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Cascading failure analysis of multistate loading dependent systems with application in an overloading piping network

Yixin Zhao [a], Baoping Cai [b], Henry Hooi-Siang Kang [c], Yiliu Liu [a,*]

[a] Department of Mechanical and Industrial Engineering, Norwegian University of Science and Technology, Trondheim, Norway
[b] College of Mechanical and Electronic Engineering, China University of Petroleum, Qingdao, Shandong, China
[c] Institute for Vehicle System & Engineering (IVeSE), Universiti Teknologi Malaysia, Johor Bahru, Malaysia

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ABSTRACT

Many production and safeguard systems consisting of multiple components are susceptible to the cascading failures, where one possibility is that the failure of a component leads to more workloads of other components. Such loading dependence can result in failure propagation, make the systems more vulnerable and maintenance decision-makings more difficult. In this study, we develop a model for analyzing the propagation process of failures in loading dependent systems considering overloading states and degradation of components. The multinomial distribution is applied to characterize the probabilities of total numbers of failed- and overloading components, and probability distributions of different stop scenarios of cascading process are derived. A practical case in piping network is investigated to illustrate the analysis procedure, and to compare the effectiveness of the proposed model with those of the existing methods. Numerical analyses are conducted for evaluating the factors influencing the probability distributions of total number of failed- and overloading components, as well as the occurrence frequencies of different stop scenarios. It is expected that design and maintenance of loading dependent systems can be optimized with the support of this new cascading analysis approach.

Notation

n	Total number of components in a system
j	Cascading generation $j = 0,1, 2, \ldots$
d	Initial disturbance amount
L^{max}	Maximum workload on a component
L^{min}	Minimum workload on a component
l_i	Initial workload on component i
l_f	The additional load from a failed component
l_o	The additional load from an overloading component
l_j	Loading increments from all the failed and overloading component in the jth generation
l_{ij}	Workload on component i in the jth generation
C^{max}	Maximum capacity of a component
C^{min}	Minimum capacity of a component
c_0	Initial capacity of component i
c_d	Capacity decrement of functioning component in every generation
c_j	Capacity of every component in the jth generation
r_{ij}	The workload-capacity ratio of component i in the jth generation
r^*	The overloading threshold for a component
p_f	The probability for a component to fail
p_o	The probability for a component to overload
p_w	The probability for a component to work normally
$\Phi(x)$	The saturation function representing the probability
n_{fj}	Number of failed components in the jth generation
n_{oj}	Number of overloading components in the jth generation
n_{wj}	Number of working components in the jth generation
s_j	The case of how many components are in each state in the jth generation
u	The total number of the failed components
v	The total number of the overloading components
t	Cascading time, and $t = 0$ when the cascading process starts
$R(t)$	The probability that the system is still working until time t
T_j	The duration of cascading process from the start to the jth generation
$F^{(J+1)}(t)$	The probability distribution function that all components fail in generation J at time t

* Corresponding author.
E-mail address: yiliu.liu@ntnu.no (Y. Liu).

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1. Introduction

Technologies bring more capabilities as well as more complexities to production, transportation, storage, and safeguard systems, which are currently composed of interacted subsystems and components. In a complex system, when one component fails, the failure may propagate, meaning to cause failures of other components. Each failed component further weakens system performance. We call this phenomenon as a cascading failure (CAF). CAF has been recognized as one of the usual causes resulting in the catastrophes of many modern technical systems [1], such as power grids system [2,3], industrial communication networks, railway networks [4], chemical clusters [5] and other complex network systems [6,7]. Typical events triggered by CAFs are blackout in American in 1996 [8] and massive blackout occurred in Italy [9] in 2003, seriously shocks the normal functions of the society. The Fukushima nuclear accident generated by a tsunami and started by earthquake in 2011 and The Amazon Web Services outage in 2012 are also classical CAFs examples [6,10]. These CAF events occur because technical systems are composed of multiple components structurally or functionally dependent with each other. Loading dependent system is one of the typical systems with dependence, where all components share the overall workload on the system.

Performance of a component always depends on its capacity and workload. In most cases, when the workload on a component is much higher than its capacity, a failure occurs. Then, the overall load on the system is re-distributed to the remaining functioning components whose workloads become higher, and these components become more vulnerable to failures. Such a re-distribution of workloads thus initiates a cascading process. For a loading dependent system, e.g., a wind farm, an energy station with several chargers, a piping network, and a medical center relying on several key devices, its performance can be related to the number of functioning components. If a cascading process starts, performance of such a system will degrade with less functioning components. For an individual component, an increasing workload can result in an immediate failure or an overloading state [11,12]. In the latter situation, the increased workload does not exceed the capacity of the component but is higher than the normal. Another non-negligible factor is the natural degradation of components in a loading dependent system, which has been studied in some research [13,14]. The degradation of components consists of their independent natural degradation and the degradation initiated by the conditions of other components [15], namely the capacity loss of components in our work. The performance of such an overloading component due to additional loads and capacity loss can thus be expected to deteriorate, and such degradation can shorten the lifetime of this component and affect the associated maintenance planning. For a loading dependent system, appropriate understanding on the overloading problems can be helpful to avoid the system-level failure or serious accidents.

Several models can be found for analyzing CAFs, such as the sand pile model [16,17], the ORNL-PSerc-Alask (OPA) model [18], the CASCADE model [8,19], the branching process model [20,21], and the topological models from the complex network theory [22,23], etc. Moon et al. [24] have proposed a load-dependent cascading failure model to evaluate the resilience of small devices' network to strategies for node removal by adopting the principle of sandpile process. Qi et al. [25] have estimated the joint distribution of two types of cascading outages with multi-type branching processes and tested with data generated by the AC OPA cascading simulations on the IEEE 118-bus system. Some methods based on the CASCADE model can be found in [16,26] for solving the self-organizing issue during cascading overload failures. The cascading process in a loading dependent system was first investigated by the CASCADE model [8,18], following an extended quasi-binomial distribution. The classical CASCADE model calculates probabilistic cascading failure for the weakening of the system as the basic cascade proceeds due to loads transformation [18]. The branching process model [21], as approximation of the CASCADE model, describes the total number of failed components as a Poisson random variable. However, the aforementioned cascading overload failure still refers to a failure mode induced by overloads, which is distinct from the notion overloading state as we proposed. To our best knowledge, most previous work focuses on direct failure spreading while ignoring the overloading phenomenon and components degradation.

Some practical challenges motivate the extension of the existing models on the issue of lacking the discussion about the overloading phenomenon and components degradation. For example, some pipes operating at higher pressures than expected might impose additional loads on other pipes in the same network. This kind of overloading state may occur due to their own degradation or other environmental factors and lead to loads transformation. The loading dependence induced by overloading components is thought to exert influence on the failure cascading process, though it is not as noticeable as that caused by failed components. The additional loads from overloading components and the natural degradation of components will undoubtedly promote component degradation and affect the evolution of the cascading process. The component reliability and system performance will be overestimated if the influence of this type of loading dependence and components own degradation on the failure cascading process is discarded. If the state of a component or system is overestimated when performing maintenance activities, delayed or inadequate maintenance may follow. In such cases, a more precise, realistic model that accounts for overloading components and component degradation supports maintenance decision makers in making more appropriate decisions.

Therefore, a more practical method is needed for analyzing the performance of loading dependent system subjected to CAFs affected by overloading components. In this new model, we consider the situation that components degrade gradually and may become overloading. Whenever a component is overloaded, it might have a negative effect on the other functioning components in the loading dependent system. It is expected that the extended model can reflect the cascading process more practically and detect more information such as the effect of overloading phenomenon and components degradation which are ignored in the existing classical CASCADE models.

The remainder of this paper is organized as follows. In Section 2, we describe the states transition mechanism in loading dependent systems and the algorithm of the classical CASCADE modeling, based on which some assumptions and algorithm of the CASCADE model are proposed. The model considering overloading components and three stop scenarios for cascading process are illustrated in Section 3. To illustrate the differences between the proposed model and classical model, an example of a piping system is provided in Section 4. In Section 5, we examine the variables affecting probability distributions of total number of failed and overloading components by discussing numerical results. Conclusions and future research directions are summarized in Section 6.

2. *Cascading failures and analysis models*

2.1. *Loading dependence as a cascading mechanism*

CAFs occur when the failure or degradation of one component weakens reliability and availability of the remaining components [27]. In this study, we classify CAFs as direct- and indirect- ones. The

Table 1
Comparison between Direct and Indirect Types of CAFs.

