Facilitating Undergraduate Students' Productive Disciplinary Engagement Through Model-Eliciting Activities (MEA)

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This study aims to describe a case of the emergence of Productive Disciplinary Engagement (PDE) in MEA and to illustrate how the PDE principles are elaborated through MEA. This qualitative case study involves four students at Undergraduate Mathematics Education Program in Indonesia engaged in MEA. Data sources in this study include video recordings of activities, the written answer to the MEA Task, and interviews. The results of this study indicate the emergence of PDE characterised by intellectual progress, in this case, checking and refining the completion and solution of the MEA Task, which is gradually becoming more sophisticated.

Keywords: engagement, productive disciplinary engagement (PDE), model-eliciting activities (MEA)

INTRODUCTION

The development of HOTS is expected to support the mastery of the four key competencies of the 21st century, namely critical thinking, creativity, communication, and collaboration (Howard, 2018). These skills cannot be forced but can only be learned through engagement with inspiring tasks and discussions (Hannula et al., 2019). Tasks that are *ill-structured*, open, contextual, familiar to students, and applied to group settings will result in better HOTS achievement (Murtafiah, Sa'dijah, Chandra, & Susiswo, 2020). If a teacher intentionally and continuously giving opportunities such as engaging students with real-world problems, class discussions, and inquiry-based experiments, there is a good chance that students will develop higher-order thinking skills (Sa'dijah et al., 2021). Students' engagement has been extensively studied in the past decade (Salmela-Aro et al., 2021). Studies related to engagement in learning include conceptualisation and measurement of engagement (Cheng et al., 2019; Sträßer, 2017; Wang, Fredricks, Ye, Hofkens, & Linn, 2016), engagement research on a large scale (Evans et al., 2020), engagement in a certain context (Tekkumru-Kisa & Akcil-Okan, 2020), the relationship between engagement with the other variable (Schnitzler, Holzberger, & Seidel, 2020), interventions to increase engagement (Reeve, Cheon, & Jang, 2019), engagement in technology-mediated learning (Attard & Holmes, 2020).

Facts in the field point out that engaging students is a challenging task. Well-designed learning does not ensure all students are engaged (Agarwal & Sengupta-Irving, 2018). An active engagement in learning also does not guarantee productivity. This is based on the results of preliminary observations by the researcher, the teacher of one of the classes they teach. Students can actively discuss the group in collaboratively designed learning in small groups. However, the results obtained by the group have not shown any significant intellectual progress. In designing learning in a certain field and considering engagement, we must also pay attention to productive engagement. Related to this, Engle and Conant (2002) proposed the term Productive Disciplinary Engagement (PDE) and explained how learning arrangements could foster this type of engagement.

For PDE, there needs to be evidence of learner engagement in classroom activities (Engle & Conant, 2002), which needs to be assessed by reference to cultural practices to demonstrate engagement relevant to a particular learner (Engle, 2011). Secondly, to be disciplined, the engagement of learners must come into contact with some ideas, practices, skills, dispositions, technologies, perspectives, and others that are relevant and valuable (Engle, 2011; Engle & Conant, 2002). Finally, to be productive, the disciplinary engagement of the peas and the learner must show evidence of making valuable intellectual progress over time (Engle, 2011; Engle & Conant, 2002). Thus, defining PDE operationally in the case of a particular class requires an understanding of the student's background knowledge and interests as well as the teacher's instructional goals (Engle, 2011).

There has been a lot of research on PDEs and learning designs that can facilitate them. Venturini & Amade-Escot (2014) examines teacher practice in ordinary physics learning, focusing on how teachers manage didactic interactions, characteristics that may be related to the principles that drive PDEs. Mortimer & Araújo (2014) shows that traditional teaching approaches fail to promote at least two of the four principles that potentially foster PDEs: the principle of authority and accountability. Meyer (2014) considers what PDE looks like in the case of inquiry-based investigations, in which students participate in answering actual scientific research questions in collaboration with practising and proposing scientists who propose that these students experience PDE at an initial and deeper level, where the student's initial PDE in scientific activities serves as a resource for PDE at the disciplinary level which is more specific. Three groups of students as they worked on a collection of fluid mechanics problems and identified examples and origins of productive disciplinary engagement, which showed that students disagreed and debated how to solve the problem, and they engaged in productive disciplinary discussions.

