



Dispersion of Solute in Casson Fluid through a Stenosed Artery with the Effect of Body Acceleration

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ABSTRACT

Body acceleration and slip velocity effects towards the dispersion of solute in Casson blood flow through a stenosed artery is investigated mathematically. Momentum and constitutive equations are solved analytically to obtain the blood velocity. Convective-diffusion equation is solved using Generalized Dispersion Model to obtain dispersion function and mean concentration of solute. A study has been conducted on how body acceleration and slip velocity disturb the dispersion of a solute in blood flow. With the increase of slip velocity and body acceleration, blood velocity increases. The impact of body acceleration on blood flow is to increase flow rate while lowering resistance to flow. Casson fluid is a suitable fluid model to examine the blood velocity and drug transport to the targeted problematic region through a narrow artery for the treatment of arterial diseases. The findings of the present study can be beneficial for pharmaceutical research to design better drug by referring mathematical analysis data that produced in this study.

1. Introduction

Stenosis of an artery is the narrowing of the area of blood supply in the artery by forming arteriosclerosis plaques due to fat deposits, cholesterol, and so on the inner wall of the artery. Healthy eating, exercise, cigarette smoke reduction, and alcohol intake management may all help to prevent CAD and CVD. It may also be used to address risk factors including high blood pressure, high cholesterol, and diabetes stated by Thomas *et al.*, [1]. The slip velocity is the difference between the velocity of the air transmitted and that of the conveyed particles. The slip velocity can be used to calculate the blood velocity pipe geometry at the wall of the artery when a slightly higher blood velocity is chosen for the practical transfer of blood and solvent across an artery at a given rate. Nagarani and Sarojamma [2] stated that the study of blood flow in arteries with body acceleration is important in the diagnosis and treatment of diseases. In narrow arteries with a single mild axisymmetric stenosis under body acceleration. The shear augmented dispersion of solute in blood flow through circular pipe and channel between two parallel flat plates with the effect of chemical reaction is investigated by Jaafar *et al.*, [3], treating the blood as non-Newtonian of Casson fluid

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model. Das *et al.*, [4] using Casson model to describe the dispersion of a solute in the flow of blood through a restricted artery with an absorptive wall, which is relevant to arterial pharmacokinetics. Rana and Murthy *et al.*, [5] studied the transport of a solute in an unsteady blood flow in small arteries with and without absorption at the wall using Casson fluid model was suitable for blood flow in small vessels. The unsteady blood flow in the artery is observed from the systemic functioning of the heart and body acceleration to a pulsatile pressure gradient that rises. Lee [6] studied numerically the effects of steady flow through double similar symmetrical bell-shaped equivalent constrictions with percentage of restrictions of 33.3, 50 and 66.67 percent in tube for the Reynolds number in the range of 5-400, where the dimensionless restriction spacing is set as 1.0. Mandal *et al.*, [7] investigated the impact of various bell-shaped stenosis structures on blood flow characteristics via the stenosis artery by considering various shapes at the inlet and outlet of the stenosis configuration. Debnath *et al.*, [8] examined the effect of heterogeneous chemical reaction on the transport of a solute in a Casson fluid flow through an annular pipe under a periodic pressure gradient.

The research aims to study the dispersion of solute in Casson fluid with the effect of slip velocity and body acceleration through a bell-shaped stenosed artery. Objectives of this research are to formulate the mathematical model of Casson fluid model in a circular straight pipe and to solve momentum and continuity equation to obtain velocity of Casson fluid model. Unsteady convective-diffusion is solved to obtain the steady dispersion function using Generalized Dispersion Model (GDM).

2. Mathematical Formulation

The governing equations of equation motion, namely the momentum, constitutive and convective-diffusion equations are discussed.