Category		Direct	Indirect
Difference	Driving force	Sudden shock and damage	Loading dependence
	Effects on components in sequence	Failures or degradation	Failures, degradation or overloading components
Similarities	Trigger	One failure or failures	
	Stop condition	There are no more new failures	

difference and similarities between two types of CAFs are listed in Table 1. A direct CAF occurs if the failure of a component or components directly induces damage to other ones or reduce their lifetime to some extent, while an indirect CAF often occurs due to loading dependence: The overall workload of the system is redistributed because some components exclude from normal operation. The loading dependence is resulted from the activities of loading balancing or loading sharing. Loading balancing is the practice of equally spreading the workload across distributed system nodes to optimize resource efficiency and task response time, which avoids a situation that some nodes are substantially loaded while others are idle or performing little work [28]. Loading sharing system is the practice of spreading the workload in a way that some loads are sent to one node in the system while the remainder is routed to others [28]. Loading dependent systems suffer from indirect CAFs.

2.2. CASCADE models

2.2.1. Classical CASCADE model

In this section, we present the mechanism of the classical nonstandard CASCADE model in loading dependent systems and the failure mechanisms of cascading process. This model is the basis inspired by which we extend our model. In current research related to classical CASCADE model, states Working and Failed are characterized for a component in a loading dependent system. When the workload is higher than the failure threshold, a failure occurs. Load redistribution then further facilitates the cascading process until that no new failures occur. Some assumptions are made in this classical CASCADE model:

1) The total number of components n in the system is finite.
2) All components in the system are identical, exchangeable and nonrepairable.
3) Each component in the system has two states: Working and Failed.

The classical CASCADE model is proceeding as the following steps:

Step 0. All components are normally working initially with random loads uniformly distributed in $[L^{min}, L^{max}]$.

Step 1. An initial outside disturbance to all components triggers the initial event followed by failure propagation. The initial failure is set as a trigger in generation 0 of a CAF.

Step 2. Check states for each component. If the load of component i exceeds L^{max}, then component i is failed. Otherwise, the component is working. Suppose that there are n_{fj} failed components in the jth generation. If $n_{fj} = 0$, there is no more new failures in the jth generation, and the cascading process stops. The stop condition of cascading process is that all components fail or the loads of the unfailed components are less than L^{max}.

Step 3. Additional loads due to failed components in this generation are allocated according to the number of failed components and added to working components in next generation.

Step 4. Go to the next generation and iterate from step 2.

This cascading mechanism is shown in Fig. 1. According to the CASCADE algorithm, the failure cascading process is triggered by an outside disturbance and stops in the jth generation if there are no more new failures in generation $j + 1$. This cascading process can stop when a) all components fail (cascading process stops, system fails); or b) the load of the unfailed component is less than the failure threshold (cascading process stops, system does not fail).

The classical CASCADE model is a tractable tool to capture the basic failure cascading process driven by loading dependence. However, the effect of some practical issues such as other states of components and components degradation on cascading property should be considered more. This encourages us to extend and improve the current classical models to tackle more practical problems. In practices, some components are functioning in the overloading state, which is often undervalued since the overloading components only seem to reduce the efficiency of the system. For example, the cascading process of a loading dependent piping network may vary if we consider not only the failures but also the overloading state of the pipelines, compared to the cascading process considering only the failures. Moreover, what about the impact on the cascading process when the inherent degradation of pipelines is also considered? This is also a subject we need focus on since most components may degrade naturally in reality, which should not be neglected. These practical problems will be addressed in the following sections.

2.2.2. Multi-state cascade model

In this section, we provide the mechanism of multi-state CASCADE model considering overloading components in loading dependent systems and the failure mechanisms of cascading process. For a component in a loading dependent system, it can actually have three states or performance levels: Normally Working, Overloading and Failed. The performance level can be determined by the ratio of workload to capacity r. When the workload is very highly, namely the ratio to capacity exceeds the failure threshold, a failure occurs. When the workload is higher than normal value, but the load/capacity ratio is still below the failure threshold, we regard the component is at an overloading state. We can also have a certain value of the load/capacity ratio as the overloading threshold, indicating that if the ratio is lower than this value, the component is Normally Working. In both Normally Working and Overloading states, a component is functioning, but it is inclined to fail when it is overloading. We use Functioning to denote the states of Overloading and Normally Working for short in this study. The failed and overloading components allocate loads to the functioning components during cascading process. Note that the overloading components also allocate loads to themselves. Here we do not consider maintenance, and the component state is generally getting worse. The states transition during cascading process are illustrated by Fig. 2.

Consider a technical system, some assumptions for our model are shown as below

1) The total number of components n in the system is finite.

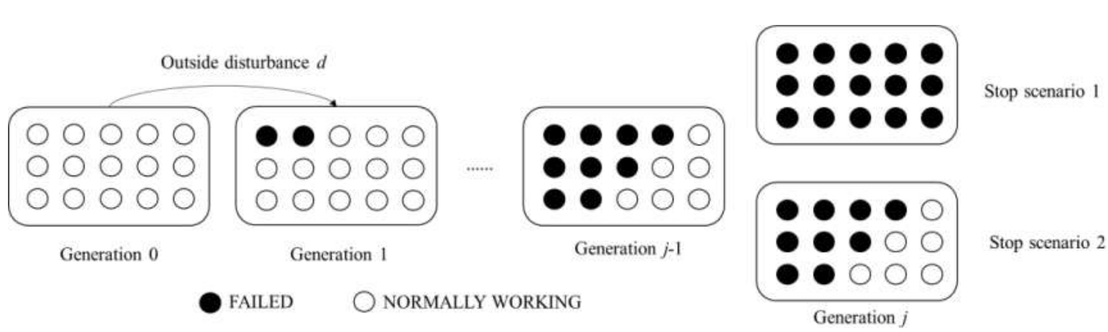


Fig. 1. Failure cascading process and stop scenarios of classical CASCADE model.

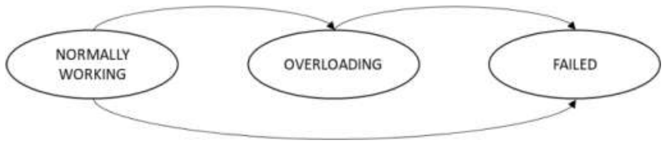


Fig. 2. States transition of components.

2) All components in the system are identical, exchangeable and nonrepairable.
3) Each component in the system has three states: Working, Overloading and Failed.
4) The capacity of every functioning component degrades naturally as the cascading failure propagate. The value of capacity decrement in every generation is c_d.

We can set that the workload on components lies in $[L^{min}, L^{max}]$, the capacity lies in $[C^{min}, C^{max}]$, and the initial disturbance D lies in $[D^{min}, D^{max}]$. When illustrating these parameters with a case of piping network, the workload can correspond to the flow rate through a pipeline, and the capacity is related with the failure limit and the expectation flow rate. An unexpected rise in flow rate is triggered by change of client requirement or work schedule, to lead to an initial disturbance. Furthermore, abrupt temperature fluctuations from the surroundings might affect workload via provoking an initial disturbance. These undesirable alterations should be observed since they are the driving force behind the start of the CAFs. The values of L^{min}, C^{min} and D^{min} are generally 0 in practice, but the values of L^{max}, C^{max} and D^{max} are not fixed. It is convenient for calculation to normalize the loads l and capacity c in [0, 1]. Based on the normalization of loads and capacity, if the initial disturbance $d \geq 1$, all components fail, and the cascading process stops immediately. If the initial disturbance $d = 0$, every component is working well and there is no failure to start the failure cascading process. Hence the following discussions assume that the range of d is normalized in (0, 1).

The nonstandard CASCADE model [8,19,20] and Modified Normalized CASCADE model [29] have been introduced for assigning workloads and initial disturbance to the components. Inspired by the existing CASCADE models, we reflect the cascading process in a similar way. To illustrate overloading state and capacity degradation, we introduce the quasi-multinomial distribution to model the three states of components. The extended multi-state CASCADE is modeled as the following steps:

Step 0. All components are normally working initially with capacity $c_0 = 1$ and random loads l_i that are uniformly distributed in [0, 1].

Step 1. An initial outside disturbance d to all components triggers the initial event followed by failure propagation. The initial failure is set as a trigger in generation 0 of a CAF.

