

Residence Time Distribution of an Irregular Octagonal Tank under Hydrodynamic Sloshing Effect

Wei Han Khor^a, Jie Heng Goh^a, Qi Ming Yew^a, Hooi-Siang Kang^{a,b,*}, Jun-Wei Lim^c, Koji Iwamoto^d, Collin Howe-Hing Tang^a, Pei Sean Goh^e

^aFaculty of Mechanical Engineering, Universiti Teknologi Malaysia, 81310 Johor Bahru, Johor, Malaysia

^bMarine Technology Centre, Institute for Vehicle System & Engineering, Universiti Teknologi Malaysia, 81310 Johor Bahru, Johor, Malaysia

^cDepartment of Fundamental and Applied Sciences, HICoE-Centre for Biofuel and Biochemical Research, Institute of Self-Sustainable Building, Universiti Teknologi PETRONAS, 32610, Seri Iskandar, Perak, Malaysia

^dMalaysia-Japan International Institute of Technology, Universiti Teknologi Malaysia, Jalan Sultan Yahya Petra, 54100 Kuala Lumpur, Malaysia

^eAdvanced Membrane Technology Research Centre (AMTEC), Universiti Teknologi Malaysia, 81310 Johor Bahru, Johor, Malaysia
 kanghs@utm.my

Sloshing is the movement of fluid in a partly filled vessel subjected to external motions. Generally, sloshing is an undesirable motion for the purpose of marine operations as it causes intense forces and instability. However, the internal motion of liquid might be useful for devices that require mixing. For offshore-based floating photobioreactors, sloshing induced by external ocean wave forces are desirable, as the movement of liquid is able to bring about mixing and mass transfer for the nutrients and gas in the microalgae culture. Therefore, the objective of this study is to investigate the correlation between the regular wave-induced sloshing and mixing behaviour of the irregular octagonal tank. The irregular octagonal tank is modelled as a non-ideal reactor with stagnant volume, and the Residence Time Distribution (RTD) of the irregular octagonal tank is measured. The RTD is dependent on the external excitation parameters, such as amplitude and frequency, besides being dependent on the filling ratio of the tank. Dissolved solids are injected into the system and the concentration of dissolved solids are measured using a Total Dissolve Solid (TDS) sensor. The experimental results are able to show the RTD curve for the irregular octagonal tank undergoing internal sloshing dependent on the filling ratio of the tank, excitation frequency and excitation amplitude of the sinusoidal motion. The mixing efficiency of the sloshing motion peaks at a filling ratio of 30 % with excitation frequency closest to the natural frequency of the irregular octagonal tank and large excitation amplitude.

1. Introduction

Sloshing is defined as the movement of fluid in a partly filled container subjected to external motions. Sloshing dynamics are often discussed in terms of inherent physical mechanics, interaction with supporting containers and methods to suppress or reduce sloshing. Mostly, sloshing is the undesirable effect of moving internally contained liquid that will cause intense forces to the containing structure. Dynamic instability, decreased performance of moving vessels, and structural damage (Pizzoli et al., 2022) can be brought upon by the hydrodynamic pressure distribution, forces, moments and natural frequencies of the free-liquid surface. Sloshing is commonly discussed due to the complexity of its physical mechanics and impacts on a wide range of applications, especially on behalf of transportations. For instance, the violent motion of the free liquid surface inside a tanker may cause deformation on the structures of the container and instability issues (Brar and Singh, 2014). Most of the time, sloshing brings hazards to supporting structures, as the violent sloshing in partially-filled containers may cause intense forces that may cause instability and accidents to ships and land vehicles besides damage to structures (Faltinsen and Timokha, 2009).

Generally, sloshing is an undesirable motion of the contained partially liquid, especially for the purpose of marine operations. However, by looking from another perspective, the internal motion of liquid might be useful for devices that require mixing. For example, efficiency of oil and gas separators on-board floating platforms will be affected by sloshing because the oil and gas continuously mix together (Kim and Kim, 2017). This is because the kinematics of free surface and flow instabilities induced by hydrodynamic sloshing can lead to mixing and mass transfer. The to-and-fro motion of the liquid inside the partially filled container due to periodic excitations in translational and rotational movements causes dynamic instability to the liquid that enhances the mixing and mass transfer of the contained fluid. A study conducted by Grotle and Aesoy (2017) investigated the thermodynamics response caused by sloshing in a liquified natural gas fuel tank using computational fluid dynamics (CFD). Sloshing is able to reduce the time needed for the methane liquid in the tank to achieve thermal equilibrium by increasing the rate of pressure drop from the operating pressure to the equilibrium pressure. The outcomes presented that the mixing of the partially filled liquid is influenced by the frequency of the external excitation force and gets intensive when the sloshing frequency is high (Khor et al., 2020).