2.1 Governing Equations

The momentum equation with body acceleration for steady flow is defined as Sankar and Lee [9]

$$\frac{1}{\bar{r}} \frac{d}{d\bar{r}} (\bar{r}\bar{\tau}) = \bar{F}(\bar{t}) - \frac{d\bar{p}}{d\bar{z}}, \quad (1)$$

where $\bar{\tau}$ is the shear stress, \bar{p} is the pressure, \bar{z} is the axial coordinate for a circular pipe, \bar{r} is the radial coordinate and

$$\bar{F}(\bar{t}) = a_0 \cos(\omega_b \bar{t} + \phi_1) \quad (2)$$

is the acceleration of the body, a_0 denotes the constant parameters of the pressure gradient, $\omega_b = 2\pi\bar{f}_b$ is the circular frequency, \bar{f}_b is the dispersion coefficient and \bar{t} is the time and ϕ_1 is the lead angle of $\bar{F}(\bar{t})$ which relates to the heartbeat movement. The circular frequency is expected to be smallest, thus ignore the wave effect stated by Chaturani and Palanisamy [10]. The boundary condition of momentum equation Eq. **Error! Reference source not found.** is given as follows:

$$\bar{\tau} = \text{finite at } \bar{r} = 0 \quad (3)$$

The constitutive equation of Casson fluid is given by

$$-\frac{d\bar{u}}{d\bar{r}} = \begin{cases} \frac{1}{\mu}(\sqrt{\bar{\tau}} - \sqrt{\bar{\tau}_y})^2 & \text{if } \bar{\tau} > \bar{\tau}_y, \\ 0 & \text{if } \bar{\tau} \leq \bar{\tau}_y, \end{cases} \quad (4)$$

where \bar{u} is the velocity of Casson fluid, μ is viscosity coefficient of Casson fluid model and $\bar{\tau}_y$ is yield stress. For the unknown velocity \bar{u} , with slip condition at the wall of the circular pipe and thus the slip boundary condition given by Sankar *et al.*, [11] is as follows

$$\bar{u} = \bar{u}_s \text{ at } \bar{r} = \bar{R}(\bar{z}), \quad (5)$$

where

$$\bar{R}(\bar{z}) = \bar{R}_0 \left(1 - \frac{\delta}{\bar{R}_0} \exp\left(-\frac{\bar{k}^2 \bar{\varepsilon}^2 \bar{z}^2}{\bar{R}_0^2}\right) \right), \quad (7)$$

where boundary conditions of convective diffusion at the circular pipe center, $\bar{r} = 0$ is

$$\frac{\partial \bar{C}}{\partial \bar{r}}(0, \bar{z}, \bar{t}) = 0 \quad (8)$$

and at the wall,

$$\bar{r} = \bar{R}(\bar{z}) \text{ is } \frac{\partial \bar{C}}{\partial \bar{r}}(\bar{R}(\bar{z}), \bar{z}, \bar{t}) = 0 \quad (9)$$

2.2 Non-Dimensional Variables

The following is the non-dimensional variables

$$\begin{aligned} C &= \frac{\bar{C}}{\bar{C}_0}, u = \frac{\bar{u}}{u_0}, u_s = \frac{\bar{u}_s}{u_0}, u_m = \frac{\bar{u}_m}{u_0}, r = \frac{\bar{r}}{\bar{R}_0}, r_p = \frac{\bar{r}_p}{\bar{R}_0}, P = \frac{-d\bar{p}}{d\bar{z}}, z = \frac{\bar{D}_m \bar{z}}{R_0^2 u_0}, \\ z_s &= \frac{\bar{D}_m \bar{z}_s}{\bar{a}^2 \bar{u}_0}, t = \frac{\bar{D}_m \bar{t}}{\bar{a}^2}, \tau = \frac{\bar{r}}{2} \left(\bar{F}(\bar{t}) - \frac{d\bar{p}}{d\bar{z}} \right), \tau_y = \frac{\bar{r}_p}{2} \left(\bar{F}(\bar{t}) - \frac{d\bar{p}}{d\bar{z}} \right), \\ R(z) &= \frac{\bar{R}(z)}{R_0}, \omega_b = \frac{\bar{\omega}_b}{\omega}, t = \omega \bar{t}, A_r = \frac{a_0}{A_0}, F(t) = \frac{\bar{F}(\bar{t})}{A_r}, \end{aligned} \quad (10)$$

where u_0 is the fluid characteristic velocity, $C, u, u_s, u_m, r, r_p, P, z, z_s, t, \tau, R(z), \omega_b, A_r, F(t)$ and τ_y are the solute concentration, velocity, slip velocity, plug core radius, axial distance, solute length, time, shear stress, and yield stress in non-dimensional forms, respectively.