Step 2. Check states for each component. The performance level is represented by ratio of workload to capacity l/c. If the ratio r_i of component $i < r^*$, then component i is working well. When the ratio r_i of component i exceeds 1, the workload of the component will be more than its capacity could endure, so the component fails. Otherwise, the component is overloading. Suppose that there are n_{fj} failed components and n_{oj} overloading components in the jth generation. If $n_{fj} = 0$, there is no more new failures in the jth generation, and the cascading process stops. We define the stop condition of cascading process that if no new failures occur in one generation, the failure cascading process stops here, regardless of whether there would be more failures occur in subsequent generations.

Step 3. The capacity of every functioning component decreases due to natural degradation, so we have the capacity of the component in the jth generation $c_j = c_0 - j \cdot c_d$ and the load/capacity ratio of the component $r_{ij} = \frac{l_{ij}}{c_j}$. The additional load due to each failure in this generation on every functioning component in next generation is l_f. The additional load on every functioning component in next generation due to each overloading component in this generation is l_o. It is natural that l_o is considered smaller than l_f. Additional loads $l_j = n_{fj} l_f + n_{oj} l_o$ are allocated according to the number of failed and overloading components and added to every functioning component. Each functioning component is assigned an additional load value of l_j.

Step 4. Go to the next generation and iterate from step 2.

This cascading mechanism is shown in Fig. 3. According to the CASCADE algorithm, if and only if there are no more new failed components in generation $j+1$, the cascading process stops in the jth generation. This is the only criterion for determining if the cascading process stops, regardless of whether there are still functioning components in the system currently. We consider it as a *new cascading process* if the remaining components tend to fail after a period and there would be *new generation 0*. We shall clarify that the stop condition of cascading process is differentiated from the stop condition of system. The former one is determined by whether new failures occur at a certain generation, whereas the latter one is determined by the system reliability. In conclusion, the cascading process stops when all components fail, but not all components fail when the cascading process stops. Following the explanation of the stop condition of cascading process, we can characterize three stop scenarios (scenarios of how the system works) when the cascading process terminates as follows. This cascading process can stop when a) all components fail (cascading process stops, system fails); or b) the load/capacity ratio of the functioning component is less than the failure threshold (cascading process stops, system does not fail). These two cases could be classified into three scenarios. In stop scenario 1, all components and the system already failed; in stop scenario 2, there exist some overloading components; in stop scenario 3, the load/capacity ratio of the functioning component is less than r^* and all components work normally.

3. Quantitative analysis with the multi-state CASCADE model

3.1. Total number of components in different states

To start the cascade, initial disturbance d is assigned to each component. If there are components failed, the failure cascading process starts, followed by that the number of failed components increases and the functioning components decreases generally. The numbers of failed components, overloading components and normally working components are n_f, n_o, n_w and $n_f + n_o + n_w \leq n$. It is natural that $n > 0$ and n_{fj}, n_{oj}, n_{wj} for $j = 0, 1, \ldots$ are restricted to nonnegative integers. The state of the component follows a multinomial distribution $X \sim PN(\mathrm{N} : p_f, p_o, p_w)$, determined by outside initial disturbance, additional loads from failed and overloading components, and overloading threshold of components. In each generation, the probability that there are n_{fj} components failed, n_{oj} components overloading and n_{wj} components normally working is

$$P[X_1 = n_f,\ X_2 = n_o, X_3 = n_w] = C_n^{n_f} C_{n-n_f}^{n_o} p_f^{n_f} p_o^{n_o} p_w^{n_w} \quad (1)$$

where $p_f \geq 0,\ p_o \geq 0, p_w \geq 0, p_f + p_o + p_w = 1$.

The probability of the total number of components in different states might be derived as follows:

In generation 0, before the initial disturbance applied, the probabilities that the component in different states depend solely on the random loads l_i. Then we could obtain $p_f = 0,\ p_o = 1 - r^*,\ p_w = r^*$. In generation 0, the cascading process has not been started yet since all components are functioning.

After the initial disturbance d is applied in generation 1, the load of component i is $l_i + d$. But the capacity of each component is still c_0 since the cascading process just started from this generation. After the cascading process begins, the capacity of components gradually

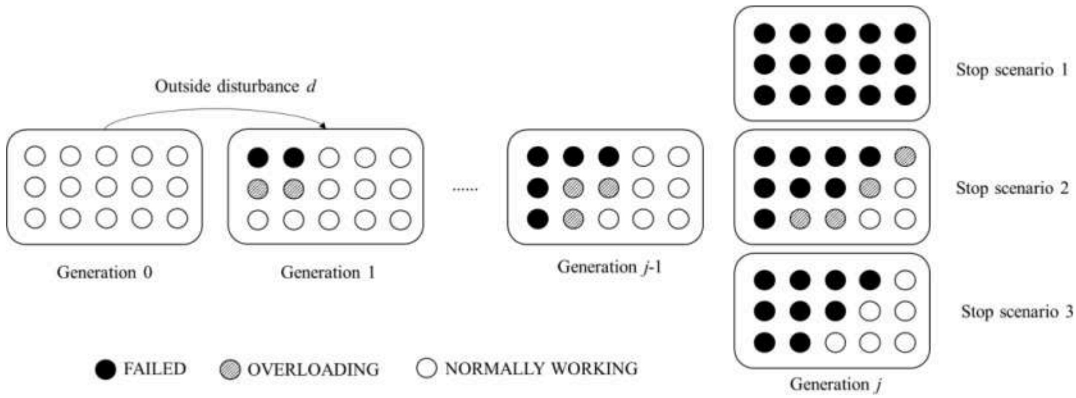


Fig. 3. Failure cascading process and stop scenarios of multi-state CASCADE model.

declines. Similar to the load redistribution principle, the component capacity loss at this generation (generation 1) will be reflected in the next generation (generation 2). For generation 1, if the load/capacity ratio of a component exceeds 1, the component fails, we have $l_i \leq 1$ and $1 < \frac{l_i+d}{c_0}$, and the probability that the component fails is the probability that l_i satisfies the constraints of the previous two equations. So, we could obtain the interval of l_i: $c_0 - d < l_i \leq 1$, and further we could easily achieve $p_f = 1 - c_0 + d$. The same holds applicable for the other two probabilities. According to our definition, if the load/capacity ratio of a component lies in [r^*, 1], this component is overloading, which could be represented by $r^* < \frac{l_i+d}{c_0} < 1$. Hence, we obtain the probability that the component in overloading state is $p_o = c_0(1 - r^*)$. When $\frac{l_i+d}{c_0} < r^*$, the component works well, and the probability is $p_w = c_0 r^* - d$. The three probabilities are respectively $p_f = d,\ p_o = 1 - r^*,\ p_w = r^* - d$ in generation 1 based on that the initially capacity is normalized as $c_0 = 1$.

In the jth generation, the cascading process has already gone through some generations, the total number of failed components and the total number of overloading components could be calculated. Let $s_j = (n_{fj}, n_{oj}, n_{wj})$, $S_j = (N_{fj}, N_{oj}, N_{wj})$ for $j = 0, 1, \dots$ and write

$$u_j = n_{f0} + n_{f1} + \cdots + n_{fj} \text{ and } v_j = n_{oj} \tag{2}$$

for $j = 0, 1, \cdots$.

Each functioning component suffers additional loads $ul_f + vl_o$ from failed and overloading components and total loads $l_i + d + ul_f + vl_o$. With the same principle for calculation of probability of components in different states in generation 1, we have $l_i \leq 1$ and $1 < (l_i + d + ul_f + vl_o)/c_j$ for the case that the component fails, and the probability that the component fails is the probability that l_i satisfies the previous two constraint equations. So, we could obtain the interval of l_i: $c_j - (d + ul_f + vl_o) < l_i \leq 1$, and further we could get $p_f = 1 - c_j + d + ul_f + vl_o$.

$$P[U=u, V=v] = \begin{cases} C_n^u C_{n-u}^v \varphi(d)\varphi\left(1 - c_j + d + ul_f + vl_o\right)^{u-1} \varphi\left(c_j(1-r^*)\right)^v \varphi\left(c_j r^* - \left(d + ul_f + vl_o\right)\right)^{n-u-v},\ u = 0, 1, \dots, n-1 \\ 1 - \sum_{u=0}^{u=n-1} P(U=u,\ V=v),\ u = n \end{cases} \tag{5}$$

The same holds applicable for the other two probabilities. Likewise, we could obtain the constraint equations of other two states after some generations: when $r^* < (l_i + d + ul_f + vl_o)/c_j < 1$, the component is overloading and the probability is $p_o = c_j(1 - r^*)$. When $(l_i + d + ul_f + vl_o)/c_j < r^*$, the component works well, and the probability is $p_w = c_j r^* - (d + ul_f + vl_o)$.

