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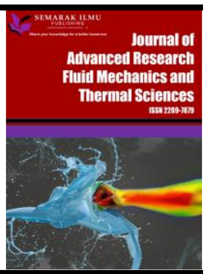

Mathematical Modeling on Magnetohydrodynamics Upper Convected Maxwell Fluid Flow Past a Flat Plate Using Spectral Relaxation Approach

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ABSTRACT

The focus of this treatise is on the scrutiny of MHD heat and mass transfer motion of upper convection Maxwell (UCM) fluid in a thermally radiated flat plate. The models of the flow equations were considered when there is a temperature difference: the Cattaneo-Christov model and the Soret-Dufour mechanisms. A magnetic field of firmness was inflicted in opposition to the flow. The flow inspection is controlled by partial differential equations (PDEs). Suitable similarity variables were utilised on the PDEs to obtain a set of nonlinear ordinary differential equations (ODEs). The simplified set of ODEs shall be answered by exploiting the spectral relaxation method (SRM). The SRM is a numerical technique that solves differential equations by utilising the repetition of a sequence of operations that is iterated iteratively by first decoupling the coupled systems of equations. The magnetism was found to decline the velocity and hydrodynamic boundary layer due to the Lorentz force. The Deborah number was found to enhance the velocity contour. The Eckert number was discovered to improve the temperature profile due to the production of heat energy within the boundary layer. An increase in Prandtl number was found to enhance the hydrodynamic and thermal boundary layer thickness. The local skin friction and Nusselt number were found to be elevated by the increase in the Dufour parameter.

1. Introduction

Magnetohydrodynamics (MHD) is acclaimed for explaining the behaviour of fluids that conduct electricity. This type of liquid is plasma, which has an impact due to its magnetism. A physical consequence occurs in magnetism. However, an instigated electric current along with magnetism is produced. In current generation, because of the transportation of an electrically conducting liquid due to magnetism, a resistive force reacts on the fluid and lowers its progression. MHD has gained much significance in industries and applications of the latest technology, such as crystal growth, reactor cooling, coating of metals, MHD accelerators, power generators, and electromagnetic pumps. Many scholars discussed based on this paper analysed MHD in non-Newtonian or Newtonian terms,

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specifically heat and mass transport. MHD liquid upward oscillatory flow in a permeable plate with radiation immersion has been resolved by Arifuzzaman *et al.,* [1]. Fagbade *et al.,* [2] resolved the MHD progression of viscoelastic liquid within an accelerating passable facet. Idowu and Falodun [3] resolved SRM on MHD heat in company with mass transport of Walters-B liquid. The resolved work of Reza-E-Rabbi *et al.,* [4] explained chemically the MHD unsteadiness movement of a Casson fluid along with thermophoresis influence.

The inquiry of boundary layer non-Newtonian flow past an upward or flat plate has acquired much awareness previously because of its usefulness in engineering and the sciences. Non-Newtonian liquids are recognised by their complex equations. Due to the nonlinearity and complexity of these equations, many scholars have presented numerical strategies to solve them previously. Examples of non-Newtonian fluids are micropolar fluids, Maxwell liquids, Walters-B fluids, Casson liquids, etc. Maxwell fluid is an engrossing non-Newtonian model that is applicable in biosystems, industrial and chemical processes, biomedical engineering, and food processing. Jena *et al.,* [5] observed the significance of chemical reactions on MHD viscoelastic liquid flow moving in a vertical stretchable sheet. The latest analysis of Idowu and Falodun [6] resolved the concurrent flow of double-diffusive flow by diversified viscosity along with thermal conductivity.

The dissipative fluid in a laminar-force convection channel was studied by Tyagi [7]. By neglecting the axial conduction effect, Basu and Roy [8] explored laminar heat transport with viscous dissipation in a tube. Desale and Pradhan [9] numerically solve the flow equations by taking into consideration the influence of viscous depletion via a horizontal plate with a fluctuating temperature. Oyelami and Dada [10] resolved the consequence of viscous depletion on heat transmission in a permeable channel with natural convection. Bhatti and Abdelsalam [11] resolved the effects of magnetic fields and radiation on the bioinspired drive of hybrid nanofluids. Abo-Elkhair *et al.,* [12] explored the peristaltic movement of a bionanofluid using an adequate Reynolds number. The Soret-Dufour mechanism for conducting nanofluid flow via a half-infinite porous medium with chemical reaction and buoyancy effect was researched by Falodun and Ayegbusi [13]. For boundary layer applications, it was thought that the Reynolds number was sufficiently low to ignore the induced magnetic field.

Likewise, there is no magnetic field opposition to the transverse flow. The Soret-Dufour effects can be disregarded because the process is chemically based on first order, and for the concentration component, they are thought to be minimal. The consequences of viscous depletion are thought to be large. Using the spectral relaxation method, Alao *et al.,* [14] scrutinised heat along with mass transfer in a chemically reacting liquid. A degenerated velocity along with the concentration of the fluid is detected for the growth of a chemical reaction. The sufficient descent condition, along with the conjugate gradient method, was used by Malik *et al.,* [15] to derive a new coefficient. Abas & Yatim [16] examined a travelling-wave similarity solution for a non-Newtonian fluid rivulet propelled by gravity. Malik *et al.,* [17] studied the new coefficient conjugate gradient method by examining convergence analysis. Hayat *et al.,* [18] narrated the chemically reactive flow of a double-layered Powel-Eying liquid. Their research indicated that the flux caused by Fourier's formulations is attributable to the lack of a thermal factor. Mondal *et al.,* [19] investigated MHD mass transfer when chemically reactive fluid was present. Oyelami and Falodun [20] used spectral relaxation to investigate the chemical reaction impact on non-Newtonian Casson fluid. Idowu & Falodun [21] investigated steady natural convection flow movement with chemical reactions. The chemical reaction was detected to reduce the concentration dispersion in the investigation. Falodun and Omowaye [22] used spectral homotopy analysis to investigate MHD double-diffusive flow with chemical reactions. Their research indicated that the rate of local heat transfer increases with higher chemical reactions.

Muhammad and Sohail [23] examined the theoretical treatment of bio-convective Maxwell nanofluid over an exponentially stretching sheet. The analysis of Cattaneo-Christov heat flux for stagnation point flow of micropolar nanofluid was investigated by Ahmad *et al.,* [24]. Ahmad *et al.,* [25] conducted a mathematical analysis of heat and mass transfer in a Maxwell fluid with double stratification. Muhammad *et al.,* [26] have investigated the transient flow of Maxwell nanofluid over a shrinking surface using numerical computations and stability analysis. A comparative study between linear and exponential stretching sheets with double stratification of a rotating Maxwell nanofluid flow was examined by Muhammad and Sohail [27]. Nadeem *et al.,* [28] studied the transportation of slip effects on nanomaterial micropolar fluid flow over exponential stretching. Asghar and Ying [29] examined three-dimensional MHD hybrid nanofluid flow with rotating stretching and shrinking sheets and Joule heating. Nadia *et al.,* [30] studied thermal radiation in nanofluid penetrable flow bounded by a partial slip condition. The effects of the imposition of viscous and thermal forces on the dynamical feature of swimming of microorganisms in nanofluids were investigated by Sharafatmandjoor & Azwadi [31]. In another study by Omar *et al.,* [32], the effect of analytical solutions on the performance of unsteady Casson fluid with thermal radiation and chemical reactions was investigated.