2.3 Method of Solution

Generalized Dispersion Model (GDM) is a derivative series expansion the approach of Gill and Sankarasubramanian [12] which is given by GDM for $C_m(z_1, t)$ as

$$\frac{\partial C_m}{\partial t}(z_1, t) = \sum_{i=1}^{\infty} K_i(t) \frac{\partial^i C_m}{\partial z_1^i}(z_1, t). \quad (1)$$

The velocity expression in the outer non-plug core region is indicated as

$$\bar{u}(r) = \bar{u}_s + \frac{1}{4\mu} \left(\bar{F}(\bar{t}) - \frac{d\bar{p}}{d\bar{z}} \right) \left[\bar{R}^2(\bar{z}) - \bar{r}^2 + 2(\bar{R}(\bar{z}) - \bar{r})\bar{r}_p - \frac{8}{3}\sqrt{\bar{r}_p}(\bar{R}^{3/2}(\bar{z}) - \bar{r}^{3/2}) \right], \quad (2)$$

where $d\bar{p}/d\bar{z}$ is the axial pressure gradient. Evaluating $\bar{r} = \bar{r}_p$ in the Eq. (2) the velocity of fluid in the plug flow region is obtained as follows

$$\bar{u}(\bar{r}_p) = \bar{u}_s + \frac{1}{4\mu} \left(\bar{F}(\bar{t}) - \frac{d\bar{p}}{d\bar{z}} \right) \left[\bar{R}^2(\bar{z}) - \bar{r}_p^2 + 2(\bar{R}(\bar{z}) - \bar{r}_p)\bar{r}_p - \frac{8}{3}\sqrt{\bar{r}_p}(\bar{R}^{3/2}(\bar{z}) - \bar{r}_p^{3/2}) \right]. \quad (3)$$

GDM is applied in convective-diffusion equation to obtain dispersion function longitudinal diffusion coefficient and mean concentration. The mean velocity is given by

$$\bar{u}_m = \int_0^{2\pi} \int_0^{\bar{R}(z)} \frac{\bar{u}r d\bar{r} d\bar{\theta}}{\bar{r} d\bar{r} d\bar{\theta}}. \quad (4)$$

The dispersion function $f_1(r, t)$ plays an important role in calculating the deviation of the mean concentration $C_m(z_1, t)$. The whole process of dispersion is

$$f_1(r, t) = f_{1s}(r) + f_{1t}(r, t), \quad (5)$$

where $f_{1s}(r)$ is the dispersion function in the steady state and $f_{1t}(r, t)$ is the dispersion function in the unsteady state that describes the time dependent nature of the dispersion of the solute. In this study, the solution of steady dispersion function is crucial to observe the solute dispersion behaviour. The dispersion of solute which is low at the center and high when closes to the wall give improved consequences in the medicine since the solute can disperse to the artery wall quicker and efficiently. Therefore, the steady dispersion function $f_{1s}(r)$ is given as follows

$$f_{1s}(r) = f_{1s+}(r) = -\frac{1}{12}r_p^3 + \frac{1}{12}BF(t)r_p^3 - \frac{1}{672}\frac{r_p^6}{R^2(z)} + \frac{1}{672}\frac{r_p^6}{R^3(z)} + \frac{1}{48}\frac{r_p^4}{R(z)} - \frac{BF(t)r_p^4}{48R(z)} + \frac{2}{21}r_p^{5/2}R^{1/2}(z) - \frac{2}{21}BF(t)r_p^{5/2}R^{1/2}(z) - \frac{1}{32}r_p^2R(z) + \frac{1}{32}BF(t)r_p^2R(z) + Cl, \text{ if } r_p \leq r \leq 1, \quad (6)$$