Gonida & Lemos (2019) provide three case illustrations. Through a richly designed learning environment, students can be encouraged to engage productively in the disciplinary practices necessary for future work-life skills. However, the extent to which students' experiences with PDE with peers leave a cognitive imprint and motivation for future productive engagement is unknown. Agarwal & Sengupta-Irving (2019) developed a CPDE framework by integrating power to create a balanced group PDE. Dasgupta, (2019) provides an approach to designing a learning environment to productively engage K-12 students in engineering design practices using suboptimal models. Finally, Koretsky, Vauras, Jones, Iiskala, & Volet (2021) explores how PDE is associated with levels of cognitive activity and collective group outcomes in collaborative learning in various contexts. However, more recent research (Koretsky et al., 2021) reveals that it is still necessary to research to determine the relationship between the nature of the problem and how collaborative groups approach PDE.

Engle and Conant (2002) outline four principles for encouraging PDEs that include introducing open problems to be solved (which may have multiple solution paths as well as multiple solutions); joint authority among teachers and students; accountability to each other and discipline; and sufficient material and symbolic resources (including sufficient time). Swenson & Wendell (2017) proposes to apply the Model Eliciting Activities (MEA) because the MEA is a client-driven real-world problem that requires students to deepen their conceptual knowledge and apply it to create a generalisable mathematical model. These more complex issues provide more opportunities to give rise to PDE (Swenson & Wendell, 2017).

MEA is a complex mathematical problem that is open, non-routine, has a real-world context, and is carried out in groups (Wessels, 2014). Therefore, learning using MEA is based on situations that occur in students' real lives to stimulate student motivation and engagement in the learning process (Gilat & Amit, 2012). Wessels (2014) also states that the use of MEA in the process of learning mathematics in schools can develop student creativity, starting with expanding students' mathematical knowledge and building a better understanding of how to build the process of increasing the creativity of students.

Engaging students with modelling activities help them learn mathematics meaningfully. Through engagement with the MEA, students get the opportunity to acquire conceptual tools that serve as mathematical models. In doing so, they engage with several cycles, including translating, deciphering, predicting data, and getting repeatable results in the solution path. This process of iterative cycles is called mathematising the situation (Lesh & Doer, 2003). In particular, the MEA helps achieve four objectives (Chamberlin & Coxbill, 2014), namely (1) helping to analyse how students think mathematically, (2) helping to equip students with a variety of abilities in doing mathematics due to the various entry points they have, (3) they help identify the broader abilities of students, and (4) they help identify and facilitate the development of creatively gifted students.

Students focus on several processes while doing the MEA. Unlike simple problems, the MEA cannot be solved with one or two simple calculations. The processes required and models created to solve the MEA are much more profound than simple algorithmic solutions and thus encourage creative thinking because they are not routine. Therefore, a high level of thinking (Wieczerkowski, 2000) is required to complete each MEA. Students must also engage in metacognition to solve the MEA (Lesh, et al., 2000).

Theoretically, creating a math class full of engagement with such activity is possible. However, there has never been a study that explicitly reveals student engagement in the MEA. Thus, it is necessary to explore more deeply the engagement in the MEA, especially, productive engagement. Thus, the purpose of writing this article is to describe how the principles of PDE are elaborated through the MEA and to show the case of emergence of PDE in the MEA implementation.

METHOD

Participants and Context of This Study

This study was carried out at a private university located in a small town in Indonesia. The participants of this study were 16 students of the Mathematics Education Undergraduate Study Program who were taking the Integral Calculus course. One of the researchers (first author) plays a role as a teacher in this class. Integral Calculus course is usually taught traditionally by the lecturer giving explanations concerning certain mathematics concepts and examples of the problems in the form of routine questions. Next, the students were given assignments in the form of practice questions related to the concept. In this research, the applied learning was using MEA. MEA was applied for the first time in the class to introduce the concept of integral as Antiderivatives through graphs. In this case, mathematical modelling is not used to apply the concepts learned but as a tool to build conceptual understanding (Shahbari, Daher, & Rasslan, 2014). The implementation of MEA is carried out by dividing students into heterogenous and voluntary groups consisting of 4 students each.