The treatise of Oyelami and Falodun [20], on heat in company with mass transfer of hydrodynamic flow towards a horizontal plate, together with the consequence of altered temperature along with viscous depletion, was a Newtonian fluid together with a variable temperature, also with viscous dissipation and chemical reaction. In their previous study, there was no Cattaneo-Christov model, and the Soret-Dufour mechanism and magnetic field strength were also neglected. The work done by Oyelami and Falodun [20] has been extended in this study to fill the missing gap. The case in this study shall focus on the MHD heat and mass transfer motion of upper convection Maxwell fluid along a flat plate with thermal radiation, variable temperature, the Cattaneo-Christov model, and the Soret-Dufour mechanism. The work done by Oyelami and Falodun [20] was on a Newtonian fluid with variable temperature, viscous dissipation, and chemical reactions. The present study hereby extends their analysis by considering a non-Newtonian fluid moving with the consequences of thermal radiation, magnetic, and Soret-Dufour dynamics. The Cattaneo-Christov theories shall be considered for the motion of upper-convected Maxwell non-Newtonian fluid flow along a flat plate.

To the best of our knowledge, no study in the literature has reported mathematical modelling on magnetohydrodynamics upper convected Maxwell fluid flow past a flat plate using the spectral relaxation method. With this fact in mind, this study explores MHD upper-convection Maxwell fluid flow past a flat plate. The present analysis is a non-Newtonian model that was modelled in the form of partial differential equations. A hybrid numerical scheme that employs the Gauss-Seldel approach has been utilised. This research work shall examine the magnetohydrodynamic heat and mass transfer on the motion of upper convection Maxwell fluid along a flat plate with thermophysical properties. Most of the aforementioned literature does not consider the motion of the upper convection along a flat plate in the presence of the Cattaneo-Christov model and Soret-Dufsour mechanisms. Hence, the motivation for this research is to extend the study of Oyelami & Falodun [20], which was on Newtonian fluid flow, to non-Newtonian upper convection Maxwell fluids flowing along a flat plate.

2. Mathematical Model

The flow of a fluid velocity U steadily towards a horizontal plate at a temperature that is consistent (uniform) T_w with concentration C_w firmly held at the surface is considered. The fluid's initial surface temperature and concentration are $T_\infty$and C_∞. The Reynolds number was thought to become low enough to ignore the created magnetic field. Additionally, the magnetic field does not oppose the transverse flow. The level of concentration was so high that Soret's and Dufour's influence could not be neglected. The viscous dissipation effect and thermal radiation are considered to be significant. The theories of Cattaneo-Christov shall be examined in terms of heat and mass flux. The physical scenario that will be modelled in this research is the movement of a non-Newtonian liquid across a horizontal plate, accompanied by constant viscosity and thermal conductivity. (see Figure 1).

The characteristics of a fluid is constant, and normally the approximation of boundary layer is valid Oyelami and Falodun [20]. The fundamental equations for the Bousenesq approximation are below

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+\lambda\left(u^2\frac{\partial^2 u}{\partial x^2}+v^2\frac{\partial^2 u}{\partial y^2}+2uv\frac{\partial^2 u}{\partial y\partial x}\right)-\frac{\sigma B_0^2(x)}{\rho}u \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}-\frac{Dk_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2-\beta_1\left[uu_xT_x+vv_yT_y+uv_xT_x+v^2\frac{\partial^2 T}{\partial y^2}+vu_yT_x+2uv\frac{\partial^2 T}{\partial y\partial x}+u^2\frac{\partial^2 T}{\partial x^2}\right]-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \quad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}-\frac{Dk_T}{T_m}\frac{\partial^2 T}{\partial y^2}=D\frac{\partial^2 C}{\partial y^2}-k_c(C-C_\infty)-\beta_2\left[uu_xC_x+vv_yC_y+uv_xC_x+v^2\frac{\partial^2 C}{\partial y^2}+v\frac{\partial u}{\partial y}\frac{\partial C}{\partial x}+2uv\frac{\partial^2 C}{\partial y\partial x}+u^2\frac{\partial^2 C}{\partial x^2}\right] \quad (4)$$

Subject to boundary conditions Oyelami and Falodun [20]

$$u=0,\ T=T_w(x),\ C=C_w(x)\ \text{ at } y=0$$

$$u=U, T=T_\infty, C=C_\infty \text{ as } y\to\infty \quad (5)$$

where u is the velocity in x axis whereas for v is the velocity (m/s) in y axis, also ν denotes fluid viscosity (m^2/s), thermal conductivity is represented by α, T and C are temperature (K) and fluid concentration (mol), for coefficient of viscosity we use the symbol μ (m^2/s), C_p is representing the main heat at uniform pressure (J/KgK), the fluid density was represented by ρ (Kg/m^3), D stand for mass diffusivity (m^2/s), k_c means chemical reaction coefficient (Kmolm^{-3}), T_∞ and C_∞ are stand for free stream temperature (K) and free stream concentration (mol), and finally we use T_w and C_w as the wall temperature (K) and wall concentration (mol) correspondingly., λ is the coefficient of Maxwell fluid, σ is electrical conductivity, B_0 is magnetic field strength, k_T is thermal diffusion ratio, c_s is concentration susceptibility, β_1, β_2 are coefficient of thermal and mass flux, q_r is radiative heat flux (W/m^2), T_m mean fluid temperature.