Note that the total number of overloading components v_j is not sum of n_{oj} in previous generations for $j = 0, 1, \dots$ since the overloading components may fail in a cascading process. If we calculate the total number of overloading components by summing up the overloading components in all generations, the total number of failed components partially overlaps the total number of overloading components. We only use the number of the overloading components in latest generation to represent the total number of overloading components. Generalize the derivation and apply this distribution to normalized load-dependent case and we can obtain the distribution of the total number of failed components and overloading components. An extended quasi-multinomial distribution is applied as following on basis of extended quasi-binominal distribution introduced by Consul [19,30]. The quasi-binomial distribution is a small "perturbation" of the binomial distribution, whose mass probability function could be defined by $P(X = k) = C_n^k p(p + k\phi)^{k-1}(1 - p - k\phi)^{n-k}$. When extended to quasi-multinomial distribution, we also strictly follows the format of the distribution, as shown in Eq. (3).

$$P[U=u, V=v] = \begin{cases} C_n^u C_{n-u}^v \varphi(d)\varphi\left(p_f\right)^{u-1} \varphi\left(p_o\right)^v \varphi\left(p_w\right)^{n-u-v}, u=0,1,\dots,n-1 \\ 1 - \sum_{u=0}^{u=n-1} P(U=u, V=v), u=n \end{cases} \tag{3}$$

In Eq. (3), $\varphi(x)$ is a saturation function representing the probability

$$p = \varphi(x) = \begin{cases} 0, & x < 0 \\ x, & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases} \tag{4}$$

We have Eq. (5) to calculate the distributions of the total number of components in different states.

When we consider the accident risk, the number of failures is more of interest than number of overloading components. The equation to denote the distributions of the total number of failed components is Eq. (6).

$$P[U=u]=\begin{cases}C_n^u\varphi(d)\varphi(p_f)^{u-1}\varphi(1-p_f)^{n-u}, & u=0,1,\ldots,n-1\\ 1-\sum_{u=0}^{n-1}P(U=u), & u=n\end{cases} \tag{6}$$

In this CASCADE model, the system reliability could be calculated as Eq. (7) when considering the cascading time t, which could be discussed in further research.

$$\begin{aligned}R(t)&=1-\sum_{j=0}^{n-1}P(U_j=n,\ T_j<t)\\&=1-\sum_{J=0}^{n-1}F^{(J+1)}(t)\cdot P[u=n]\end{aligned} \tag{7}$$

where $R(t)$ is the probability that the system is still working until time t. T_j is duration of cascading process from the start to generation J. $F^{(J+1)}(t)$ is the probability distribution function that all components fail in generation J at time t. This equation is independent of the number of overloading components, as only failed components are typically included when investigating system reliability.

3.2. Distributions of stop scenarios

In the previous subsection, the probability that there are n_{fj} components failed, n_{oj} components overloading and n_{wj} components normally working in the jth generation is

$$P[X_{1j}=n_{fj},\ X_{2j}=n_{oj},X_{3j}=n_{wj}]\ =C_n^{n_{fj}}C_{n-n_{fj}}^{n_{oj}}p_f^{n_{fj}}p_o^{n_{oj}}p_w^{n_{wj}} \tag{8}$$

However, in the cascading process, the sojourn probability of components in different states is not constant as the failure propagates and loads are reallocated. Since the workload of components is mounting due to loading dependent and the capacity of components is decreasing due to natural degradation gradually, the probability of the number of components in different states should be recalculated after each generation according to the loading increments. It is convenient to use equations of $\alpha_j=\varphi(p_{fj}),\ \beta_j=\varphi(p_{oj}),\ \gamma_j=\varphi(p_{wj})$ for calculating in the subsection.

$$P[S_j=s_j,\ldots,S_0=s_0]=\frac{n!}{n_{f0}!n_{o0}!n_{w0}!}\alpha_0^{n_{f0}}\beta_0^{n_{o0}}\gamma_0^{n_{w0}}\frac{(n-u_0)!}{n_{f1}!n_{o1}!n_{w1}!}\alpha_1^{n_{f1}}\beta_1^{n_{o1}}\gamma_1^{n_{w1}}\cdots\frac{(n-u_{(j-1)})!}{n_{fj}!n_{oj}!n_{wj}!}\alpha_j^{n_{fj}}\beta_j^{n_{oj}}\gamma_j^{n_{wj}} \tag{12}$$

In generation 0, $p_{f0}=0,\ p_{o0}=1-r^*,\ p_{w0}=r^*$, and we could obtain $\alpha_j=0,\ \beta_j=1-r^*,\ \gamma_j=r^*$ for $j=0$.

In generation 1, with the initial workloads given as described in step 0 and the initial disturbance applied as in step 1, the CASCADE algorithm starts. In step 2, for a loading dependent system considering decreasing capacity, the probability that the initial disturbance triggers one component fails or overloads in generation 1 is $\alpha_1=\varphi(1-c_0+d)$, $\beta_1=\varphi(c_0(1-r^*))$, $\gamma_1=\varphi(c_0r^*-d)$, and could be written as $\alpha_1=\varphi(d)$, $\beta_1=\varphi(1-r^*)$, $\gamma_1=\varphi(r^*-d)$ since $c_0=1$. The probability that there are n_{f0} failed components and n_{o0} overloading components is

$$\begin{aligned}P(S_0=s_0)&=P[X_1=n_{f0},\ X_2=n_{o0},X_3=n_{w0}]\\&=C_n^{n_{f0}}C_{n-n_{f0}}^{n_{o0}}\alpha_0^{m_{f0}}\beta_0^{m_{o0}}\gamma_0^{(n-n_{f0}-n_{o0})}\end{aligned} \tag{9}$$

In the jth generation, the capacity of each functioning component decreases due to natural degradation after several generations, and the additional loads are accumulated and added to each functioning component as cascading process proceeds. Additional loads from failed components in generation j-1 to the functioning components in the jth generation is $n_{f(j-1)}l_f$. Additional loads from overloading components in generation j-1 to the functioning components in the jth generation is $n_{o(j-1)}l_d$. The additional loads from failed and overloading components could be assigned to the functioning components including itself in generation $j+1$.

$$l_j=n_{f(j-1)}l_f+n_{o(j-1)}l_o \tag{10}$$

For loading dependent system considering capacity decrement of the components, we have

$$\begin{aligned}\alpha_j&=\varphi\left(\frac{1-d-u_{(j-2)}l_f-v_{(j-2)}l_d-c_j+l_j}{1-d-u_{(j-2)}l_f-v_{(j-2)}l_d}\right),\\ \beta_j&=\varphi\left(\frac{c_j(1-r^*)}{1-d-u_{(j-2)}l_f-v_{(j-2)}l_d}\right),\\ \gamma_j&=\varphi\left(\frac{c_jr^*-l_j}{1-d-u_{(j-2)}l_f-v_{(j-2)}l_d}\right)\end{aligned} \tag{11}$$

for $j=2,3,\ldots$, and $u_{-1}=0,\ v_{-1}=0$.

The probability that the number of failed components and overloading components in every generation follows $(s_0,\ s_1,\ldots s_j)$ until the jth generation is given by Eq. (12).

Suppose that cascading process stops in the jth generation and $d+u_{(j-1)}l_f+v_{(j-1)}l_d\geq c_j$, then all components fail in the jth generation. Cascading process stops according to stop scenario 1. In this case

$$P[S_{j+1}=s_{j+1}|S_j=s_j,\ldots,S_0=s_0]=1 \tag{13}$$

for $n_{f(j+1)}=0$.