The MEA problem used in this research is called the MEA Task. Every MEA task consisted of four sections. The first two sections addressed the context of the problem, and the last two sections presented the problem. The first part of MEA is a reading passage. This section contains newspaper articles written to arouse student's interest and stimulate discussion related to the context of the problem. In this instrument, the reading article activity was replaced by playing a video from *Youtube*, which contains the climbing trajectory of Mount Budheg to make the activity more enjoyable. This activity was carried out individually before the class started. The second section of MEA is the readiness question. The question was a warm-up to get to the main problem. Readiness question is done in groups outside the class. The students could choose face-to-face discussion, video conferencing, or group chat.

Furthermore, the students were asked to submit the recordings of their discussion. The third section of MEA is data section. The form of this section varies, including diagrams, charts, maps, timetables or performances, sales tables, etc. The fourth section of MEA is problem-solving task. These questions or statements generally consist of one paragraph, and the students were asked to solve a complex mathematical problem for a hypothetical client. The last two sections were unstructured problems for students to solve, showing mathematical modelling (Chamberlin & Moon, 2005). The third and fourth sections were carried out face-to-face in class.

The MEA task developed was adapted from The Tramping Problem promoted by Yoon, Dreyfus, and Thomas (2010) by changing the context to "Budheg Mountain Climbing." As a result, the problem has met the six principles of MEA (Yoon et al., 2010) presented in Table 1.

TABLE 1
PRINCIPLES OF MEA TASK

No	Principles of MEA	Integration within problem
1	Reality principle	The problems were arranged in the context of the hiking trail, starting with the presentation of a video about the phenomenon of the hiking trail on Mount Budheg, Indonesia.
2	Model construction principle	A gradient graph of a hiking trail was presented as information. Next, the students were asked to develop a method to determine the distance-height graph of the actual trajectory.
3	Model generalising principle	The problem also required students to generalise the resulting method so that it could be applied to any gradient graph.
4	Simple prototype principle	Mathematically, the problems equivalent to producing a method to determine the antiderivative of a given gradient graph and the insertion of this mathematical task in the context of the climbing trajectory met the simple prototype principle.
5	Model documentation principle	The problems required students to write the method in the form of a letter to the client to determine whether the trajectory was suitable for their purpose.
6	Self-assessment principle	Students were instructed to implement their method to illustrate the distance-altitude graph, allowing them to test and revise their method. Therefore, it fulfils the self-assessment principle.

Data Collection

The data collection was gathered through videotape recording all students' activities during MEA. In addition, written answers and doodles produced by the students during the MEA implementation were also collected. Finally, retrospective interviews were also conducted to complete the data.

Data Analysis

Studies dealing with the sequence of the learning process point to how learners interact, not only in certain situations but also across situations. In distinguishing the infinite interconnections within the data, we describe an analytical solution that implicates two analysis levels (de Sousa & Rasmussen, 2019). At the first level, the overall learning process is examined, and the students' progress within the temporal boundary of the unit is analysed. Contextual aspects central to engagement were used to characterise the

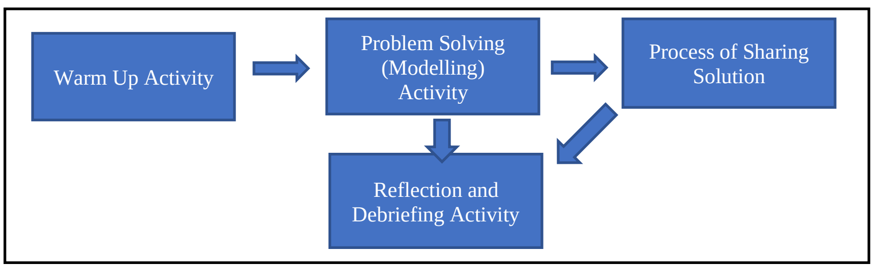
PDE moments (interactional level) and determine the characteristics over time. At the interactional level, the moment-to-moment social development of engagement in connection to MEA implementation is investigated. This level supports us in comprehending how meanings are produced through the teacher and students' interactions with the available resources. The two levels of analysis inform each other and provide insights into how a particular activity or knowledge became relevant at a point in time and how and why it remained relevant throughout an activity. This involved analysing the data to identify the dialogical aspects that characterised disciplinary engagement using microanalytic approaches. Utterances were also sequentially analysed to illustrate how the inter-animation of different voices allowed meaning to emerge and develop as students engaged in meaning-making about the ethical theories presented in the unit.This article's purpose is not to generalise but to provide an overview so that it only focuses on the case of 1 group that gave rise to PDE. The group consisted of 4 people consisting of 1 man with medium to upper abilities and three women (1 high, 2 medium to low). The names used in this article are pseudonyms.