Considering the free surface evolution, velocity field, magnitude velocity, vorticity, turbulent kinetic energy and dissipation rate of the turbulent kinetic energy when exposed to sinusoidal excitation, Bouabidi et al. (2016) showed that the hydrodynamic properties of the battery is significantly influenced by the vertical baffle and sinusoidal excitation, where recirculation zones on the liquid and vorticity are observed to be maximum. The results indicated that addition of baffles increase the vorticity of the sloshing liquid which is also beneficial for mixing. Chen et al. (2018) showed in their study that the swirling waves caused by the sloshing motion could also enhance mixing. The mixture had the largest particle circulation and mixing efficiency with the installation of paired baffles at symmetrical locations. The vortices developed at the tip of the baffles could improve the fluid exchange between the top and bottom. The research showed that the location and geometry of the baffles have high influence on the vorticity and fluid flow in the tank.

For photobioreactors, the movement of liquid is desirable, specifically for homogenous mixing of nutrients and efficient mass transfer of gases in the culture medium (Grobelaar, 2000). Mixing is vital for the even dispersion of nutrients, dissolved gases, temperature and pH for the microalgae culture, besides enhancing light distribution (Huang et al., 2017). For land-based photobioreactors, mixing consumes up to 30% of the energy input form microalgae cultivation and is commonly achieved by aeration systems, pumps or motors (Chisti, 2008). By employing photobioreactors into the ocean, the utilization of the free wave energy proposes a new possibility to cut down the production cost for microalgae cultivation in photobioreactors (Khor et al., 2022). Despite the possibility of sloshing motions offering beneficial effects for the mixing of microalgae medium in a floating photobioreactor, the effects of the external motion on the mixing of the photobioreactor is still unclear. Consequently, this research has been aimed to study the effects of sloshing hydrodynamics on the effects of mixing inside an externally excited irregular octagonal tank. The relationship between regular wave-induced sloshing and the residence time distribution of an irregular octagonal tank is determined.

2. Theory

Residence Time Distribution (RTD) is an analysis tool commonly used to diagnose and inspect the mixing behaviour of chemical reactors. It is important to inspect the flow pattern in the reactors to prevent the formation of stagnant regions, short-circuiting, bypassing and internal recycle flows. RTD is the probability distribution of the time of solid or fluid materials spend in the operation of a continuous flow system (Lopes et al., 2002). It determines the time that the "fluid element" resides inside the reactor. The experimental RTD function, $E(t)$ can be obtained experimentally by using stimulus-responses technique. For example, the tracer injection technique where an impulse of the tracer is injected at the entrance as an input while the exiting concentration, $C_T(t)$ is quantified as a function of time. $E(t)$ can be expressed by the normalization of the response at the outlet with respect to the whole amount of tracer used, given by:

$$E(t) = \frac{C_T(t)}{\int_0^{\infty} C_T(t) dt} \quad (1)$$

The effective mean residence time is given by:

$$\bar{t} = \int_0^{\infty} tE(t)dt \quad (2)$$

The average residential time, or renowned as space time, τ is the correlation between the volume of the reactor, V and the volumetric flow rate, v , where:

$$\tau = \frac{V}{v} \quad (3)$$

For ideal flow patterns of with a volumetric flow that is constant with no stagnant zones or dispersion, the mean residence time, \bar{t} will be similar to the average residential time, τ . However, for non-ideal cases, the experimental value of \bar{t} will deviate drastically from the nominal space time. Nonideal flow patterns lead to ineffective contact of the molecules and stagnant zones hold back molecules therefore leading to longer residence time.

3. Method

3.1 Experimental setup

To determine the RTD of a non-ideal reactor of a sloshing tank with stagnant volume, a communication system is established by constructing two holes at opposite ends of the tank. Clean medium is pumped in at the entrance and the mixed solution is pumped out from the exit, as illustrated in Figure 1. At the entrance, a salt solution of fixed concentration mixed with methylene blue is used as a tracer and impulsively injected into the tank. The tracer salt concentration at the exit is measured using a TDS sensor and the purpose of the methylene blue is to observe the mixing of the tank.

