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ORIGINAL ARTICLE

# An Approach to Logical Compatibility Determination for Solution Principles in Morphological Matrix-Based Conceptual Design

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**ABSTRACT** – Morphological matrix-based conceptual design has been proven to enable the generation of high-quantity, variety, and novelty concepts. Indeed, many studies have proposed methods for enhancing it. However, logical relations for determining the compatibility of solution principles (SPs) in combinatorial solution chain is yet to be established in the literature. This study attempts to develop a logical relation for determining the compatibility of SPs in solution variants chain. The features of the interaction boundary of each solution principle were characterised and vectorially numerated for onward matching of adjacent SPs in a combinatorial solution chain to determine compatibility. The consistency of the compatibility determinant (CD) was tested on engineering designer students. It was found that the students could determine the CD with excellent consistency. Besides, the applicability of logical relation for determining the feasibility of solution variants was tested and found applicable in many trials. Despite the limitation of the study, the results indicate that the logical compatibility determinant is consistent and applicable for the complete exploration of the morphological matrix design space. Therefore, this study provides a foundation for the development of a reliable CAD system for conceptual design.

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## INTRODUCTION

Conceptual design is a phase in the product life cycle where the emphasis is on transforming the function that an artefact is expected to perform into forms that can fulfil such functions[1]. Three main tasks involved in conceptual design are concept generation, concept evaluation and concept selection [2]. Of the many known methods of concept generation [3] [4][5][6], the morphological matrix method [7] is one of the most effective when novelty, quantity, and variety, among others, are the focus. This is because it enables the exhaustive generation of large number of concepts [8][9]. As claimed by Ullman [10], the more number of feasible concepts generated in a design, the more chances to develop a good concept. Concept generation in the morphological matrix method involves decomposing the main function to obtain the simplest form of the functional element [11][12][13], for which solution principles (SPs) can easily be generated [14][15][16]. Such generated SPs are arranged in rows against the respective sub-functions in a morphological matrix [17]. From the morphological matrix, combinatorial chains of SPs are made across the rows. In doing so, one SP is chosen from each row to form each combinatorial chain of SPs called solution variant (SV) [7]; such SVs are the generated concepts. By using this approach, many concepts can be generated. One of the flaws of this method of concept generation is that most of the SVs generated out of the morphological matrix are theoretical [18]. In practice, some of the adjacent SPs in the combinatorial SV chain are either not existing or practically incompatible[19]. The challenge is, how could the feasible concepts be completely sorted out of the entire theoretical concept generated?

Several research efforts were made to enhance the concept generation possibilities using a morphological matrix. On the one hand, Weber and Condoor [20] developed a scheme for extracting concepts from the morphological matrix. To ensure that only compatible SPs are included in the combinatorial chain formation, they instructed that the designers should use their own digression to identify and eliminate SVs with incompatible SPs. In addition, a study was carried out by Weiss and Gilboa [21], in which manual screening of SVs comprising incompatible SPs was also considered. Furthermore, in another study by Arnold et al. [22], a computer programme was developed for extracting components from a repository to build a morphological matrix. Screening the SVs was done by the designer while running the interactive computer programme. Additionally, the work of Ölvander et al. [23] is another hallmark of morphological matrix-based concept screening approach. They considered selective assertion of SPs into the morphological matrix, to avoid formation of SVs with incompatible SPs. Moreover, Bo [24] developed a binary encoding system to identify empty cells in the morphological matrix. Equally, Kang and Tang [16] later developed a morphological matrix-based concept generation scheme in which SVs with incompatible adjacent SPs are manually screened by designers. More recently, Chawla and Summers [8] studied the impact of function ordering on concept generation from the morphological matrix. Thereafter, Summers [18] explored the effect of the ordering of function on the ease of extracting SVs from a morphological matrix.

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On the other hand, Chakrabarti and Bligh [14] proposed a representation for physical effects of functions. The representation is based on three entities (i.e., type, sense, and orientation). The type was represented by the name of the physical effect, e.g., force, torque, speed. Also, the orientation was represented by the coordinates i, j, k while + or – represented the sense of the coordinates. Besides, He et al. [25] represented motion elements of mechanisms with parameters such as R and T for rotational or translatory type, respectively. Furthermore, -X, -Y and -Z or +X, +Y and +Z were adopted for negative or positive direction of X, Y and Z, respectively. They also represented the constancy of the motion with 0 or 1.

Furthermore, He et al. [26] proposed physical parameters to represent the physical effects of concepts in the catalogue and functions. They proposed six representations for the physical parameters. Moreover, they developed space matrix, for attaching the physical parameters to functions. Then, they created a cell within the space matrix, in which working structures from a design repository that correspond to the physical effect with the respective physical parameter can be inserted. Nevertheless, a more generic procedure, for eliciting directional numeric representation of compatibility determinant (CD) features, on the surface of the SPs that form the boundary between two adjacent SPs (structures) in an SV chain is lacking in the literature. The traditional practice is to make random picking of SVs out of the morphological matrix [27]. Evaluation and final selection of concepts are done on these randomly picked concepts.