Suppose that cascading process stops in the jth generation and $d+u_{(j-1)}l_f+v_{(j-1)}l_d<c_j$, meaning to satisfy the stop scenarios 2 or 3. In addition, the loads of functioning components are uniformly distributed in $[d+u_{(j-1)}l_f+v_{(j-1)}l_d,\ c_j]$ conditioned on $n-u_j$ not have failed in generation $j+1$. The probability that there are $n_{o(j+1)}$ overloading components and $n_{w(j+1)}$ normally working components is given by Eq. (14).

$$P[S_{j+1}=s_{j+1}|S_j=s_j,\cdots,S_0=s_0]=C_{n-u_j}^{n_{o(j+1)}}\beta_{j+1}^{n_{o(j+1)}}\gamma_{j+1}^{n_{w(j+1)}} \tag{14}$$

Table 2
Load of components in an example of classical CASCADE model.

j	1	2	3	4	5	Loading increments to next generation	Notes
0	0.75	0.5	0.45	0.25	0.9	/	Initial workloads
1	0.95	0.7	0.65	0.45	1.1	0.1	Initial disturbance d added; 5 fails
2	1.05	0.8	0.75	0.55	/	0.1	1 fails
3	/	0.9	0.85	0.65	/	0	No new failure occurs, and the cascading process stops

Multiplying Eqs. (12) and (14) we could obtain Eq. (15) to verify the distribution for the stop scenarios.

$$P\left[S_{j+1}=s_{j+1},\cdots,S_0=s_0\right]=\frac{n!}{n_{f0}!n_{o0}!n_{w0}!}\alpha_0^{n_{f0}}\beta_0^{n_{o0}}\gamma_0^{n_{w0}}\frac{(n-u_0)!}{n_{f1}!n_{o1}!n_{w1}!}\alpha_1^{n_{f1}}\beta_1^{n_{o1}}\gamma_1^{n_{w1}}\cdots\frac{\left(n-u_{(j-1)}\right)!}{n_{fj}!n_{oj}!n_{wj}!}\alpha_j^{n_{fj}}\beta_j^{n_{oj}}\gamma_j^{n_{wj}}\cdot C_{n-u_j}^{n_{f(j+1)}}\beta_{j+1}^{n_{o(j+1)}}\gamma_{j+1}^{n_{w(j+1)}} \tag{15}$$

In case cascading process stops according to stop scenario 1, all components fail in the jth generation. In the case cascading process stops with stop scenario 2, some components (or all functioning components) are overloading in the jth generation, and the number of failed components in generation $j+1$ is 0. In case cascading process stops according to stop scenario 3, there are still $n-u_j$ components normally working well in the jth generation, and the number of failed components in generation $j+1$ is also 0.

4. A practical case with model comparison

In this section, we apply both the classical CASCADE model and the proposed multi-state CASCADE model to a generic petrochemical piping network for comparing their effectiveness in the analysis of cascading failures due to loading dependence. We consider a part of a piping network system consisting of 5 gas pipes. Each pipe is designed with a failure limit of 20 m/s and expectation flow rate 16 m/s. Fouling would emerge inside the pipe as the pipe transfers gas, increasing the pressure, reducing the gasses throughput, and lowering system operation efficiency, which is the process we called natural degradation. The volume of gas transported in the pipes is the indicator of working load, and the capacity of the pipe is determined by the degree of fouling. If one pipe stops functioning due to exogenous disturbance, sudden changes in temperature for example, other pipes share the workload of the failed pipe.

It is possible to use the classical CASCADE model to study the cascading failure in such a system. According to the classical CASCADE model, the components fail or normally working during the cascading process without degradation. We normalize the workloads and capacity to [0, 1]. The initial loads of components are randomly valued in [0, 1]. Assume that the initial disturbance $d=0.2$, loading increments from failed components $l_f=0.1$ without losing generality. Table 2 and Fig 4 show the changes of the workloads of all components and the cascading process. The loading increments in this model depends on the number of the failed components, the load of which exceeds 1. The failure cascading process ends in the third generation with components 1 and 5 failed, and the system is still working.

The classical CASCADE model investigates the loading dependence due to malfunction of some pipes. The congestion due to filth accumulation, which is inevitable during system operation, also require additional gas on the remaining functioning pipes. The extra gas speed up fouling of functioning pipes and let them undergo accelerated degradation. When the gas is transferred in the pipe at a rate more than the expectation flow rate 16 m/s but under the failure limit of 20 m/s, we think the pipe is overloading since the workload exceeds its expectation capacity. We could consider that there is the overloading threshold $r^*=0.8$. In one specific circumstance, we can assume that the value of overloading threshold is constant. However, when the component degrades, it stores less capacity, hence a lower workload will overload the component with the same overload threshold. Some pipes become overloading with excessive workloads, and their performance suffers severely, which is why overloading components need to be addressed in the proposed model. Based on the assumption about the initial loads, initial disturbance, and loading increments from failed components when using classical CASCADE model, the loading increments from overloading components l_o is set to be 0.05 without losing generality. In addition, assume that the capacity decrement of functioning component in every generation $c_d=0.01$. The load/capacity ratio r of components and cascading process are listed and performed in Table 3 and Fig 5. The loading increments in this model depends on the number of the failed components and overloading components. The capacity of the functioning components decreases in every generation. Using this model, load/capacity ratio r is utilized to determine states of components. When the failure cascading process stops in the fourth generation, all components fail, and the system fails.

From the example, we can see that the system and the pipes function in radically different states under the same circumstances. The cascading process of classical CASCADE model ends in the third generation, but the system continues to function. The cascading process of the proposed multi-state CASCADE model stops in the fourth generation, and all components fail. Furthermore, we can see that the load/capacity ratio values in Table 3 are generally bigger than those in Table 2 (if we consider the component capacity in the example in Table 3 to be constant at 1). This implies that the components in multi-state CASCADE model operates in somewhat worse state than those in classical CASCADE model. The primary difference between the two conclusions is that the degradation of components and the effects of overloading components are considered, which is more compatible with how the system works in engineering industry. In practice, if we neglect components degradation and the influence of overloading components, we may overestimate the performance of components and the system, negatively affecting maintenance decision making.

5. Model parameter analysis

The model proposed can be used to analyze the cascading process in a large complex system with loading dependence. These systems can be wind plants, power systems, piping networks, key medical devices, road systems, etc., where the system performance is related to the number of functioning components. To investigate the usefulness of this CASCADE model in the optimization of controllable variables in design and operation, this section examines several examples of the effects with varying parameters of CASCADE distribution on failures and stop scenarios of a general loading dependent system.

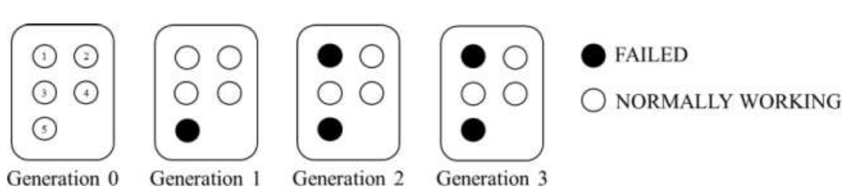


Fig. 4. Failure cascading process of a piping system using classical CASCADE model.

Table 3
Load/capacity ratio of components in an example of multi-state CASCADE model.

j	1	2	3	4	5	Loading increments to next generation	Capacity of the functioning components	Notes
0	0.75	0.5	0.45	0.25	0.9	/	1	Initial workloads/ Initial capacity
1	0.96	0.71	0.66	0.45	1.11	0.15	0.99	Initial disturbance d added; 5 fails
2	1.12	0.87	0.82	0.61	/	0.2	0.98	1 fails
3	/	1.08	1.03	0.82	/	0.25	0.97	2 and 3 fail
4	/	/	/	1.09	/	0.15	0.96	4 fails; the system fails; the cascading process stops

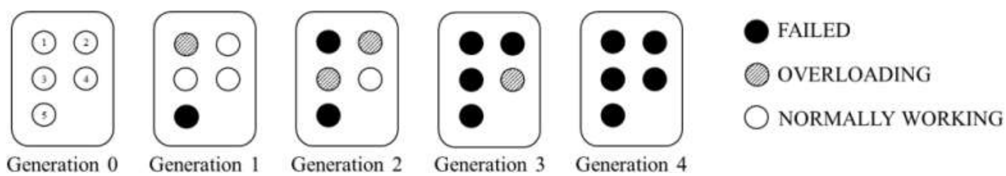


Fig. 5. Failure cascading process of a piping system using multi-state CASCADE model.