According to research by Fagbade *et al.,* [2], approximation Roseland is utilized to calculate the heat flux on the fluid motion.

$$q_r=-\frac{4\sigma_s}{3k_e}\frac{\partial T^4}{\partial y} \quad (6)$$

In which the Stefan-Boltzman constant=σ_s, also we use k_e for mean absorption coefficien. It is concluded that temperature variations in fluid movement are minor, and T^4 might be represented in a linear function by decomposing T^4 over T_∞ using Taylor series and forgone elements of higher order to produce

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{7}$$

Substituting (7) into (6) yields

$$-\frac{\partial q_r}{\partial y} = \frac{16\sigma_s T_\infty^3}{3k_e}\frac{\partial^2 T}{\partial y^2} \tag{8}$$

The plate's temperature at the plate is assume to be

$$T_w(x) - T_\infty = Ax^n \tag{9}$$

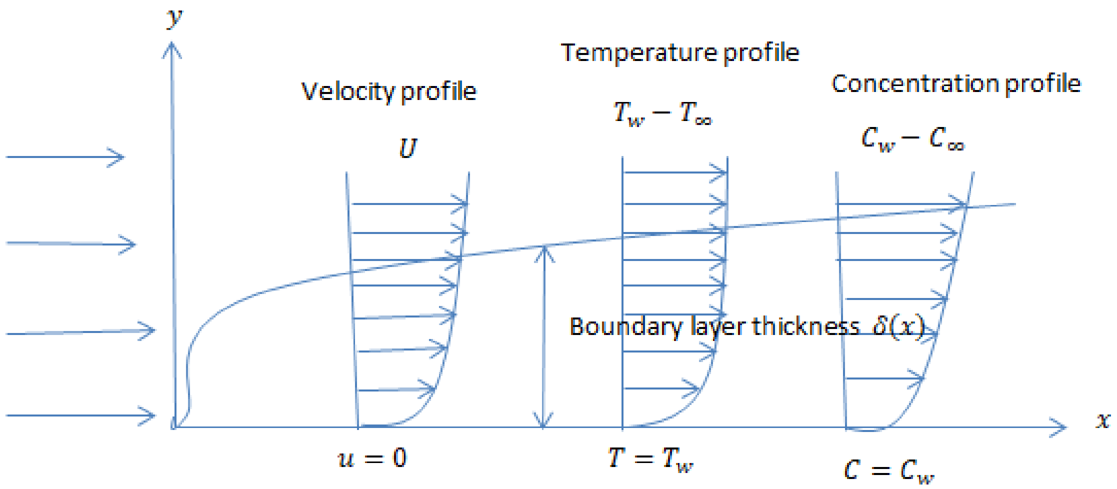

Fig. 1. Systematic diagram of Boundary layer for heat along with mass transfer

The similarity variable are given as follows Oyelami & Falodun [20,33].

$$\eta = y\sqrt{\frac{B}{\nu}}, \psi = \sqrt{B\nu} f(\eta), \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \phi(\eta) = \frac{C-C_\infty}{C_w-C_\infty} \tag{10}$$

$$u = \frac{\partial\psi}{\partial y} = Bxf' \ and \ v = -\frac{\partial\psi}{\partial x} = -\sqrt{B\nu} f(\eta) \tag{11}$$

The symbol η stands for similarity variable or distance dimensionless term, while we use ψ for dimensionless stream function and also $\theta(\eta)$ and $\phi(\eta)$ for temperature and concentration respectively.

It should be noted that prime signifies differentiation with respect to η. By utilizing the similarity transformation defined in Eq. (10) and Eq (11) on the governing equations to obtain

$$f''' + f'' - (f')^2 + \beta(2ff'f'' - f^2 f''') - Mf' = 0 \tag{12}$$

$$\left(\frac{1+R}{Pr}\right)\theta'' + f\theta' + Do\phi'' + Ec(f'')^2 - \gamma_1[ff'\theta' + f^2\theta''] = 0 \tag{13}$$

$$\phi'' + ScSo\theta'' + Scf\phi' - ScKr\phi - \gamma_2(ff'\phi' + f^2\phi'') = 0 \tag{14}$$

Subject to

$$f' = 1,\ f = f_w,\ \theta = 1,\ \phi = 1,\quad at \quad \eta = 0 \tag{15}$$

$$f' \longrightarrow 0,\ \theta \longrightarrow 0,\ \phi \longrightarrow 0,\quad as \quad \eta \longrightarrow \infty \tag{16}$$

The governing parameters in the transformed equations above are: β is Deborah number, M is Magnetic parameter, Pr is the Prandtl number, R is the Thermal radiation parameter, Do is Dufour parameter, Ec is Eckert number, γ_1 is heat flux relaxation parameter, Sc is Schmidt number, So is Soret number, Kr is chemical reaction parameter, γ_2 is mass flux relaxation parameter. The engineering quantities of interest are the local Nusselt number (Nu_x), local Sherwood number (Sh_x), and skin friction coefficient (Cf). These quantities are defined as follows

$$Cf = \frac{\tau_w}{\rho(U_w)^2},\ Nu_x = \frac{q_w}{K(T_w - T_\infty)},\ Sh_x = \frac{S_w}{D(C_w - C_\infty)},$$

where

$$\tau_w = \left[\mu\left(1 + \frac{1}{\beta}\right)(u_y)|_{y=0}\right],\ q_w = -K(T_y)|_{y=0} - 4\sigma_0(3Ke)^{-1}(T_y^4)|_{y=0},$$

3. Spectral Relaxation Approach (SRM)

The proposed method of solutions is Spectral Relaxation Method. SRM employs the procedure of Gauss-Seidel techniques of relaxation to decouple and linearize the transformed coupled differential equations simultaneously. This method employed the Chebyshev pseudo-spectral techniques to discretize the linearized set of equations. The linear tern is considered at the current iteration to be $r + 1$ with nonlinear terms considered at the previous iteration denoted by r.

The basic steps of the spectral approach are

i. firstly, use Gauss-Siedel methods to decouple and linearize the nonlinear equations;
ii. Further discretization of the linearized equations; and
iii. Chebyshev pseudo-spectral approach is used to iteratively solve the discretized equations.

$$f'''_{r+1} + f''_{r+1} - (f'_r)^2 + 2\beta f_r f'_{r+1} f''_{r+1} - 2f_r^2 f'''_{r+1} - Mf'_{r+1} = 0 \tag{17}$$

$$\left(\frac{1+R}{Pr}\right)\theta''_{r+1} + f_{r+1}\theta'_{r+1} + Do\phi'_r + Ec(f''_{r+1})^2 - \gamma_1 f_{r+1} f'_{r+1}\theta'_{r+1} - \gamma_1 f_{r+1}^2\theta''_{r+1} = 0 \tag{18}$$

$$\phi''_{r+1} + ScSo\theta''_{r+1} + Scf_{r+1}\phi'_{r+1} - ScKr\phi_{r+1} - \gamma_2 f_{r+1} f'_{r+1}\phi'_{r+1} - \gamma_2 f_{r+1}^2\phi''_{r+1} = 0 \tag{19}$$

subject to

$$f_{r+1}(0,\eta) = f_w,\quad f'_{r+1}(0,\eta) = 1, \theta_{r+1}(0,\eta) = 1,\quad \phi_{r+1}(0,\eta) = 1 \tag{20}$$

$$f'_{r+1}(\infty,\eta) \longrightarrow 0, \quad \theta_{r+1}(\infty,\eta) \longrightarrow 0, \quad \phi_{r+1}(\infty,\eta) \longrightarrow 0 \tag{21}$$