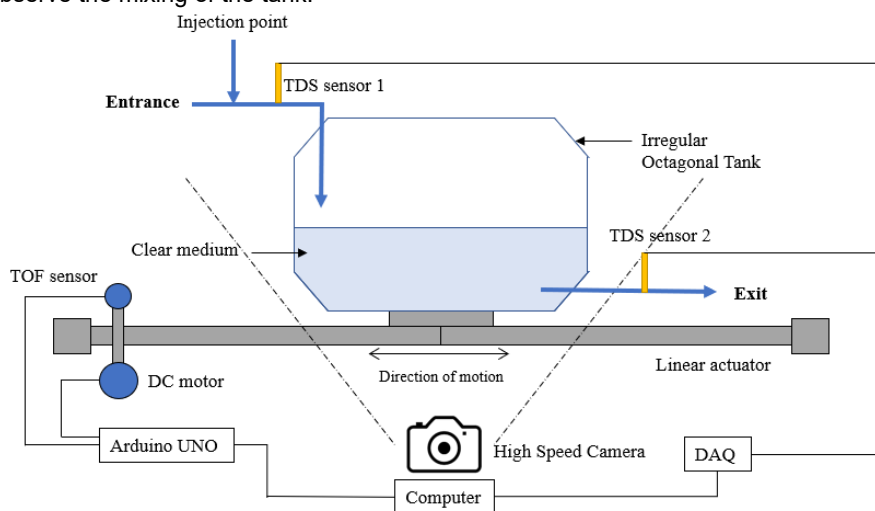


Figure 1: Experimental setup to determine RTD in irregular octagonal tank

The experimental arrangement as presented in Figure 1 includes an irregular octagonal prism tank, direct current linear actuator, TDS sensors and data acquisition system. The dimensions of the irregular octagonal prism tank made from acrylic sheets is 39 cm in length, 28 cm in height, and 12 cm in width with a maximum capacity of 12 L. The TDS sensors (Model: SEN0244) are connected to the DAQ to measure the salt concentration at the entrance and at the exit of the tank. A direct current (DC) linear actuator is used to generate sinusoidal motion in the horizontal direction parallel to the length of the tank to imitate the surge motion of the floating photobioreactor in the head sea position. The displacement of the linear actuator is calibrated by comparison with data obtained from the time of flight (TOF) sensor (Model: VL53L0X). The linear actuator and DC motor is controlled with a proportional integral (PI) controller via Arduino Uno. A camera is used to capture and observe the effects of mixing when the photobioreactor is subjected to external motion.

The experiment is repeated with different filling ratio (R), excitation frequency (f) and excitation amplitude (A), where the filling ratio is the parameter of tank while the excitation amplitude and frequency are the parameters of an external sinusoidal wave motion, while. The tank is first filled up to the corresponding filling ratio, for example 30% filled. The external pumps are initiated to establish the weak connection system between the entrance and the exit, and the linear actuator is started to slosh the liquid inside the tank. The RTD of the sloshing tank was determined by injecting the tracer using the pulse method, where the tracer solution is injected into the injection point using a syringe and the data from the TDS sensors are recorded and further analysed.

3.2 Sensor calibration

Total Dissolved Solid (TDS) sensors measured the quantity of dissolved solids in liquid. The calibration of the TDS data is conducted by manipulating the amount of NaCl in a fixed volume of distilled water. The relationship between the quantity of dissolved solids with the mass of solute and solvent. By manipulating the mass of solute

and mass of solvent, the TDS can be calculated and compared to the measured data from the TDS sensors. The output for the TDS sensor is as shown in Figure 2. Based on calculation, the amount of total dissolved solids directly varies with the mass of solute, in this case, NaCl. However, the measurement from the TDS sensor deviates away from calculation results when mass of solute increases. The amount of error increases up to 37 % when the mass of solute is increased to 0.7 g. Considering the range of the sensor as mentioned in the data sheet is only up to 1000ppm, it is more suitable to keep the measurement of the TDS sensor in the range of 0 to 800ppm, where the error is less than 20 %.