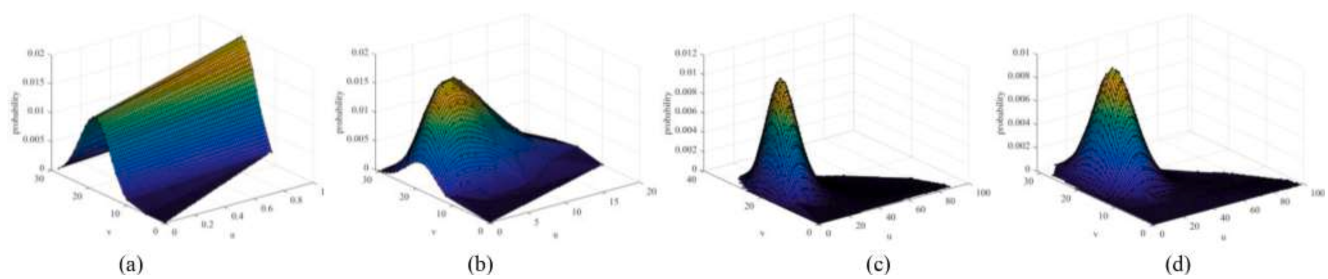


Fig. 6. Total number of failed and overloading components with different d. (a) $d = 0.001$. (b) $d = 0.01$. (c) $d = 0.05$. (d) $d = 0.1$.

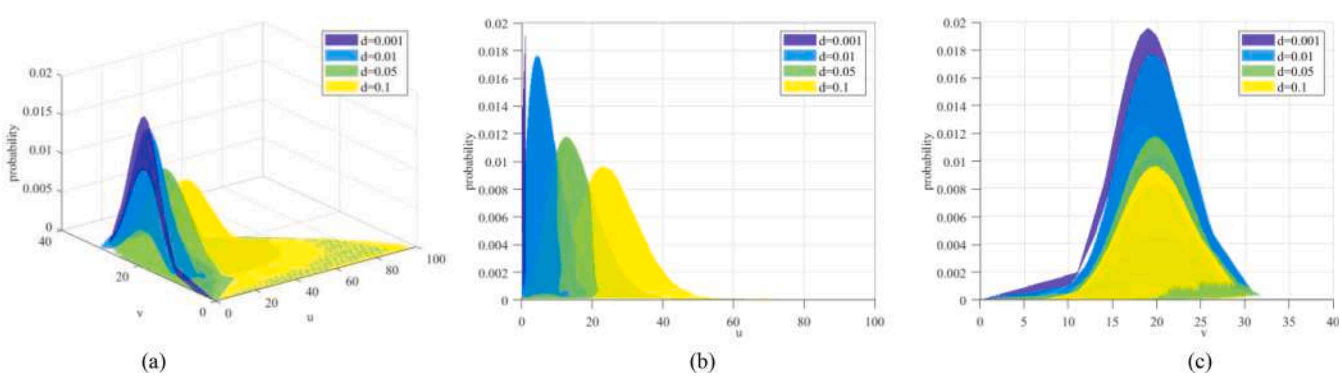


Fig. 7. Integration of probability distributions with different d. (a) Three-dimensional profile. (b) *p-u* profile. (c) *p-v* profile.

5.1. Effect of initial disturbance

This subsection illustrates the change of CASCADE distribution as the initial disturbance varies by comparing the probabilities of total numbers of failed and overloading components. We consider a system in which the number of components $n = 100$. Without losing generality, we firstly assume that the overloading threshold of a component is $r^* = 0.8$, the loading increments from failed and overloading components are respectively $l_f = 0.005$, and $l_o = 0.001$. The changes of probability distributions of total numbers of failed and overloading components are observed with different initial disturbance $d = 0.001, 0.01, 0.05, 0.1$.

The probability distributions of total numbers of failed and overloading components are calculated and shown in Figs. 6 and 7. The nodes on the surfaces in Fig. 6 denotes the probabilities of total numbers of failed and overloading components of numerical results. When the initial disturbance increases, the workloads of components tend to exceed the failure threshold, which is the reason the value of u grows up. For $d = 0.001$, the initial disturbance value is relatively small, causing only a small number of failures. The low number of failed components also results in fewer additional loads to drive the cascade process. The cascading process ends quickly when there are still some functioning components, and the system is still operating (stop scenario 2). The short cascading process leads to that only few nodes can be observed to compose a surface in Fig. 6(a), which is more like a folded plane. As d increases, the number of obtained nodes in Fig. 6(b), (c) and (d) gradually rises, the surface becomes smoother and shows obvious peaks. This peak represents the highest probability of a scenario with a certain total number of failed components and a certain total number of overloading components in this case. The phenomenon that all components fail emerges in Fig. 6(d), indicating that stop scenario 3 occurs.

Fig. 7 integrates the five surface to illustrate the variation tendency better. Fig. 7(a) illustrates the trend of a lower overall probability

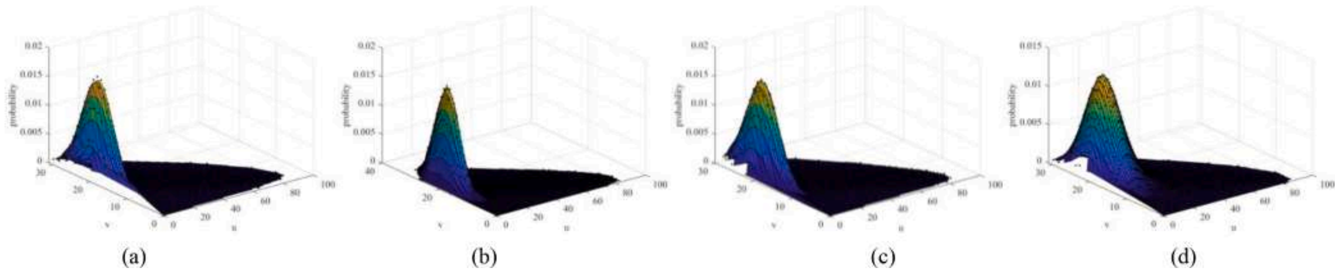


Fig. 8. Total number of failed and overloading components with different l_f. (a) $l_f = 0.0001$. (b) $l_f = 0.0005$. (c) $l_f = 0.001$. (d) $l_f = 0.005$.

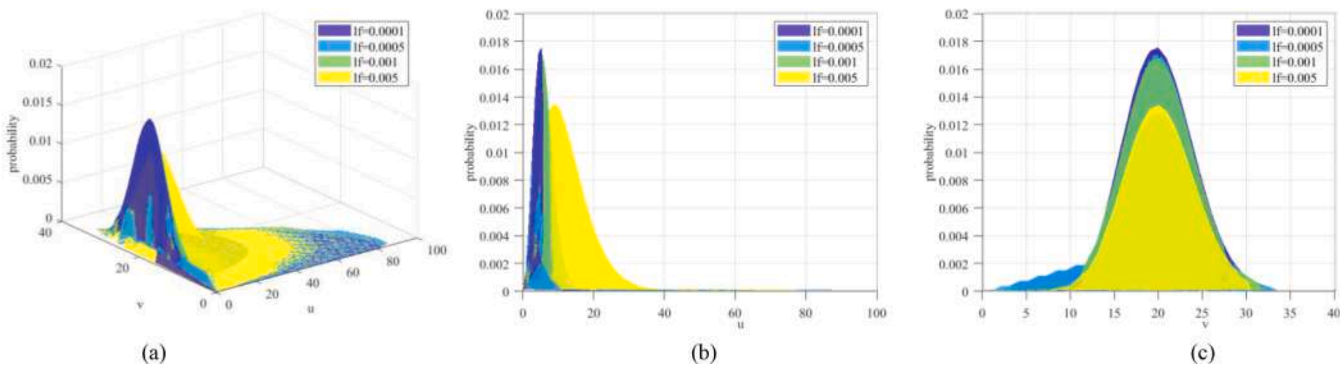


Fig. 9. Integration of probability distributions with different l_f. (a) Three-dimensional profile. (b) *p-u* profile. (c) *p-v* profile.

distribution of total numbers of failed and overloading components. Fig. 7(b) verifies the conclusion that there is a critical value of *u* that maximizes the probability. As *d* increases, this peak value of probability gradually decreases. In addition, the probability distribution range corresponding to *u* gradually shifts to the direction that *u* becomes larger when *d* becomes larger. It could also be observed from Fig. 7(c) that for different *d*, the number of overloading components is basically concentrated from 15 to 25, and the probability peak decreases gradually as *d* increases. Besides, the peaks of probabilities for *u* and *v* both show approximate power law behavior near the peak value.