Setting

$a_{0,r} = -(f'_r)^2,\ a_{1,r} = 2\beta f_r f'_{r+1},\ a_{2,r} = -2f_r^2,\ b_{0,r} = \left(\frac{1+R}{Pr}\right),$

$b_{1,r} = f_{r+1},\ b_{2,r} = Do\phi''_r,\ b_{3,r} = Ec(f''_{r+1})^2,\ b_{4,r} - \gamma_1 f_{r+1} f'_{r+1},$

$b_{5,r} = -\gamma_1 f_{r+1}^2,\ c_{0,r} = SoSc\theta''_{r+1},\ c_{1,r} = Scf_{r+1},\ c_{2,r} = -\gamma_2 f_{r+1} f'_{r+1},\ c_{3,r} = -\gamma_2 f_{r+1}^2$

The coefficient parameters defined above is invoked into (17) - (19) to give

$$f'''_{r+1} + f''_{r+1} + a_{0,r} + a_{1,r}f''_{r+1} + a_{2,r}f'''_{r+1} - Mf'_{r+1} = 0 \tag{22}$$

$$b_{0,r}\theta''_{r+1} + b_{1,r}\theta'_{r+1} + b_{2,r} + b_{3,r} + b_{4,r}\theta'_{r+1} + b_{5,r}\theta''_{r+1} = 0 \tag{23}$$

$$\phi''_{r+1} + c_{0,r} + c_{1,r}\phi'_{r+1} - ScKr\phi_{r+1} + c_{2,r}\phi'_{r+1} + c_{3,r}\phi''_{r+1} = 0 \tag{24}$$

subject to

$$f_{r+1}(0,\eta) = f_w, \quad f'_{r+1}(0,\eta) = 1, \theta_{r+1}(0,\eta) = 1, \quad \phi_{r+1}(0,\eta) = 1 \tag{25}$$

$$f'_{r+1}(\infty,\eta) \longrightarrow 0, \quad \theta_{r+1}(\infty,\eta) \longrightarrow 0, \quad \phi_{r+1}(\infty,\eta) \longrightarrow 0 \tag{26}$$

Making use of the Gauss-Lobatto collocation defined as follows

$$\xi_j = cos\frac{\pi j}{N}, \quad j = 0,1,2,\ldots,N; \quad 1 \le \xi \le -1 \tag{27}$$

and N =collocation number. The original domain of the problem is $[0,\infty)$. This domain is first transformed into $[-1,1]$ in other to solve the linearized equations. The transformed interval is mapped by utilizing

$$\frac{\eta}{L} = \frac{\xi+1}{2}, \quad -1 \le \xi \le 1 \tag{28}$$

In Eq. (28), L connote scaling function which was utilized to implement the given constraints at infinity. Furthermore, the first approximation of unknown functions are obtained at $\eta = 0$. This is choosing to conform with the constraints (25) and (26). Therefore $f_0(0,\eta), \theta_0(0,\eta)$ and $\phi_0(0,\eta)$ are defined as

$$f_0(y,t) = f_w - e^{-\eta}, \quad \theta_0(0,\eta) = \phi_0(0,\eta) = e^{-\eta} \tag{29}$$

Starting from the initial guess defined in Eq. (29), Eq (22) – Eq. (24) are solved iteratively for all unknown functions. However, the schemes Eq. (22), Eq. (23), and Eq. (24) are iteratively solved for $\phi_{r+1}(0,\eta), f_{r+1}(0,\eta)$ and $\theta_{0,\eta}(0,\eta)$ as $r = 0,1,2$. To solve (22)-(24), we first discretized by employing the Chebyshev spectral collocation approach as well as finite difference approach. The finite difference technique is further employed with centering about an average unknown functions. Thus, utilizing the centering in functions $\phi(0,\eta), \theta(0,\eta)$ and $f(0,\eta)$ alongside their derivative leads to:

$$\left(\frac{df}{d\eta}\right)^{n+\frac{1}{2}} = \frac{f_j^{n+1}-f_j^n}{\Delta\eta},\ \left(\frac{d\theta}{d\eta}\right)^{n+\frac{1}{2}} = \frac{\theta_j^{n+1}-\theta_j^n}{\Delta\eta},\ \left(\frac{d\phi}{d\eta}\right)^{n+\frac{1}{2}} = \frac{\phi_j^{n+1}-\phi_j^n}{\Delta\eta} \tag{30}$$

The use of spectral collocation approach is by employing matrix differentiation D to simplify the unknown derivatives given as

$$\frac{d^r f}{d\eta^r} = \sum_{k=0}^{N} D_{ik}^r u(\xi_k) = D^r f,\ \frac{d^r\theta}{d\eta^r} = \sum_{k=0}^{N} D_{ik}^r \theta(\xi_k) = D^r\theta, \quad i = 0,1,\ldots N \tag{31}$$

$$\frac{d^r\phi}{d\eta^r} = \sum_{k=0}^{N} D_{ik}^r \phi(\xi_k) = D^r\phi, \quad i = 0,1,\ldots N \tag{32}$$

Using Chebyshev collocation approach on Eq. (22) – Eq. (24) and later apply the finite differences gives.

$$D^3 f_{r+1} + D^2 f_{r+1} + a_{0,r} + a_{1,r} D^2 f_{r+1} + a_{2,r} D^3 f_{r+1} - MDf_{r+1} = 0 \tag{33}$$

$$b_{0,r} D^2\theta_{r+1} + b_{1,r} D\theta_{r+1} + b_{4,r} D\theta_{r+1} + b_{5,r} D^2\theta_{r+1} + b_{2,r} + b_{3,r} = 0 \tag{34}$$

$$D^2\phi_{r+1} + c_{1,r} D\phi_{r+1} - ScKr\phi_{r+1} + c_{2,r} D\phi_{r+1} + c_{3,r} D^2\phi_{r+1} + c_{0,r} = 0 \tag{35}$$

subject to

$$f_{r+1}(0,\eta) = f_w, \quad {f'}_{r+1}(0,\eta) = 1, \theta_{r+1}(0,\eta) = 1, \quad \phi_{r+1}(0,\eta) = 1 \tag{36}$$

$$f'_{r+1}(\infty,\eta) \longrightarrow 0, \quad \theta_{r+1}(\infty,\eta) \longrightarrow 0, \quad \phi_{r+1}(\infty,\eta) \longrightarrow 0 \tag{37}$$

4. Results and Discussions

The transformed coupled nonlinear ODEs (12) and (14) along the boundary constraints (15) and (16) were resolved by using SRA. The outcome of Deborah's number (β) on the velocity contour is presented in Figure 2. The physical situation of constant viscosity shows that an increase in β raises the velocity contour. Therefore, at a lower Deborah number, the behaviour of the substances is fluidlike, but at higher elasticity, the material dominates and a solidlike behaviour is observed.

