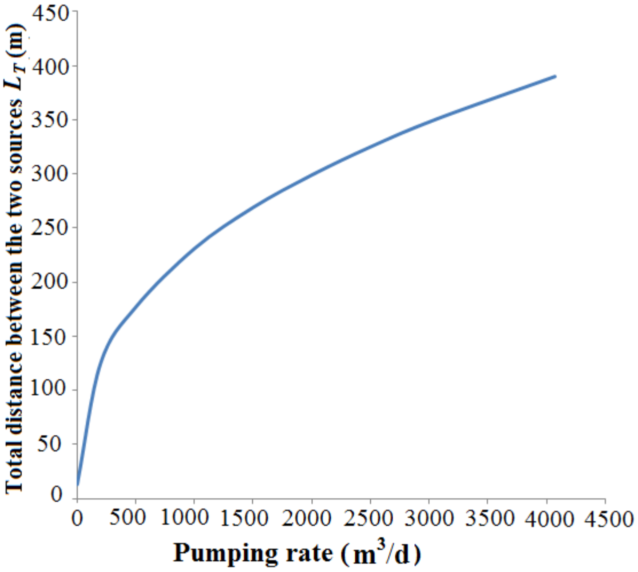

Fig. 8. Relation between pumping time period and total distance between the two rivers.

Fig. 9 shows the values of $x_{\min_1}$ and $x_{\min_2}$ at different pumping time periods. $x_{\min_1}$ is calculated by applying C_{0_1} =16 mg/L at the river, while $x_{\min_2}$ is calculated using C_{0_2} =50 mg/L at the second river. Practically, increasing the pumping time period may widen the distance between pumping well and contaminant source. In addition, the difference between $x_{\min_1}$ and $x_{\min_2}$ becomes slightly higher when the pumping time period is increased. In particular, after 3 years of pumping period, the difference between $x_{\min_1}$ and $x_{\min_2}$ can be around 30 m. However, by duplicating the time period to 6 years, this difference becomes around 36 m. After 20 years of pumping, this value reaches 50 m. This shows that, despite the initial concentration of second river was 3 times more than the first one, the difference between the $x_{\min_1}$ and $x_{\min_2}$ is not so high.

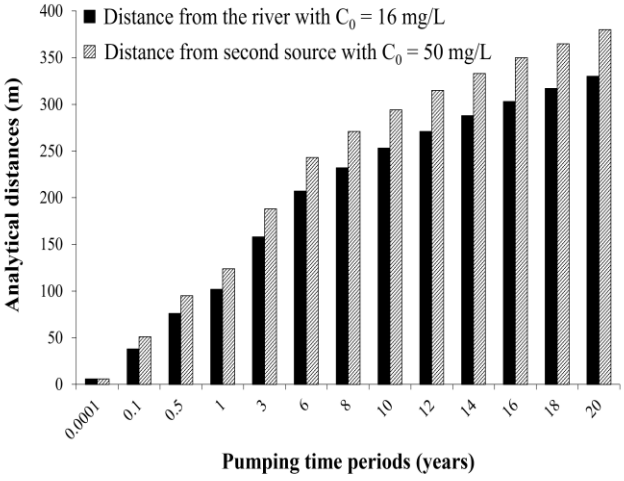

Fig. 9. Analytical distance between the pumping well and the two rivers at different pumping time periods.

IV. CONCLUSION

This study developed analytical model for locating a pumping well between two parallel sources of contaminants. The results are tested by MODFLOW numerical simulation for different pumping rates and different initial contaminant concentrations for two contaminant sources. The results confirmed the suitability of using the proposed model to determine the location of pumping well between two polluted water sources. It is found that the pumping time periods has more effect on the results compared to the pumping rate or initial contaminant concentration. However, the effects of initial contaminant concentration and pumping rate are still significant and should be taken into consideration. The model developed in this article can be used in planning new pumping wells.

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