Overall, when the initial disturbance value is small, the number of failed components is small, but the maximum probability of its occurrence is large. When the initial disturbance value is large, more components fail, but the maximum probability of its occurrence is small. The initial disturbance can be sudden shock or short-term increase in flow. Since the initial disturbance is an external factor, it is difficult to be controlled in system design, but we can still obtain some managerial implications, such as avoiding disturbances that can directly trigger

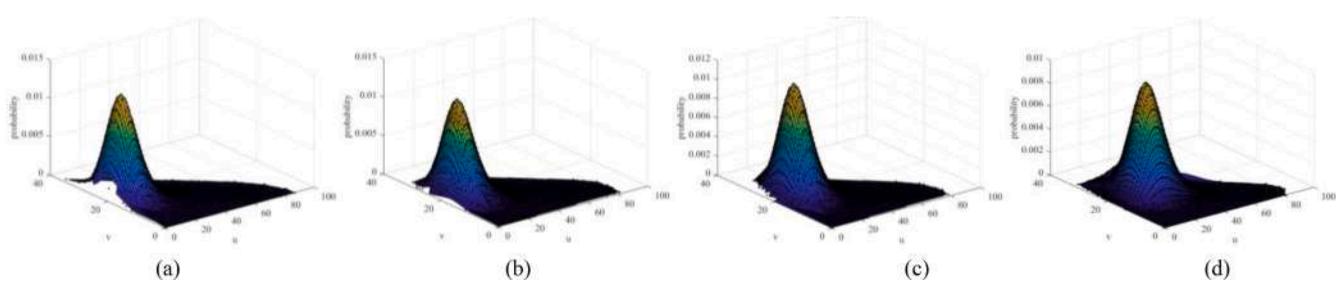


Fig. 10. Total number of failed and overloading components with different l_o. (a) $l_o = 0.0001$. (b) $l_o = 0.0005$. (c) $l_o = 0.001$. (d) $l_o = 0.005$.

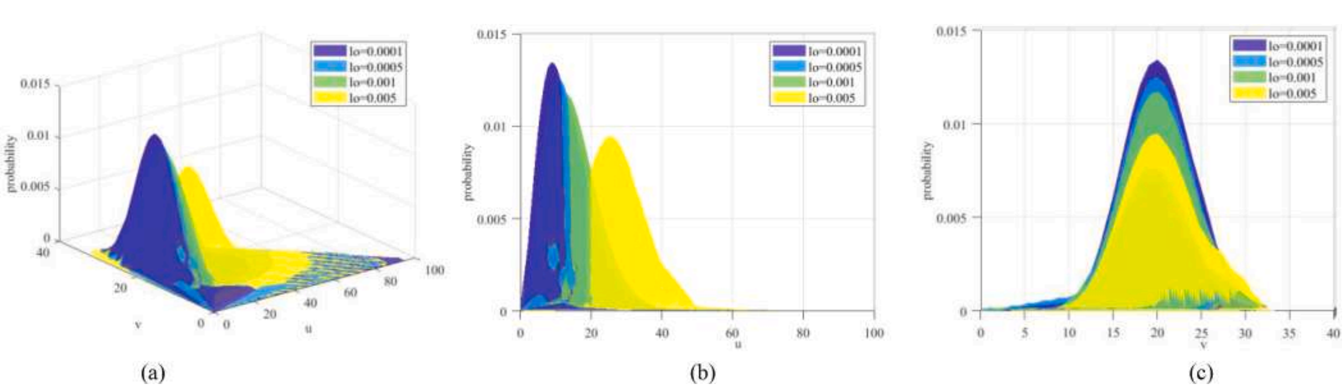


Fig. 11. Integration of probability distributions with different l_o. (a) Three-dimensional profile. (b) *p-u* profile. (c) *p-v* profile.

failure of components. In practices, such efforts can lead to high cost to ensure that none of the components in the system fail. In addition, it is unwise to ignore outside disturbances due to the low probability of occurrence that many components fail. When the system can accept a certain range of number of failed components, we can get an acceptable external disturbance value accordingly. In a bridge system, for example, the value of a sudden increase in traffic caused by holiday trips can be limited to an acceptable range to ensure long-term good operation of the system. During inspection, if the external disturbance is lower than this value, we do not need take more actions.

5.2. Effect of loading increments

To compare the effects of two kinds of loading increments, we use different values of l_f and l_o for different configurations in this subsection. To match the configurations with reality, l_o cannot exceed l_f. For the case that $n = 100$, we set $d = 0.05$, $r^* = 0.8$. Firstly, we evaluate the loading increments l_f as: 0.0001, 0.0005, 0.001, and 0.005 when fixing $l_o = 0.0001$. Then, we set $l_f = 0.005$, and observe different loading increments l_o as 0.0001, 0.0005, 0.001, and 0.005.

Figs. 8 and 10 respectively describe the probability distributions of total numbers of failed and overloading components under different settings of parameters l_f and l_o. It can be found that such changes have little influence on the shape of the surface. From integration results in Figs. 9 and 11, surfaces cannot be easily differentiated when l_f and l_o varying from 0.0001 to 0.001, while the surface apparently changes when l_f and l_o assumed to be 0.005. A reasonable explanation can be provided that when the loading increments are small, the effect of their changes on the probability distributions could be ignored, but when it reaches to a certain value, it still can affect the probability distributions of total numbers of failed and overloading components. This conclusion recommends that more attention should be paid to the timely maintenances of overloading components in practice. It is also worth mentioning that the probability distributions range of the number of overloading components is almost same as in the previous section.

Actually, the values of two kinds of loading increments could be impacted by management or strategies. Given that an initial failure has already been triggered, we try to avoid subsequent failures by developing a more rational strategy for workload distribution, that is, to manage how much workload should be reallocated to which component during system operating. Generally, the loading increments are not fixed in the design period, hence the measures to manage workload distribution would be preferred. Taking a road system as an example, when a road section cannot be used or gets blocked due to overloading, other roads will bear more traffic and pedestrian flow, or in other words, bear additional workloads. This kind of additional workloads can be adjusted by taking current limiting and reasonable diverting measures.

5.3. Effect of overloading threshold

The overloading state of components has been introduced in the proposed extended multi-state CASCADE model, accompanying with the new parameter overloading threshold considered to distinguish the state of overloading components from normally working components. Here we discuss the influence of this new parameter. Consider that the overloading threshold r^* varies from 0.6 to 0.9 as shown in Figs. 12 and 13, in which $n = 100$, $d = 0.05$, $l_f = 0.005$, $l_o = 0.001$.

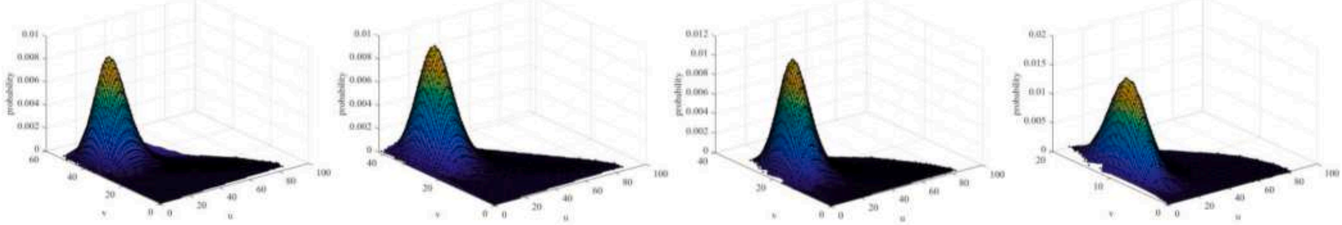
The shape and trend of each surface are still consistent with our previous discussion: each surface has an obvious peak, and the approximate power function law appears near the peak. In addition to this, the similarities and differences of the surfaces deserve more discussions. In Fig. 12, the probability distributions of total numbers of failed and overloading components, as well as the shape and trend of the surfaces are roughly same. The curved surfaces in Fig. 13 gradually shifts in the direction of v decreasing, as the overloading threshold increases. Different from the previous discussions, the distribution range of the number of failed components are almost same in this example, concentrated in 0 to 20, and the probability peaks when the overloading threshold r^* is 0.9. The results indicate that change of the overloading threshold mainly affect the probability distribution range of the number of overloading components but can barely affect that of failed components. It should be noted that even though the probability distribution range of the number of failed components is slightly affected by the overloading threshold, the maximum probability value ascends as the overloading threshold value increases, which demonstrates that as the overloading threshold value increases, it would be easier for components to fail.