Figure 3 presents the outcome of the magnetic term (M) on the velocity contour. In Figure 3, a growth of M is detected to cause degeneration of the velocity contour. The imposition of a magnetic field on Maxwell fluid flow gives rise to the Lorentz force. The force is a drag-like force that causes degeneration of the fluid flow within the layer. Physically, for constant viscosity along with thermal conductivity, a higher M decreases the fluid velocity along with the hydrodynamic boundary layer thickness due to the Lorentz force. In the absence of the buoyancy force and the Maxwell fluid, the result shown in Figure 3 is in very good agreement with Falodun *et al.,* [23].

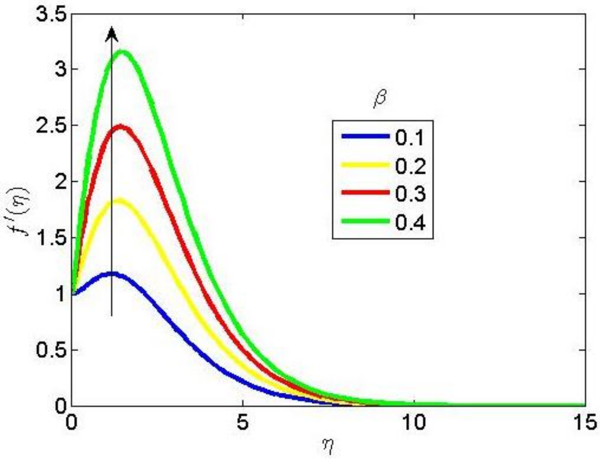

Fig. 2. Outcome of Deborah number on the velocity profile

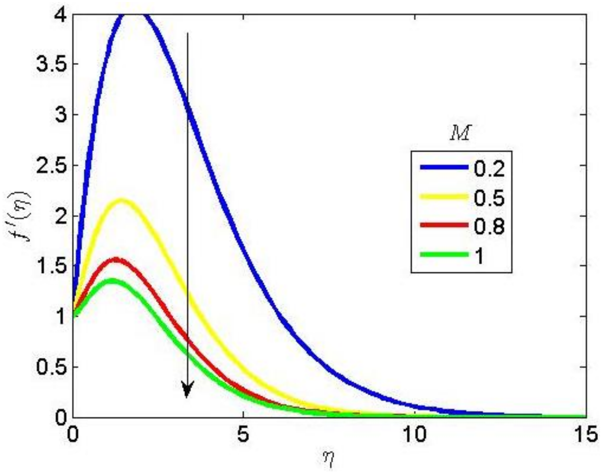

Fig. 3. outcome of magnetic parameter on the velocity contour

Figure 4 exhibits the outcome of radiation (R)on the temperature and velocity profiles. The thermal radiation explains the production of electromagnetic radiation due to the heat motion of the upper convection Maxwell fluid. A growth of R is noticed to enhance both velocity and temperature profiles. Due to the increase in temperature in Figure 4, the thermal energy increases. This increase in thermal energy enhances the thickness of the thermal layer along with the fluid thermal condition. The significance of thermal radiation can be found when temperatures are very high in thermal engineering.

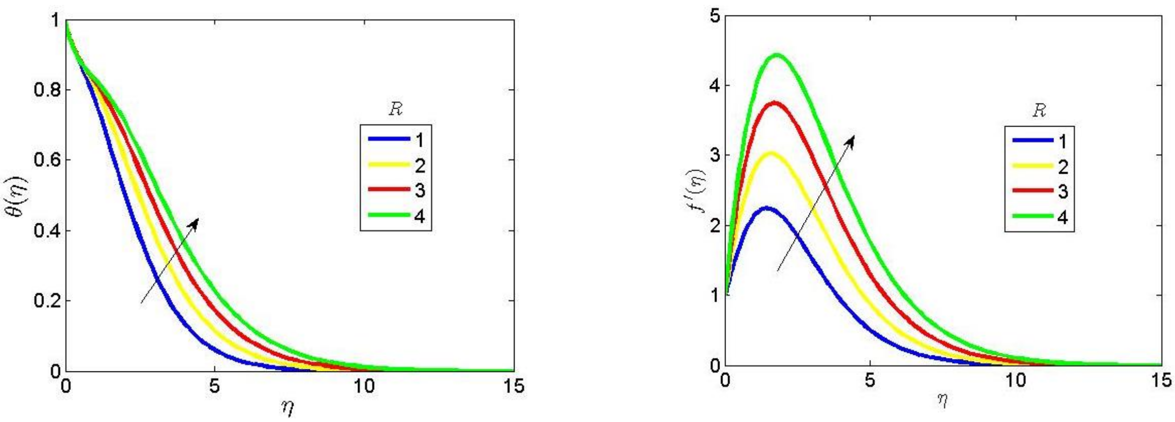

Fig. 4. Outcome of thermal radiation parameter on the velocity along with temperature contour

Figure 5 exhibits the outcome of the Eckert number (Ec) on the temperature along with velocity contours. An increase in Ec is detected to enhance the fluid temperature along with velocity contours. Physically, the heat energy rises the moment there is an increase in Eckert term.

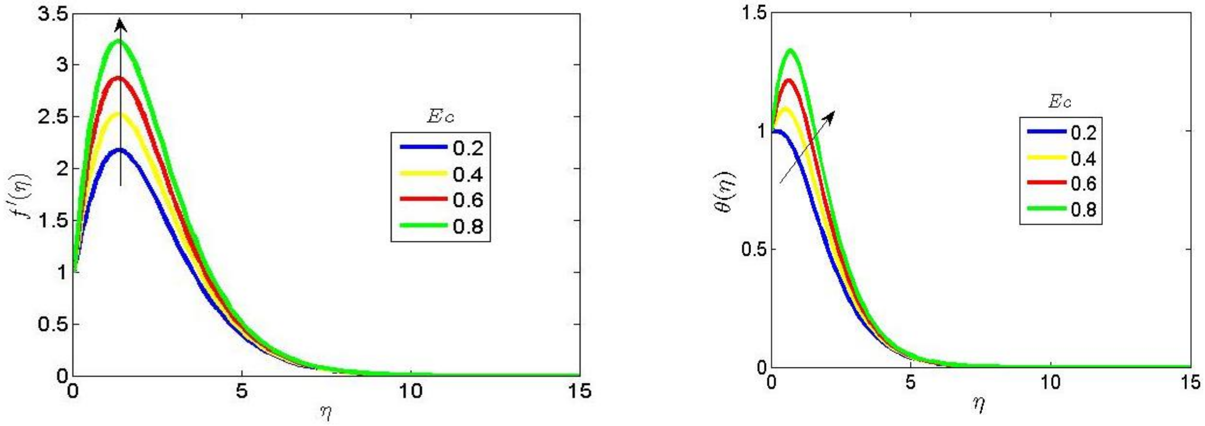

Fig. 5. Outcome of Eckert number on the velocity along with temperature profile

Figure 6 exemplifies the significant effect of the Prandtl number (Pr) on the velocity along with the temperature contours. A significant increase in Pr is noticed to decrease the velocity along with the temperature contours. The Prandtl number describes the ratio of Maxwell fluid viscosity to its thermal conductivity. Consider that viscosity is higher than thermal conductivity; this will lead to a very high Pr, which enhances both velocity and temperature. Therefore, $Pr < 1$ means a highly conducive fludic.