$$f_{1s}(r) = f_{1s-}(r) = -\frac{1}{12}r^2r_p - \frac{1}{672}\frac{r^2r_p^4}{R^3(z)} + \frac{1}{64}\frac{r^4}{R(z)} - \frac{1}{32}r^2R(z) - \frac{8}{147}\frac{r^{7/2}r_p^{1/2}}{R(z)} + \frac{4}{21}r_p^{1/2}R^{1/2}(z)\log(r_p) - \frac{1}{16}R(z)\log(r_p) - \frac{r_p^4\log(r_p)}{336R^3(z)} + \frac{\log(r_p)}{16R(z)} - \frac{4r_p^{1/2}\log(r_p)}{21R(z)} + \frac{1}{16}R(z)\log(r) - \frac{r_p\log(r)}{6R(z)} - \frac{1}{6}r_p\log(r_p) - \frac{\log(r)}{16R(z)} + \frac{r_p\log(r_p)}{6R(z)} + \frac{4r_p^{1/2}\log(r)}{21R(z)} + \frac{r_p^4\log(r)}{336R^3(z)} + \frac{1}{6}r_p\log(r) + \frac{2}{21}r^2r_p^{1/2}R^{1/2}(z) + \frac{1}{18}\frac{r^3r_p}{R(z)} + \frac{115r_p^4}{28224R(z)} - \frac{4}{21}r_p^{1/2}R^{1/2}(z)\log(r) +$$

$$\begin{aligned}
 BF(t) & \left(\frac{1}{12} r^2 r_p + \frac{1}{672} \frac{r^2 r_p^4}{R^3(z)} - \frac{1}{64} r^4 + \frac{8}{147} \frac{r^{7/2} r_p^{1/2}}{R(z)} - \frac{1}{18} \frac{r^3 r_p}{R(z)} - \frac{115 r_p^4}{28224 R(z)} \right. \\
 & - \frac{2}{21} r^2 r_p^{1/2} R^{1/2}(z) + \frac{1}{32} r^2 R(z) \frac{1}{6} r_p \log(r) - \frac{r_p^4 \log(r)}{336 R^3(z)} + \frac{\log(r)}{16 R(z)} - \\
 & \frac{4 r_p^{1/2} \log(r)}{21 R(z)} + \frac{r_p \log(r)}{6 R(z)} + \frac{4}{21} r_p^{1/2} R^{1/2}(z) \log(r) - \frac{1}{16} R(z) \log(r) + \frac{1}{6} r_p \log(r_p) + \\
 & \frac{r_p^4 \log(r_p)}{336 R^3(z)} - \frac{\log(r_p)}{16 R(z)} + \frac{4 r_p^{1/2} \log(r_p)}{21 R(z)} - \frac{r_p \log(r_p)}{6 R(z)} - \frac{4}{21} r_p^{1/2} R^{1/2}(z) \log(r_p) + \\
 & \left. \frac{1}{16} R(z) \log(r_p) \right) + CI, \text{ if } 0 \leq r \leq r_p,
 \end{aligned} \tag{7}$$

where CI is as follows:

$$\begin{aligned}
 CI = BF(t) & \left\{ -\frac{1}{12} r_p + \frac{r_p^6}{672 R^5(z)} - \frac{r_p^2}{32 R^3(z)} + \frac{2 r_p^{5/2}}{21 R^3(z)} - \frac{r_p^3}{12 R^3(z)} - \frac{r_p^4}{672 R^3(z)} - \right. \\
 & \frac{r_p^6}{770 R^3(z)} + \frac{r_p^3}{12 R^2(z)} - \frac{2 r_p^{5/2}}{21 R^{3/2}(z)} + \frac{1}{32 R(z)} - \frac{2 r_p^{1/2}}{21 R(z)} + \frac{r_p}{12 R(z)} + \frac{r_p^2}{32 R(z)} + \\
 & \frac{47 r_p^4}{14112 R(z)} + \frac{2}{21} r_p^{1/2} R^{1/2}(z) - \frac{R(z)}{32} - \frac{7}{360} r_p R^2(z) + \frac{15}{539} r_p^{1/2} R^{5/2}(z) - \frac{1}{96} R^3(z) \\
 & - \frac{1}{6} r_p \log(r_p) - \frac{r_p^4 \log(r_p)}{336 R^3(z)} + \frac{\log(r_p)}{16 R(z)} - \frac{4 r_p^{1/2} \log(r_p)}{21 R(z)} + \frac{r_p \log(r_p)}{6 R(z)} + \frac{4}{21} r_p^{1/2} R^{1/2}(z) \\
 & \log(r_p) - \frac{1}{16} R(z) \log(r_p) + \frac{1}{6} r_p \log[R(z)] + \frac{r_p^4 \log[R(z)]}{336 R^3(z)} - \frac{\log[R(z)]}{16 R(z)} + \frac{4 r_p^{1/2} \log[R(z)]}{21 R(z)} \\
 & \left. - \frac{r_p \log[R(z)]}{6 R(z)} - \frac{4}{21} r_p^{1/2} R^{1/2}(z) \log[R(z)] + \frac{1}{16} R(z) \log[R(z)] \right\}.
 \end{aligned} \tag{8}$$

3. Results and Discussions

3.1 Velocity and Mean Velocity of the Blood Flow

The effect of stenosis, slip velocity, and body acceleration on velocity is graphically computed in this section. The results of velocity and mean velocity are obtained and discussed by fixing various parameters in the flow analytic expression after solving the momentum equation and defining the yield stress.

Figure 1 shows variation of velocity, u for different values of yield stress, τ_y in the blood flow with $A_0 = 2, \omega = 1, t = 1, \phi_1 = 0, P = 2, a = 1, b = 2.3, z = 3$ and $B = 2.5$. The existence of a stenosis inhibits blood flow in the narrow artery, resulting in yield stress changes. The increment of yield stress 0.1 to 0.5, the velocity decreases. As yield stress rises, the amplitude of velocity falls, allowing the plug flow to emerge. Yield stress is significant in viscoelasticity modification because it works as a path for human blood to circulate.

Figure 2 shows variation of velocity, u for different values body acceleration, A_0 with $\omega = 1, t = 1, \phi_1 = 0, \tau_y = 0.1, P = 2, a = 1, b = 2.3, z = 3$ and $B = 2.5$. The velocity increases when the body acceleration increases as the cross-sectional area narrows. The impact of body acceleration on blood flow is to increase flow rate while lowering artery resistance to flow. The body's acceleration has an effect on the average velocity and flow rate, lowering the body's acceleration. The body acceleration parameter has been discovered to play a significant role in blood flow, causing not only quantitative but also qualitative alterations in velocity profiles.

From Figure 3 illustrates variation of velocity, u for different values of slip velocity, u_s with $A_0 = 0, \omega = 1, t = 1, \phi_1 = 0, \tau_y = 0.1, P = 2, a = 1, b = 2.3, z = 3$ and $B = 2.5$. When slip velocity increases, axial velocity increases as well. Increasing of slip velocity lead to decreasing in flow resistance. Slip velocity plays a crucial role in blood flow modelling in a stenosed artery, according to Casson model. It's also possible to deduce that with slip, vessel wall damage could be reduced.