The above results can provide references for practical system engineering design and operation. In a loading dependent system where the overloading components also influence the failure propagation, the overloading threshold should be a moderate value, neither not too high to make failures occurring easily, nor too low to prompt too many overloading components. The practical overloading threshold is a critical value beyond which the component operates in poor conditions and requires maintenance action. It could be controlled through providing different expectation values of safety margin in design. For a component designed with a failure limit of 200 MPa and normally working under its design expectation stress, it is overloading below the failure limit but in excess of the design expectation stress. Its threshold is 0.8 when expectation stress set to be 160 MPa and is 0.7 when expectation stress set to be 140 MPa. Apart from design in practical, some guidance could be provided during operation. For a repairable loading dependent system, periodical inspections and imperfect repair could be carried out during operation to restore the performance of overloading components under the threshold.

5.4. Stop scenarios and occurrences

In the previous analysis, we only consider the probability distributions of the total number of failed and overloading components in the meantime when the cascading process is not stopped yet. We now explore the stop scenarios of the cascading process and their possibilities of occurrences.

It has been summarized in previous examples that the initial disturbance d has a relatively large impact on the number of failures, and the number of failures largely determines how the system operates when



(a) (b) (c) (d)

Fig. 12. Total number of failed and overloading components with different r^*. (a) r^*=0.6. (b) r^* =0.7. (c) r^* =0.8. (d) r^* =0.9.

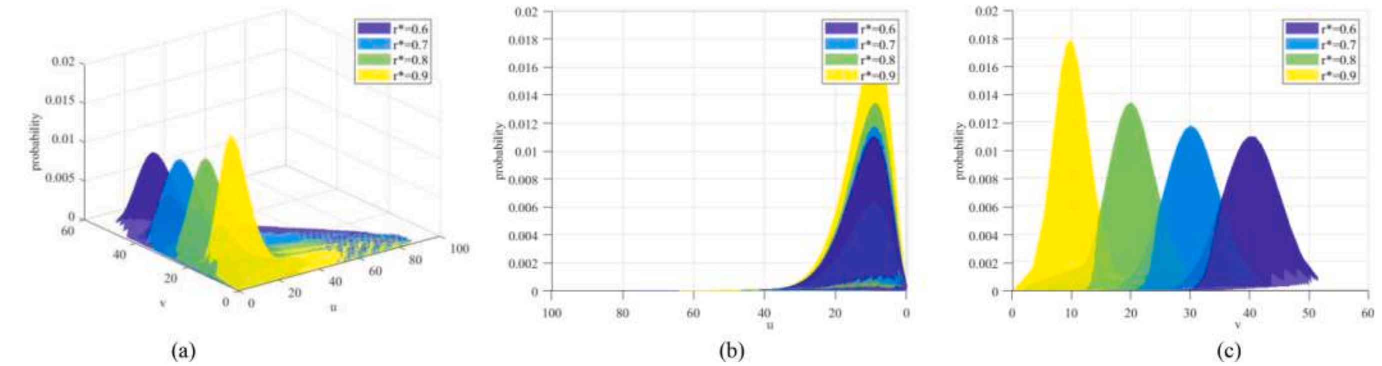


Fig. 13. Integration of probability distributions with different r^*. (a) Three-dimensional profile. (b) *p-u* profile. (c) *p-v* profile.

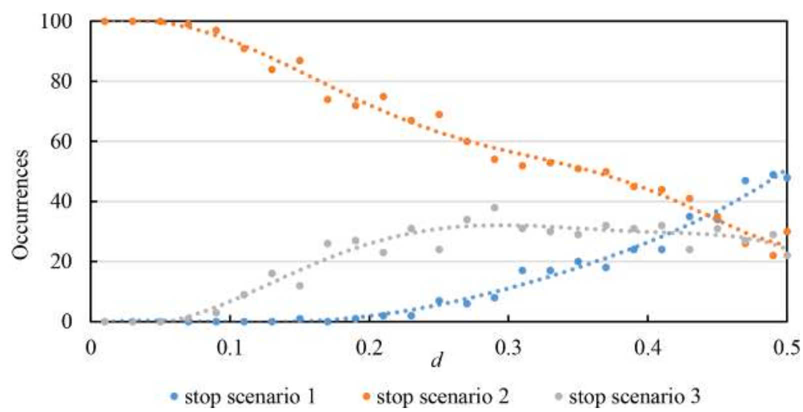


Fig. 14. Occurrences of three stop scenarios.

the cascading process stops, which is the so-called stop scenarios. Besides, the case that all components fail only occurs in Fig. 6(d), denoting that stop scenario 3 only happen in this configuration. Therefore, only the initial disturbance d is changed to conduct the investigation in this section.

Suppose $n = 100$, $r^* = 0.8$, $l_f = 0.005$, $l_o = 0.001$, and we perform 100 numerical calculations respectively for each d from 0.005 to 0.5 and examine how many times every stop scenario occurs. According to classification of stop scenarios in Section 2.2.2, there are three kinds of stop scenarios that may occur when the cascading process stops for each d. The dots in the results plotted in Fig. 14 show the occurrences times for three stop scenarios per 100 calculations for each d, denoting their possibilities of occurrences for each d. The sum of occurrences times of three stop scenarios is thus 100 for each d. The findings could be briefly summarized as follows: The cascading process of the system basically stops according to the stop scenario 2 if there is no sufficient initial disturbance. As the initial disturbance is larger, stop scenarios 1 and 3 are more likely to appear. More specifically, if the initial disturbance is small, the system is generally still working and there exist some overloading components when the cascading process terminates. When the cascading process stops, the possibility of the system being in one of two other stop scenarios grows as the initial disturbance increases: the system fails (stop scenario 1), or the system is running with all the remaining components working normally (stop scenario 3).

The difference between stop scenarios 1 and 3 is that the mounting trend of the occurrences of stop scenario 3 emerges earlier than that of stop scenario 1, which indicates that stop scenario 1 occurs with a larger initial disturbance. The occurrence times of stop scenario 1 ascends at a gradually increasing rate, while the occurrence times of stop scenario 3 initially rises rapidly, then tends to stabilize, and even shows a slight downward trend at the end of the trendline. Since the system stops running only when the stop scenario 1 occurs, the trendline of stop scenario 1 also reflects the failure probability variation of the system.

6. Conclusion remarks and future works

In this paper, we have developed a novel probabilistic model, multistate CASCADE, with the extended quasi-multinomial distribution, for loading dependent systems with CAFs where the cascading process could be affected by overloading components. Three cascading process stop scenarios are identified and interpreted. The contribution of this work lies in the involvement of overloading components and degradation of components, extending the existing studies. The results of the practical case indicate that the performance of components and the system would be overestimated if we neglect components degradation and the influence of overloading components. The proposed model can provide a more accurate characterization of the cascading process of the multistate loading dependent systems. Consequently, we can help maintenance crew and managers to make more reasonable maintenance policies. The more precise information regarding the performance of components and the system serves as the backbone to improve the decision-making process when people consider maintenance optimization for a loading dependent system with CAFs. For example, the interval between maintenance activities can be shortened to ensure that proper maintenance actions are performed on time, or that overloading components can be also considered when taking maintenance actions.

In addition, numerical examples are given to illustrate the proposed model by analyzing the influencing factors of the probability distributions of total numbers of failed and overloading components. The findings in the numerical cases have shown that the initial disturbance and loading increments affects the probability distributions. More failures may occur as the initial disturbance and loading increments increase, but the maximum values of probability distributions decrease. A novel finding is that the overloading threshold affects the probability distribution range of number of overloading components rather not the failed components. For stop scenarios of cascading process, system always operates when there are still normally working and overloading components (stop scenario 2) if the initial disturbance is quite small. As the initial disturbance increases, the cascading process tends to stop in scenarios 1 and 3.

The proposed model will encounter some issues which may be worth to investigate in the future. Firstly, since our proposed model is still limited in the multi-component system in simple configuration, further investigations on multi-state CASCADE model for *k*-out-of-*n* system and engineering application are stimulated. Secondly, it may demonstrate the necessity and practical significance of the model more intuitively to apply a practical example with maintenance activities included. Thirdly, a comparison with other models, such as modeling the situation of three states and a finite number of components by a Markov chain with transition probabilities, is suggested in our future work.

CRediT authorship contribution statement

Yixin Zhao: Writing – review & editing, Writing – original draft, Methodology, Data curation, Conceptualization. **Baoping Cai:** Writing – review & editing, Validation, Methodology. **Henry Hooi-Siang Kang:** Writing – review & editing. **Yiliu Liu:** Writing – review & editing, Supervision, Project administration, Funding acquisition, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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