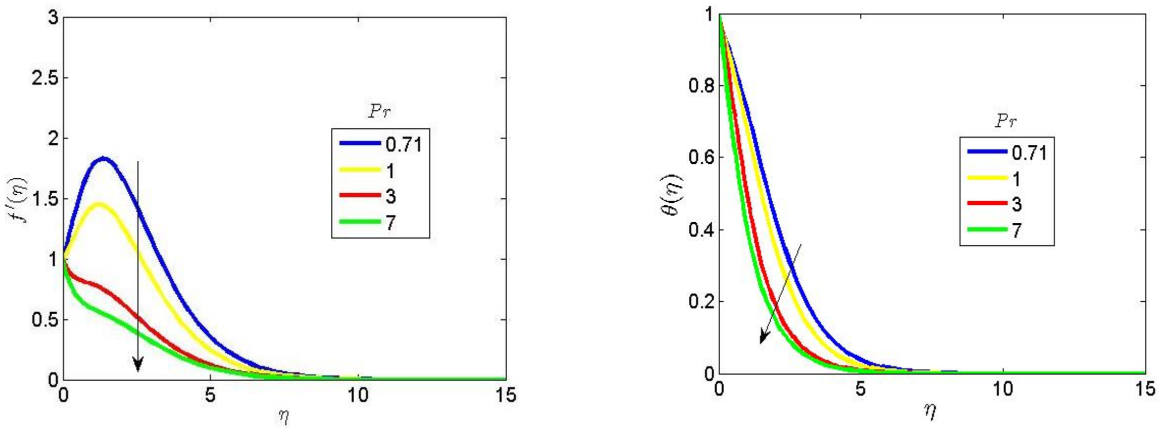

Fig. 6. Outcome of Prandtl number on the velocity along with temperature profiles

Figure 7 shows the outcome of Dufour term (Do) on the velocity along with temperature contours. A higher Dufour is noticed to enhance the fluid velocity and temperature profiles. As Do increases the Maxwell fluid particles diffuses to the thermal region which lead to increase in thermal and hydrodynamic boundary layer thickness.

Also, Figure 8 shows the outcome of Soret parameter (So) on the velocity along with concentration contours. Soret growth in Figure 8 shows an increase in the velocity along with concentration contours. The significant outcome of So as seen in Figure 8 display that the Maxwell fluid particles diffuses from the thermal to cold region.

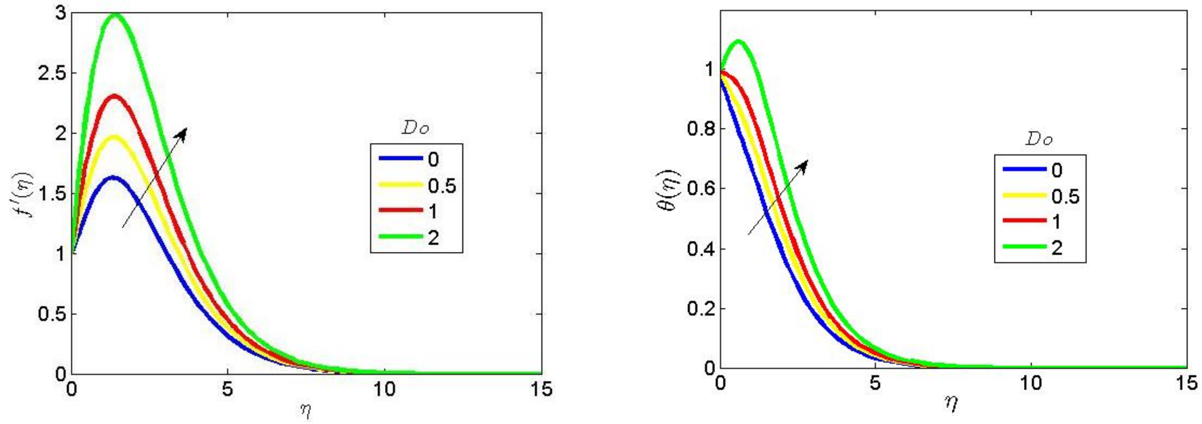

Fig. 7. Outcome of Dufour parameter on the velocity along with temperature contour

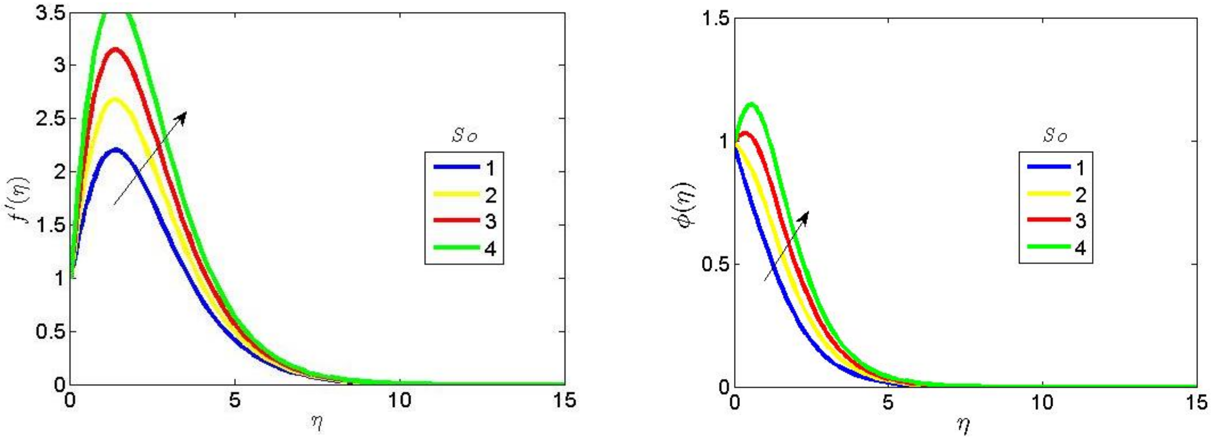

Fig. 8. Outcome of Soret parameter on the velocity along with concnetration contour

Figure 9 illustrates the outcome of heat flux relaxation parameter (γ_1) on velocity along with temperature contours. An improvement in the velocity and temperature profiles is observed due to increase in γ_1. The presence of thermal radiation alongside heat flux parameter helps to boost the overall temperature within the boundary layer.