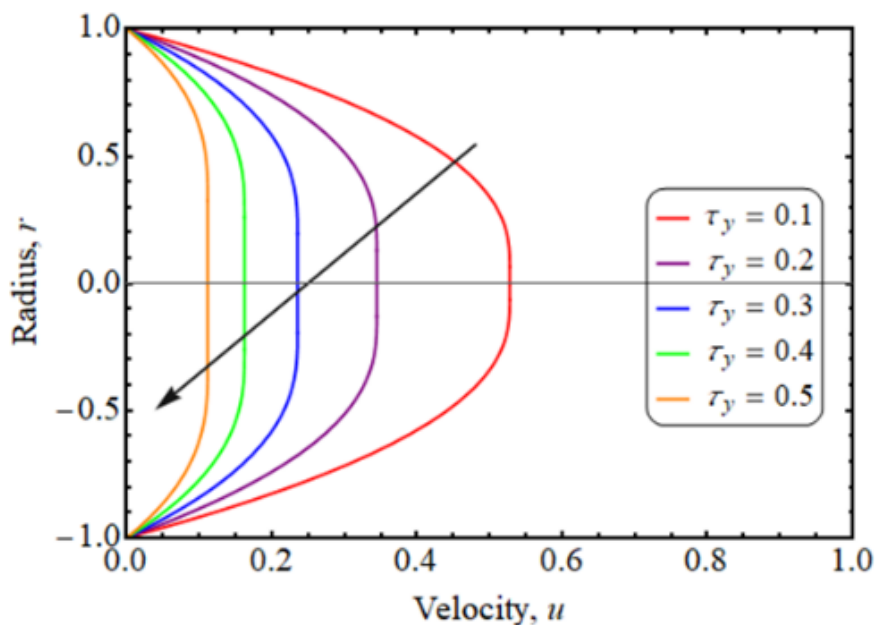


Fig. 1. Variation of velocity, u for different for values of yield stress, τ_y in the blood flow with body acceleration, $A_0 = 2, \omega = 1, \phi_1 = 0, t = 1, P = 2, a = 1, b = 2.3, z = 3$ and $B = 2.5$

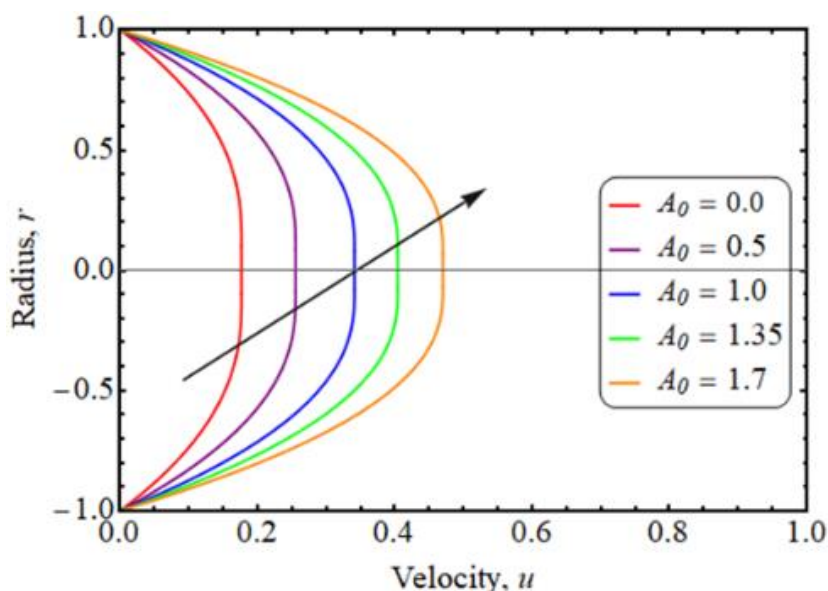


Fig. 2. Variation of velocity, u for different values of body acceleration, A_0 in the blood flow with $\omega = 1, t = 1, \phi_1 = 0, \tau_y = 0.1, P = 2, a = 1, b = 2.3, z = 5$ and $B = 2.5$.