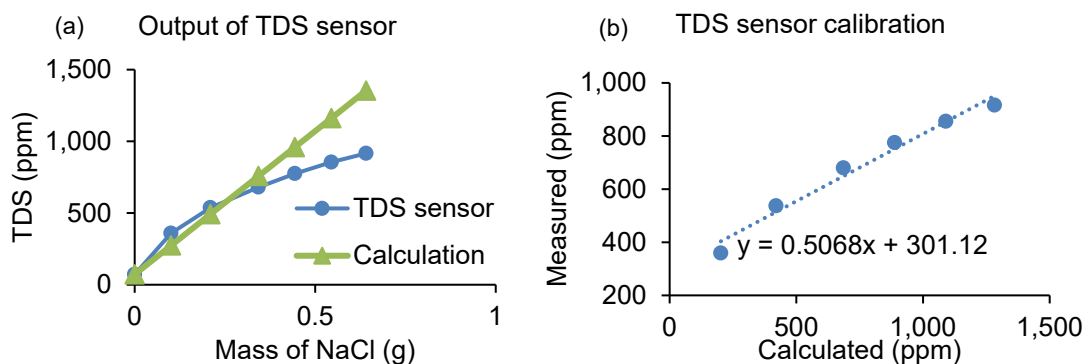


Figure 2: TDS sensor calibration data (a) Full range data of comparison between TDS sensor measurement and calculated TDS; (b) Limited range measurement for TDS sensor

4. Results

Figure 3(a) shows the outcome for the sloshing effects on the RTD for varying excitation amplitude with filling ratio constant at 30 %, and excitation frequency constant at 1.0 Hz. With decreasing excitation amplitude, it takes a longer time to achieve the peak. This implies that more time is needed for the solution to reach a homogenous solution of TDS. It is slower for the sloshing in the tank to reach homogenous mixing, which is the peak of the curve. A bigger excitation amplitude implies a longer travelling distance from one end to the other, therefore a greater external excitation force is applied on the tank (Md Arif et al., 2020), where homogenous mixing can be achieved faster. For instance, the peak is achieved at 32 s after the trace solution is injected into the photobioreactor for the excitation amplitude of 3 cm. The peak is achieved at 38 and 67 s for the excitation amplitude of 2 and 1 cm.

Figure 3(b) shows the RTD for varying frequency from 0.9 to 1.1 Hz, with filling ratio constant at 30 % and excitation amplitude constant at 3 cm. For the comparison between the mixing efficiency of varying frequency, 0.9 and 1.0 Hz has slightly better mixing compared to 1.1 Hz. The time for the solution to reach its peak is 38, 20 and 59 s, for the frequency of 0.9, 1.0 and 1.1 Hz. Although the mixing time reduced when increased from 0.9 to 1.0 Hz, the mixing time became longer when the frequency was changed to 1.1 Hz. This is because the mixing efficiency is also dependent on the natural frequency of the tank, where 1.0 Hz is closer to the natural frequency for the tank with filling ratio of 30 %. Therefore, the sloshing intensity is also highest at the frequency of 1.0 Hz allowing the solution to mix homogeneously faster.

As shown in Figure 3(c) are the results for the filling ratio of 30, 50 and 70%, with the excitation frequency constant at 1.0 Hz and excitation amplitude constant at 3 cm. In this experiment, the time duration of the measurement is limited to 15 min, therefore the curve can only show partially the mixing behaviour for the filling ratio of 50 and 70%. However, from Figure 3(c), it is illustrated that for higher filling ratios or larger stagnant volume, the mixing behaviour is almost similar to the filling ratio of 30 %, but with a longer duration to complete the whole curve. The intensity of sloshing also decreases with increasing filling ratio (Qiu et al., 2022), which implies that for the same amount of force applied to a heavier object, the object is less reluctant to move because the inertia of the heavier object is larger. With less sloshing intensity, a larger stagnant volume appears and a longer mixing time is needed. Taking into account that the irregular octagonal tank is with a stagnant volume, the flow pattern as observed for the RTD results look very similar to the RTD of a stagnant volume, where the curve is skewed towards the left and has a very long tail. This is because for the measurement of this experiment, only a weak communication system is installed for the purpose of measuring the RTD. In fact, when the filling ratio increases, the stagnant volume also increases, causing a further bias to the left in the RTD curve. Under an ideal condition, all the elements entering the tank through the inlet stream will spend the same amount of time inside the tank. Assuming that the tank is an ideal reactor, the theoretical nominal residence time with different filling ratios is 172.5, 287.5 and 402.5 s for the filling ratio of 30, 50, and 70 %.