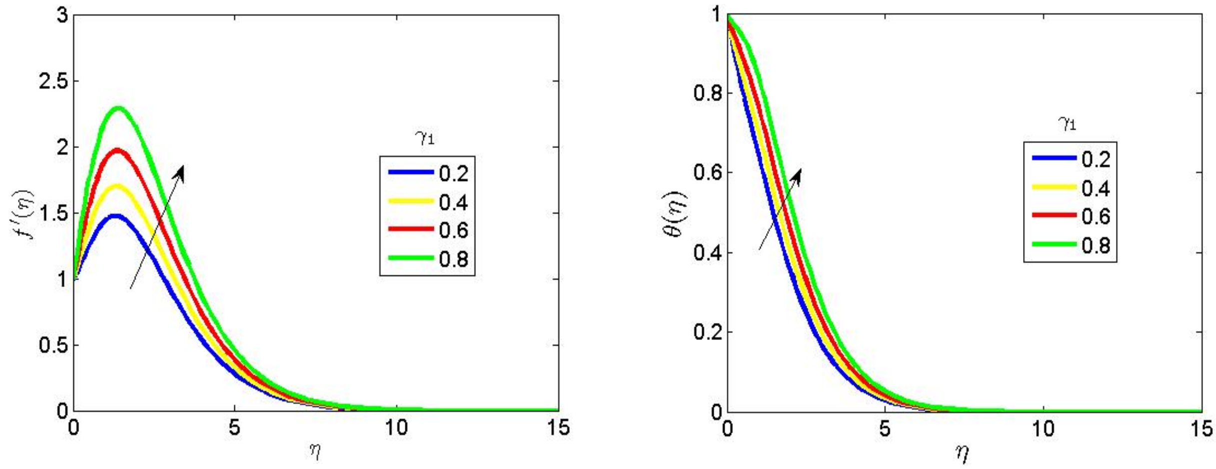

Fig. 9. Outcome of heat flux relaxation parameter on the velocity along with temperature contour

Figure 10 illustrates the effect of mass flux relaxation term (γ_2) on the velocity along with concentration contours. An increase in mass flux relaxation paramter shows a significant decrease in the velocity and concentration contours. Physically, the result in Figure 10 shows that the thickness of hydrodynamic and concentration boundary layer degenerates due to increase in γ_2.

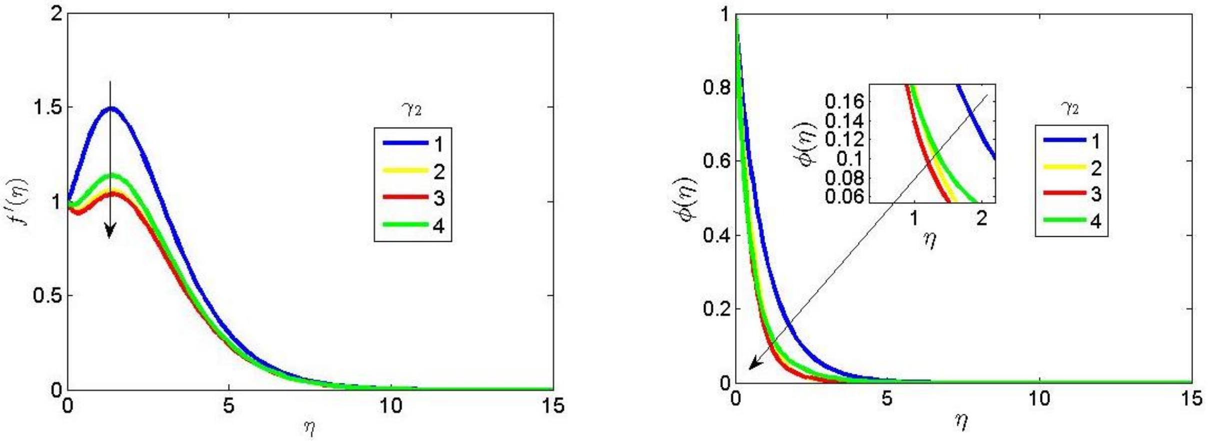

Fig. 10. Outcome of mass flux relaxation parameter on the velocity along with concentration profile

Figure 11 shows the outcome of Schmidt term (Sc) on the velocity along with concentration profiles. A growth of Sc is noticed to decrease the velocity along with concentration profiles. The Schmidt term controls the rate of mass transfer process. Therefore, for $Sc = 0$ it means no mass transfer phenomena. Hence, Sc is the correlation between thickness of hydrodynamic boundary layer alongside mass transfer.

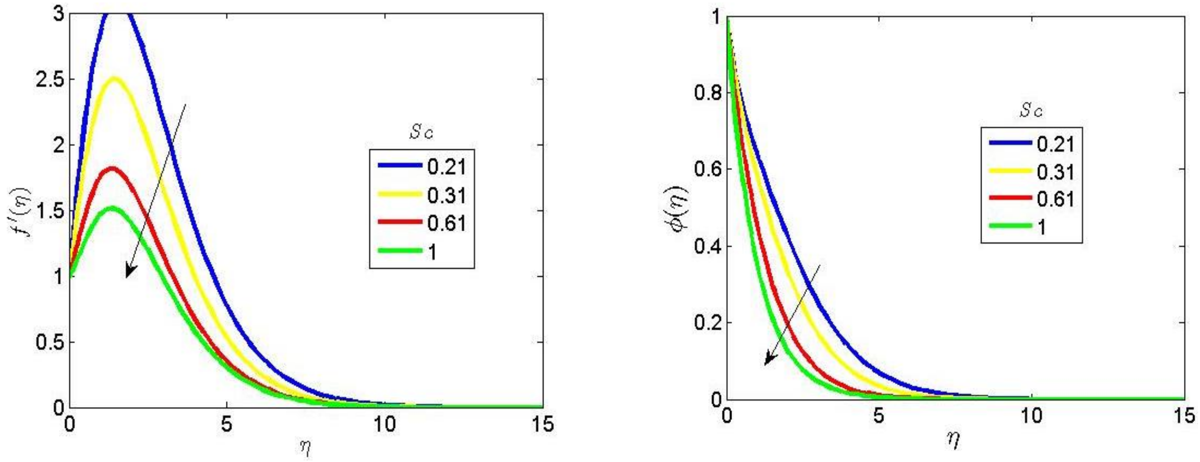

Fig. 11. Outcome of Schmidt number on the velocity along with concentration profile

Figure 12 depicts the effect of chemical reaction (Kr) on the velocity along with concentration profiles. An improvement of Kr is noticed to decrease the velocity along with concentration profiles. Hence, the thickness of specie and hydrodynamic boundary layer increases.