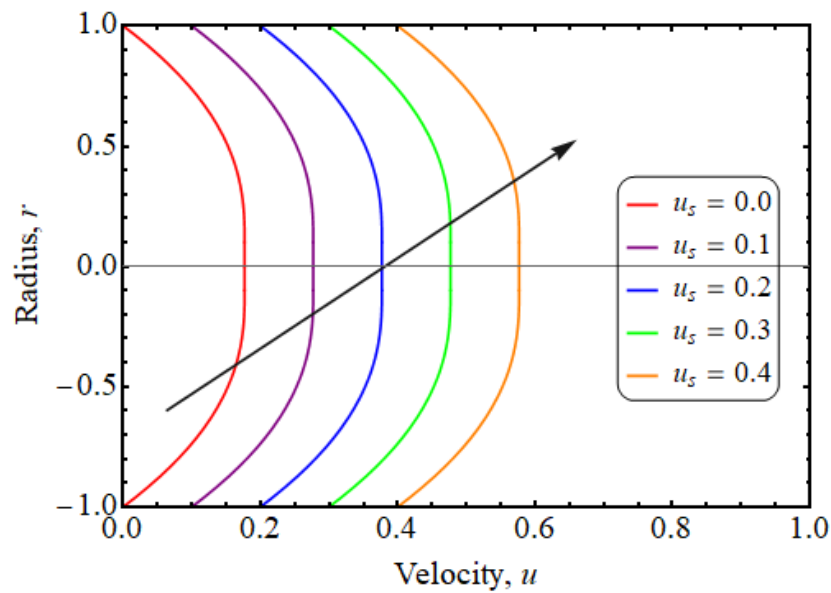


Fig. 3. Variation of velocity, u for different values of slip velocity, u_s in the blood flow with $A_0 = 0, \omega = 1, t = 1, \phi_1 = 0, \tau_y = 0.1, P = 2, a = 1, b = 2.3, z = 3$ and $B = 2.5$.

3.2 Steady Dispersion Function

The purpose of analysing the steady dispersion function in this study is to observe the changes of the steady dispersion function when the input of yield stress and body acceleration increases.

Figure 4 shows the variation of steady dispersion function, f_{1s} for different values of yield stress, τ_y in the blood flow with $A_0 = 0.5, \omega = 1, t = 2.8, \phi_1 = 0.02, P = 1, a = 0.01, b = 0$ and $z = 0.05$. When yield stress increases, the steady dispersion function decreases near the wall and opposite behaviour at the center of artery. Casson fluids are non-Newtonian fluids with a yield stress, making them ideal for narrow arteries. It is due to the flow rate of Casson fluids decreases as the viscosity of the fluid increases, and the value of the dispersion function decreases.

Figure 5 shows variation of steady dispersion function, f_{1s} for different values of body acceleration, A_0 in the blood flow with $\omega = 1, t = 2.8, \phi_1 = 0.01, P = 1, \tau_y = 0.01, a = 0.01, b = 0$ and $z = 0.05$. The dispersion function increases as the amplitude of body acceleration increases at the center of artery and decreases in the outer region near the wall. As the amplitude of body acceleration increases, the blood flow drops, causing the dispersion function to decrease and the solute dispersion to be impacted by the fluctuating blood flow. With the increase of body acceleration, the amplitude increases, lowering the value of relative axial diffusivity in the blood flow.