RESULTS AND DISCUSSION

Elaboration of Productive Disciplinary Engagement Principles Through MEA

The implementation of the MEA, in this case, is presented in Figure 1.

FIGURE 1
THE SCHEME OF IMPLEMENTATION OF MEA



Warm-up activities aimed at helping students to be confident with the modelling activity's context and at eventually introducing or testing minimum prerequisites. Warm-up activities focused on boosting students' confidence, especially in the modelling activity and when introducing or testing minimum prerequisites. Problem-solving (modelling) activities, students perform modelling cycles to produce a model that describes the starting situation. During problem-solving activities, students establish a model representing the starting situation while implementing modelling cycles. Sharing solutions are whole-class activities in which students make formal presentations about the results of their work. Each group has to present its work to the rest of the class. After the presentation, each group will be engaged in a questions and answers session, where teachers and peers could ask questions about the presented model. The process of sharing solutions is a whole class activity where each group of students formally present their work results to the rest of the class. After the presentation, teachers and peers shall give questions related to the presented model to each group to engage them in the questions and answers session. In the last session, students work individually in reflection and debriefing activities, thinking back about their experiences during the modelling process. In the last session, reflection and debriefing activities, students were asked to do an individual work and think back their experiences during the modelling process.

The specific feature of the MEA Task in this case that supports PDE are summarised in Table 2.

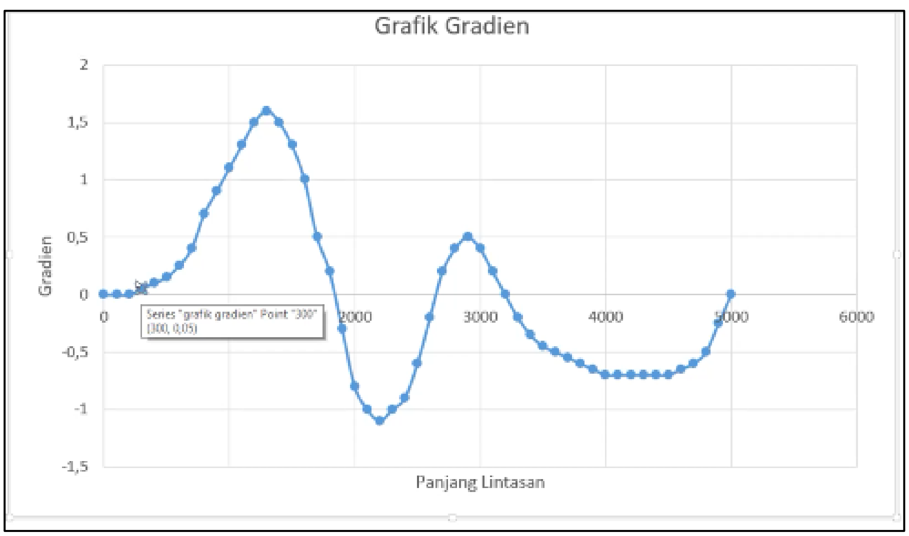
TABLE 2
THE SPECIFIC FEATURE OF THE MEA TASK THAT SUPPORTS PDE

PDE Principles	Feature MEA Task
Problematizing	Real-word problems that require students to model problems related to climbing trajectories if given info about gradient graphs, then create a general method that can be used for other climbing tracks.
Authority	Authority to produce a mathematical model of a problem and generalise its methods.
Accountability	Accountable to other group members when problem-solving tasks. Accountable to the instructor when collecting written reports about the group's progress at the end of each meeting. Accountable to others while sharing solutions and presenting the completion of the group in front of the class. Accountable to discipline by simplifying each step of the work given mathematically.
Resources	Time and place to work on, internet resources, materials from instructors (1 package of MEA TASK worksheets), reference books, laptops, math software, and discussion rooms through group chat.