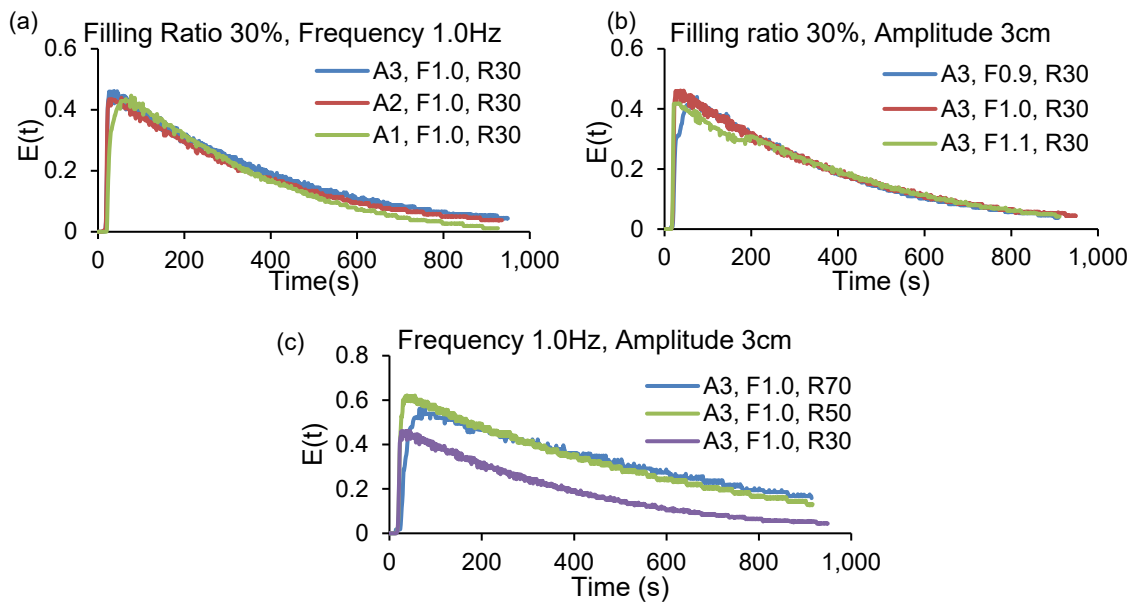


Figure 3: RTD curve (a) varying excitation amplitude; (b) varying frequency; (c) varying filling ratio.

The mean residence time for varying sinusoidal motion settings at the filling ratio of 30 % is shown in Table 1. The mean residence time for 30 % filling ratio deviates distinctly from the nominal space time, which might be due to non-ideal behaviour of the mixing in the tank. Suppose if the mixing in the tank is an ideal one, the outlet data would have been identical to the input data. However, it is shown clearly in Table 1, that for varying excitation amplitude and excitation frequency, the mean residence time deviates by more than 1.5 times from the nominal space time. Deviation from the ideal case can be due to stagnant zones and zones of uneven turbulence. Observation of stagnant and turbulent zones can be highlighted by the dye motion moving inside the tank during sloshing motion of different intensity. During the sloshing process, liquid run-ups and hydraulic jumps can be observed in the condition with filling ratio of 30 %.

Table 1: Mean residence time for varying excitation amplitude and frequency for 30 % filling ratio

Amplitude (cm)	Frequency (Hz)	Mean Residence Time (s)
1	0.9	294.2
	1.0	274.2
	1.1	341.2
2	0.9	262.9
	1.0	302.6
	1.1	259.2
3	0.9	307.8
	1.0	315.4
	1.1	315.9

5. Conclusion

From the experimental study, it is demonstrated that the residence time distribution is dependent on the manipulation of the external sinusoidal wave motion parameters, such as amplitude and frequency, besides being dependent on the filling ratio of the irregular octagonal tank. The mixing efficiency of the sloshing motion peaks at a filling ratio of 30 % with external wave motion frequency nearest to the natural frequency of the irregular octagonal tank and large excitation amplitude. An increase in excitation amplitude results with faster mixing, while higher filling ratios result with an increase in the stagnant volume. However, the study still lacks in terms of accuracy of TDS sensor and the RTD measurement system. Further studies can be conducted to reduce the long tail of the RTD curve by improving the flow rates and the position of the exit and entrance of the communication system to measure the RTD.

Nomenclature

$C_T(t)$ = outlet concentration function
 \bar{t} = mean residence time, s
 A = excitation amplitude, cm
 f = excitation frequency, Hz
 R = filling ratio, %
 RTD = residence time distribution

TDS = total dissolve solid, pp,
 τ = average residential time, t
 $E(t)$ = residence distribution function
 V = reactor volume, l
 v = volumetric flow rate, l/s

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