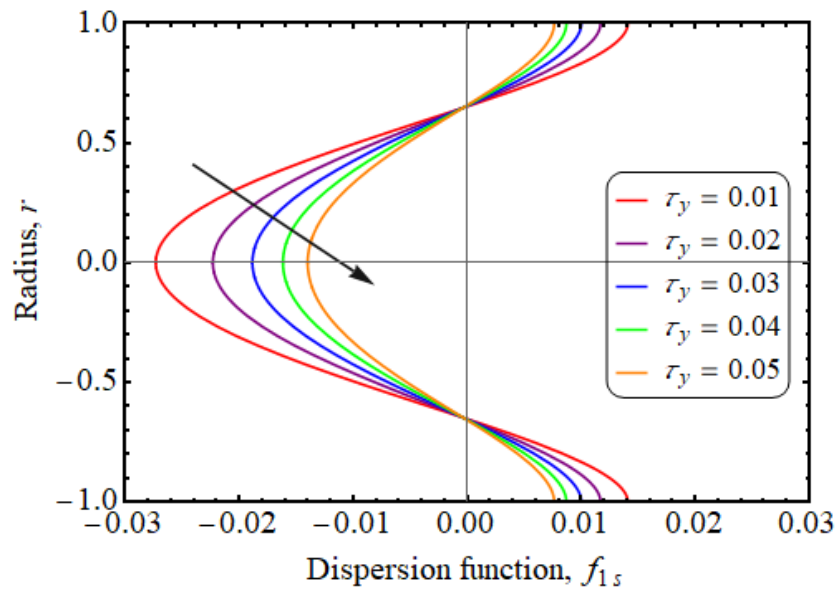


Fig. 4. Variation of steady dispersion function, f_{1s} for different values of yield stress, τ_y in the blood flow with $\omega = 1$, with $A_0 = 0.5, t = 2.8, \phi_1 = 0.02, P = 1, a = 0.01, b = 0$ and $z = 0.05$.

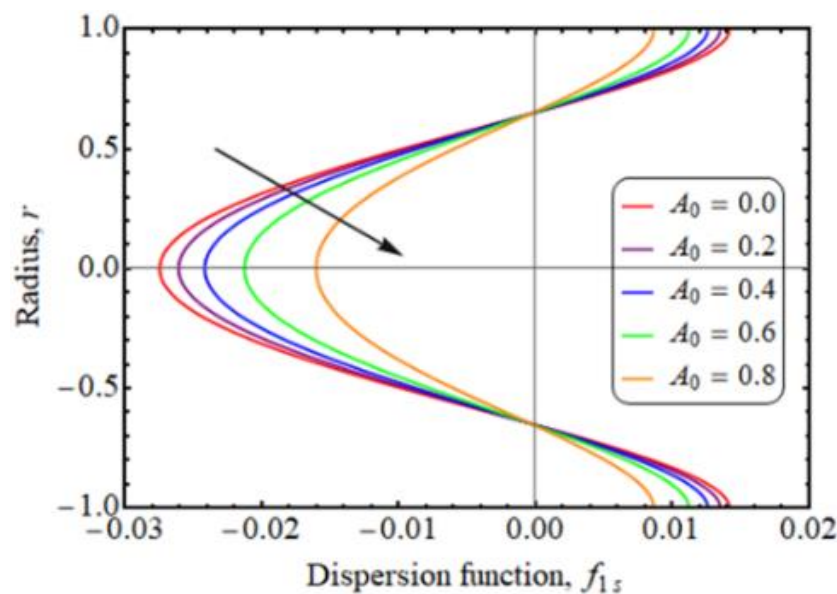


Fig. 5. Variation of steady dispersion function, f_{1s} for different values of body acceleration, A_0 in the blood flow with $\omega = 1, t = 2.8, \phi_1 = 0.01, \tau_y = 0.01, a = 0.01, b = 0, z = 0.05$ and $P = 1$

4. Conclusions

With the increase of body acceleration, the velocity increases as the cross-sectional area narrows. The impact of body acceleration on blood flow is to increase flow rate while lowering artery resistance to flow. The dispersion function increases, body acceleration increases at the center of artery and decreases in the outer region near the wall. The unsteady dispersion function grows as body acceleration rises. The value of the unstable dispersion function decreases when the radius enhances. It is noticed that the dispersion function increases with body acceleration increase and it approaches zero when the value of body acceleration keep increasing. As the height of the stenosis

increases the velocity of the stenosis reduces then the artery becomes extremely narrow, then it is the reason the velocity decreases. Limitation in this research is the dispersion of solute in blood flow is solely investigated theoretically, not empirically. Another limitation of the study is no adequate experimental data is available and the results of solute dispersion cannot be compared to genuine data from earlier literature experiments. It is hoped that this study would help researchers to have better understanding about the behaviour of blood flow and dispersion of solute in blood flow, as well as provide insight into the issue of cardiovascular disease. The current findings are helpful in addressing the issue of dispersion in the cardiovascular system. This study helps doctors diagnose and treat cardiovascular disorders by observing blood flow characteristics. In future, to research can be extended to two Casson fluid model. It should also be noticed that the two-fluid blood flow model's velocity and flow rate are significantly higher than the single-fluid blood flow model's. Future to research also can be conducted on the experimental side on the real situation if the dispersion and diffusion of blood flow in arteries are known.

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