This excerpt illustrates how four principles of PDE are elaborated by MEA implementation during problem-solving activity in a small group.

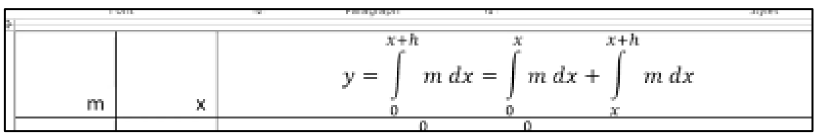
I	:	*That's x_2-x_1. How come it's 400-300?*	*Problematizing*
W	:	*Well that x_2-x_1 taken this interval (pointing to the figure)* *first point, second point, third point, fourth point, the fifth point, sixth point*	*Resources* *Accountability* *Authority*
T	:	*When it reaches 300 it's still 0 heights, it starts to climb that 400, right?*	*Problematizing*
W	:	*Are you sure, right we start from 0, but the interval is not.* *One, 0, 100, 200, 300 well here lo 300 that appears (pointing to the dots on the chart)(Figure 2).*	*Authority,* *Accountability,* *resources*

FIGURE 2
GRAPH GENERATED BY STUDENTS FROM THE TASK



T	:	*That's the important interval, which is 300.. whose gradient of 0.05 is 300 or 400?*	*Problematizing*
W	:	*0.05 300 woi, 100 200 300 this you know*	*Authority, accountability*
T	:	*Then yes 300-200 to, how come it can be 400*	*problematising*
W	:	*We briefly (to calculate) the gradient (needs) 2 points that way of looking for the bottom definitely with the top right*	*Authority, accountability*
T	:	*We are looking for y_2, it is to mean we are looking, means x_2 is 300 not 400, x_2-x_1 is 300-200 to....* *Emm, use that integral method, you know*	*Authority, accountability to math*
T	:	*It's our formula that y_2-y_1 is equal to $m(x_2-x_1)$ that integrally can be written with y equal to integral.. how to read it... integral with an interval from 0 to x + h mdx is equal to an integral of 0 of x m dx plus an integral of x to h mdx, some are confused not here, x + h it is, you know the back one is h only not x + h (Figure 3)*	*Authority Accountability*

FIGURE 3
FIRST POINTED ANSWER BY STUDENT

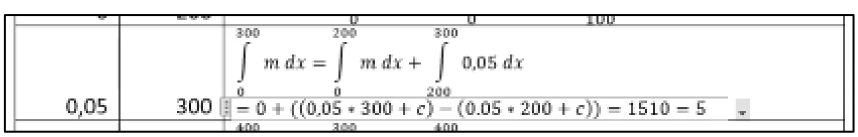


Briefly, the back one is x to h instead of x+ h, eh yeah.....interval x to x + h mdx, the first one if the intervale is like that yes the result explains if the one from 0 to 100 it is an integral interval of 0 to 100 mdx equals the interval 0 mdx oh wait…

The interval is 0 to 0 mdx plus the integral 0 of 100 0 dx equals 0, why is I use m = 0, the interval 0 to 0 uses mdx because the gradient value e fits the interval 0 to 0 different gradients with the gradient fitting in 100,

the current gradient of 0 is 0, but this will be different later because like when the interval is 0 300, how to yes explain it, the integral 0 to 300 mdx is equal to the interval integral 0 to 200 mdx + the integral 200 to 300 0.05 dx. This is because when it is 200, the m is still 0 but if it is 300 gradient e maleh 0.05 then I use mdx instead of 0 dx. How do you explain it, this is what are you confused about (Figure 4)