Table 1 illustrates the significance outcome of pertinent flow parameters on the engineering interest. Increase in the Deborah number β in Table 1 is observed to decrease the skin friction in the boundary layer. The impact of β is observed to be negligible on Nusselt and Sherwood number in Table 1. An increase in Ec, Pr, R, γ_1 and Do is noticed to increase the skin friction as well as Nusselt number. This shows that increase in Ec, Pr, R, γ_1 and Do speed up the rate of heat transfer and thickness of hydrodynamic boundary layer. In Table 1, increase in the value of γ_2 and So gives an increase to the skin friction and Sherwood number. An increase in Sc and Kr is observed to decrease the skin friction and Sherwood number. The impact of Sc and Kr is found to be negligible on the

Nusselt number. Table 2 shows the comparison of the present study and that of Oyelami and Falodun [20]. The outcomes in Table 2 shows the correctness of the present method of approach.

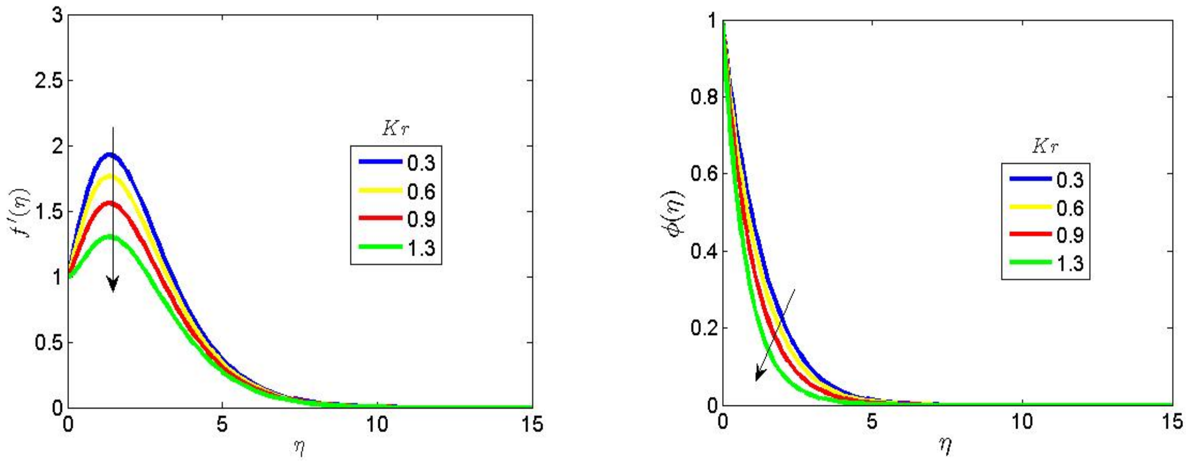

Fig. 12. Outcome of chemical reaction parameter on the velocity along with concentration profile

Table 1
Effect of flow parameters on the local skin friction, Nusselt and Sherwood numbers

β	M	R	Pr	Do	Ec	γ_1	γ_2	Sc	So	Kr	Cf	Nu_x	Sh_x
0.1											1.180468	0.637141	0.977600
0.3											0.662109	0.637141	0.977600
0.5											0.143751	0.637141	0.977600
	0.0										0.170865	0.344752	0.600569
	0.5										0.982688	0.344752	0.600569
	1.0										1.291960	0.344752	0.600569
		1.0									0.943126	0.359789	0.610569
		2.0									1.856981	0.377709	0.610569
		3.0									2.691186	0.394612	0.610569
			0.71								0.662109	0.337141	0.700121
			3.0								1.216874	0.338035	0.700121
			7.0								2.304052	0.411622	0.700121
				0.0							0.443156	0.106654	0.601021
				0.5							0.104228	0.222613	0.601021
				1.0							1.198997	0.271288	0.601021
					0.1						0.556558	0.258435	0.569420
					0.2						1.166045	0.556298	0.569420
					0.3						1.775531	0.854161	0.569420
						1.0					0.871006	0.389404	0.522410
						2.0					0.434384	0.281241	0.522410
						3.0					0.086433	0.158181	0.522410
							1.0				1.226653	0.701214	1.481460
							2.0				1.919491	0.701214	1.677293
							3.0				2.634231	0.701214	1.718407
								0.31			0.325546	0.416857	0.640112
								0.61			0.682044	0.416857	0.803441
								1.0			1.191147	0.416857	1.027441
									0.0		0.755547	0.337141	0.120518
									0.5		1.543134	0.337141	0.207438
									1.0		2.330722	0.337141	0.448475
										0.3	0.766113	0.714168	0.838687
										0.6	1.111877	0.714168	0.983914
										0.9	1.533358	0.714168	1.191504

Table 2
Comparison of the present study with Oyelami and Falodun [20] when $\beta = M = Do = \gamma_1 = \gamma_2 = R = 0$

		Oyelami and Falodun [20]		Present outcomes	
Sc	Pr	Sh_x	Nu_x	Sh_x	Nu_x
0.22		0.6794	0.6400	0.6793	0.6401
0.61		0.7730	0.6400	0.7729	0.6401
1.00		0.8809	0.6400	0.8808	0.6401
	0.71	0.7701	0.8252	0.7700	0.8251
	1.00	0.7701	0.9608	0.7700	0.9607
	7.00	0.7701	2.5119	0.7700	2.5118

5. Conclusion

The model of this paper is based on upper-convection Maxwell fluid motion with heat and mass transfer influences. The Maxwell non-Newtonian fluid was set into motion in the presence of magnetohydrodynamic, chemical reaction, thermal radiation, and Cattaneo-Christov theories. The Rosseland diffusion approximation has been utilised to explain the radiative heat flux loss in the Maxwell fluid flow. The analysis of Maxwell's non-Newtonian fluid flow phenomenon has been solved by utilising the spectral relaxation method. From the numerical calculations, the following findings are deduced

i. The imposed magnetism parameter accelerates the Lorentz force, which reduces the motion of an electrically conducting fluid such as Maxwell fluid.
ii. The viscous dissipation term influences the fluid temperature by producing heat energy.
iii. The fluid environment experienced high temperatures due to thermal radiation.
iv. The chemical reaction term influences the fluid velocity and specie concentration, and
v. The Cattaneo-Christov model has a great impact on mass and heat transfer.

This study will be of help in reducing turbulent flow due to the presence of magnetic field parameters. It finds application in polymer additives, MHD acceleration, food processing, and thermal engineering.

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