FIGURE 4
SECOND POINTED ANSWER BY STUDENT



W	:	*Well this is what we see from this graph (showing the graph on the screen) we get this formula, the formula in general so it's like this, (pointing to the picture of the addition integral formula) has something to do with the integral of course.*	*accountability authority resources*
T	:	*Emm.. right when the gradient is 0.05, how to determine the value of y, you have to find the integral interval 0 to 300, well before that we already know that it turns out that the integral when it is 0 to 200 it is 0, just look for an*	*Authority Accountability*

		integral value that is an interval of 200 to 300, which is right 200 to 300 we know that the gradient value is 0.05, so it is operated like this. *Ehm our answer does seem to be in the end the same as it used to be, the difference is that this uses integrals*	
N	*:*	*Go on, That's the same way, right?* *That's the answer that is 0 0, it means that it is still flat, the slope e has not risen like 100 and 200, yes. Understood.*	*Accountability, Resources*

Problematizing content's basic tenet is that instructors should foster their students' inquiries, suggestions, challenges, and other intellectual contributions rather than merely expecting them to memorize facts, procedures, and other "solutions" (Engle & Conant, 2002). Students expressing doubt about technique concerns over cost, tool limitations, or remedies that bring fresh disciplinary issues are a few examples of problematization. On the other hand, uncertainty related to task surface characteristics, such as missing data or mistakes in procedures, is typically not a sign of problematization (Agarwal & Sengupta-Irving, 2019). By presenting controversial problems through MEA, problematizing can be facilitated because the problems they encounter run contrary to their preexisting thought patterns. Students will experience cognitive conflict, so this will encourage students to engage in metacognitive strategies to solve these problems (Walida et al., 2022). Students are encouraged to tackle intellectual challenges that simultaneously: (a) are entirely ambiguous for them as learners; (b) are sensitive to their own interests, aims, or convictions; and (c) express some key tenet of the discipline in question (Forman et al., 2014).

To establish authority, a student must first share their own thoughts on a subject, then be acknowledged publicly as the creator of those ideas, then contribute to the ideas of others, and lastly, gain local authority on subjects where they have made significant and pertinent contributions (Engle & Conant, 2002). Student engagement can be significantly influenced by the kind and degree of student authority in the classroom. Students are first given a more active intellectual role, which may motivate them to participate more. The quality of their work may reflect their identity to the extent that they are also public stakeholders, contributors, and specialists about discipline issues, which may encourage them to engage more effectively than is appropriate (Forman et al., 2014).

To be held accountable, students must defend their positions in light of the beliefs and practices of others (Agarwal & Sengupta-Irving, 2019). When teachers and other members of the learning community hold students accountable "to others and disciplinary norms," they are fostering their responsibility to make sure that their intellectual work is responsive to the content and practices established by intellectual stakeholders both inside and outside of their direct learning environment, as well as any applicable disciplinary norms, to the extent that this responsibility can be realized in the classroom (Engle & Conant, 2002). If accountability is balanced with authority, engagement will become more productive and disciplinary. Accountability entails the duty to justify one's beliefs concerning other intellectual agents. Accountability to one's self, supporting peers, critical peers, and disciplinary authorities in and out of the classroom can all fall under this category (Forman et al., 2014).

The degree of accountability placed on students by others might also encourage effective disciplinary engagement. Students who take into account the opinions of others may be better able to convince other students to discuss their own ideas, which will encourage more engagement. In addition, students have the chance to learn from and have their views taken into consideration by other students in their class when they pay attention to what they have to say. This makes classwork more effective than it should be. Additionally, students may be more likely to contribute to a better standard if they feel like their thoughts will be considered (Engle & Conant, 2002).

Resources include everything that aids in putting PDE and other concepts into practice, including time, location, technology tools, cultural artifacts, institutional support, class norms, scaffolding, and more (Agarwal & Sengupta-Irving, 2019). Examples include tools that present suppressed identities and ideologies as material resources, such as technology-based epistemic scaffolding, relational tools that animate the diversity of identities and histories, and ideational tools, such as a teacher's cursive attitude toward students and their ideas (Agarwal & Sengupta-Irving, 2019).

The PDE framework was initially developed to provide guiding principles for designing and understanding learning environments that encourage deep student engagement and productive engagement in disciplinary work, in contrast to many pedagogical frameworks and models based on specific scripts. With a focus on math and science classrooms, the framework was established in response to the need to bring coherence and coherence with many current design concepts to produce an effective learning environment (Engle, 2011; Engle & Conant, 2002).

Evidence of Productive Disciplinary Engagement in MEA

Undergraduate Students' Engagement

The engagement of students referred to in this study is engagement at the group level. To analyze engagement refers to the multidimensional engagement framework proposed by (Rogat et al., 2022) consisting of behavioral Engagement (BE), metacognitive Engagement (ME), collaborative engagement (CE), dan socioemotional engagement (SE). Behavioural engagement is characterized by commitment, perseverance, investment, and effort, even when facing difficulties. Metacognitive engagement is characterized by coregulation and socially shared regulation focused on content and practice, backed by regulation intended to keep behavior on-task, watch over group dynamics, manage time efficiently, and adhere to task instructions. Collaborative engagement is characterized by coordination and responsiveness. Finally, a pleasant climate characterizes socioemotional engagement and requires the negotiation and upkeep of inclusive and polite interactions, cohesive teamwork, and psychological safety. The summary of the engagement of students in the MEA that appeared, in this case, is presented in Table 3.

TABLE 3
UNDERGRADUATE STUDENTS' ENGAGEMENT IN MEA

Engagement	Description
Behavioural Engagement (BE)	- Doing a conversation related to the solution of the MEA task - Creating a table and drawing graphs manually - Using software to draw graphs - Taking notes of the results of the answers - When facing a problem, the students try to find other alternative solutions to the problem, for example, looking for additional references from the internet
Metacognitive Engagement (ME)	- Discuss the goal or things that will be done together - Discuss the solved problems related to certain mathematical content - Discuss how to divide the task - Check problem-solving progress, what the results are, and what is still missing - Conditioning the groups to always be on-task, for example, by saying hello, or calling the names of the less active members - Checking whether the resulting solution has used the correct mathematical concepts, whether the strategies and reasons used are reasonable, whether all the things to be collected have been fulfilled - Checking whether all group members understand

Engagement	Description
Collaborative Engagement (CE)	- Contribute to each other within the groups according to their respective roles - Ask members' opinions or approval for the proposed solving strategy - Give feedback if one proposes an idea - Ask questions that aim to get other friends re-check the proposed idea. - Discuss together if there is a different argument. - Explain to other members who do not understand.
Socioemotional Engagement (SE)	- Respect each other's opinion during discussion - Sometimes using vernacular language during the discussion creates a comfortable and familiar atmosphere - When one of the group members felt down and thought they could not do it, the rest of the members encouraged them - When one of the group members making a mistake in making a solution strategy, the rest accept it - When there is a pretty tense debate, the rest of the group members try to slowdown - When they find out that the result is not following the purpose of the question, they return to persevere and try to find other alternatives - Laughs together - Gives praise

The level of a group's participation and commitment to specified tasks, as well as how often they deviate from them, is known as behavioral engagement (Fredricks, et al., 2004). Long-term group engagement offers the chance to learn from other's perspectives, while brief off-task conversations can rekindle friendly interactions when the group returns to work (Barron, 2000; Langer-Osuna et al., 2020). The group's interpersonal interaction quality and climate are determined by socioemotional engagement. A positive environment is characterized by the negotiation and upkeep of inclusive and respectful interactions, team cohesion, and psychological safety (Rogat & Linnenbrink-Garcia, 2011; Rogat & Adams-Wiggins, 2015). According to studies on how people learn in collaborative groups, friendly, encouraging, and risk-taking relationships improve the quality of group task completion (Barron, 2000; Kreijns et al., 2002). When constructing knowledge, collaborative engagement considers the task and conceptual coordination of groups and the distribution of contributions among group members. High-quality collaborative engagement supports establishing a shared problem space and constructing joint knowledge that accounts for diverse views (Roschelle & Teasley, 1995).

Regulatory tactics used by groups, such as planning, monitoring, and evaluation, are called metacognitive Engagement (Rogat & Linnenbrink-Garcia, 2011; Järvelä et al., 2016; Schoor et al., 2015). Recent research has shown that effective, shared regulation, which is goal-focused toward understanding and progress on the task, content, and disciplinary practices distinguish high-quality metacognitive engagement (Rogat & Linnenbrink-Garcia, 2013; Molenaar & Chiu, 2014; Khosa & Volet, 2014). Furthermore, this shared regulation is supported by, rather than the sole focus of, regulation of behavior, time, group process, and task completion.

Disciplinary Engagement in the Context of MEA

Disciplinary engagement relates to the content of collaborative talk or physical activity that brings new contributions which aim to make intellectual progress (Rogat et al., 2022). It consists of conceptual and disciplinary activities that are integrated. From the cases presented in this article, disciplinary engagement is seen in several forms.

First, the norm that can be found in the group is the socio-mathematical norm, when they solve math task instances, reason and justify their thinking, enhance each other's concepts, and modify their justification in response to fresh information or criticism. Second, the problem-solving process is related to the cycles and phases of mathematical modeling, including formulation of a possible mathematical approach to solve a real-context problem, the test of the designed solving strategy, interpretation and discussion of the testing results, and revision of the starting approach. Third, it is related to mathematical concepts or terms used, among other conversations about interpreting graph and connecting it to real problems, discussing the relation of graphic shapes with the gradient, using gradient formula, and linking it to the integral concept. Fourth, discipline-related activity and material are referred to as “disciplinary engagement,” with high-quality disciplinary engagement demonstrating links toward integrating conceptual and disciplinary competencies to address lesson difficulties. Low-quality disciplinary engagement exhibits disjointed topic discussion, little elaboration, and a concentration on memory, which may indicate an early understanding of the task or unit or an indication of task or instructional restrictions. Prior studies indicate that high-quality disciplinary engagement increases disciplinary achievement (Hmelo-Silver et al., 2015).

Productive Disciplinary Engagement in MEA

At first, the group approached the problems by directly examining the graphs given in the task, considering the graphs representing the shape of the mountain, and determining peaks, valleys, gently sloping sections, and steep by inferring directly from the graph. However, explanation related to client suggestions still tends to be based on actual context, not mathematically based. After re-checking and realizing that the graph given in the question is a gradient graph, they need to know the distance-height graph next to find out the actual shape of the mountain. Next, through discussion and using resources, the group agreed to draw the distance-height graph using the gradient formula. By taking points in a specific interval and applying them to the gradient formula, taking notes of the resulting points and representing them in a table, and then drawing the distance height manually. Because they found it difficult and the result was not good while drawing manually, the group agreed to use Microsoft Excel to make a table and draw the distance-height graph. Next, they make process-based reasons for the process to suggest to the client.

After evaluating and discovering that even though they have implemented mathematical concepts, it has not been linked to the topic being discussed, which is about integral. The group discusses the relation between the problems with the integral concept and which concept can be used to solve the problems. By utilizing the existing concept, reading references from Calculus books, and asking the lecturer, the group agreed to utilize the concept of definite integral addition. The group recalls the previous knowledge about the concept of gradient as the first derivative of a curve, so if they want to determine the curve, they have to do the inverse, which is antiderivative. Using a similar method to the second method, but by applying integral, the group drew the distance-height graph representing the mountain’s actual shape. Furthermore, the group justifies its argument based on more sophisticated mathematical reasons.

CONCLUSION

Model Eliciting Activities provide opportunities for students to engage in modeling activities where they problematize content by asking questions about the goal of the MEA Task, the strategy for solving it, and how to relate it to a real-world context. The principles of authority and accountability can be reflected when students discuss solving MEA problems. They also use many resources that support engagement in MEA.

This study shows a case of the emergence of PDE in MEA which is characterized by the presence of intellectual progress. In this study the intended progress is at the group level. For the progress of each individual further research needs to be done. For the group that is the focus of this research, there is intellectual progress in terms of mathematical modeling processes. Initially they only used a primitive approach, namely directly referring to the graph, there was no mathematization process. In the next stage of development, the mathematical modeling stage has been carried out. But in terms of the model produced and associated with the mathematical concepts used, there is still a lack of conceptual understanding of antiderivatives. This may be related to students’ views of mathematics. Of the words used, they often mention the word “formula.” In solving MEA they still think like solving ordinary real world math problems. They think they will be able to apply certain formulas to solve problems. Maybe this is related to the cultural history of students' mathematics learning, who usually study mathematics in a traditional way. This needs further study.

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