Besides, as the morphological matrix contains SPs, no matrix has been developed to contain the SVs that are developed out of the morphological matrix. Automation of conceptual design activities may be done on such matrix. However, such SV matrix or concept matrix is yet to be developed in the literature. Furthermore, Pahl et al. [7] asserted that, for a morphological matrix method to be effective, there is a need to develop a logical relation for determining the compatibility of the set of SP in an SV chain. This enables the automation of screening of the large number of theoretical SV to obtain a feasible solution. Yet, such logical relations are yet to be evidently established in the literature. Designers resort to selecting any few feasible SV out of the many possible feasible SVs that can be extracted from the morphological matrix for evaluation and selection [27]. This can be regarded as underutilisation of the design space. On this note, an SV screening feasible concept generation approach is therefore proposed in this study. The approach entails development of a theoretical SVs matrix out of a morphological matrix; thereafter, establishing mathematical relation between the SPs in the elements of the SV matrix and their respective positions in the morphological matrix. The approach entails a procedure for eliciting CD features of the boundary surface of the SPs in an SV chain. Then based on such features, directional numerical logical relations are developed to determine the compatibility of adjacent SPs in an SV chain. Furthermore, the condition for compatibility is placed, to screen the elements of the SV matrix, to yield the feasible concept.

## RELATED WORKS

The morphological matrix approach to concept generation has gained lots of research attention due to its propensity to generate concepts with high quantity, quality variety and novelty. Weber and Condoor [20] developed a method of conceptual design that considers the synergistically compatible SPs in a morphological matrix. In their work, they introduced a scheme that demands that the designer ought to observe the morphological matrix and consider SPs at different functional levels that can perform more than the function they stand for within the matrix. In their work, they highlighted steps for generation of feasible SVs out of the morphological matrix. To ensure that only compatible SPs are included in the combinatorial chain formation, they instructed that the designers should use their own digression to identify and eliminate incompatible SVs. This is only practicable when the morphological matrix has very few columns and rows. For morphological matrices with larger number of sub-functions and SPs, this approach is complicated and is not really feasible.

Later, a study was carried out by Weiss and Gilboa [21], in which they also considered a manual screening of SVs comprising incompatible SPs. Furthermore, in another study by Arnold et al. [22], an interactive tool for a morphological matrix-based concept generation was developed. The supporting tool they developed is repository based. They made an automatic selection of components from a repository to build a morphological matrix. These components in the repository are past designs with functional information. The tool they developed picks only functionally compatible components from the repository to develop the morphological matrix. However, design knowledge is wide-ranging, yet such method of concept generation is limited to the use of the components in the repository.

Furthermore, the work of Ölvander et al. [23] is another hallmark of the morphological matrix-based conceptual design approach. Concept generation in their work is based on a selection of compatible structures. In their study, they consider the screening of the SPs in the same manner proposed by Weber and Condoor [20]. In their approach, the morphological matrix is customised to contain only compatible SPs. Yet, the method is not really practicable when the system has many interrelated SPs. The methods will of course screen out SVs with incompatible adjacent SPs. However, many of the feasible solutions will as well be eliminated if there are incompatible SPs at any functional level. Secondly, being repository based, the design is limited to the content of the repository. Besides, repository-based designs are experience-based and are better applied in improvement designs [28]. In most cases there are needs for special forms of SPs for a function. Such forms may not be available in the repository. This kind of situation makes the content of the morphological matrix and the synthesis to become complicated.

Moreover, Bo [24] developed a binary encoding system to identify empty cells in the morphological matrix. They represented the SPs for a sub-function as 00, 01, 10, 11 and so on, for the first, second, third, fourth SPs and so on, respectively. For instance, to form a coded SV, the binary numbers from each functional level in the SV chain are arranged in series. So, for a morphological matrix with five sub-functions and a maximum SP of four, the SV will be represented in ten digits (e.g., 00100110010). The ten digits comprises of five set of two digits. Each set of two digit represents the coding for the SPs of a functional level representation in the SV chain. If one of the sub-functions for instance, has only three SPs, then the SV formed will be theoretically infeasible. His study indeed is helpful in screening out non existing theoretical concept. Nevertheless, the compatibility of adjacent SP is not addressed.

Furthermore, Engida Woldemichael and Mohd Hashim [29] developed a framework for computer-supporting tools based on the morphological matrix. Still, the assertion of Pahl et al. [7] is not fulfilled. In the case of Engida Woldemichael and Mohd Hashim [29], their supporting tool is run to obtain all the theoretical concepts. Then the feasible concepts are obtained by the designer through observation and deleting the non-feasible ones. For morphological matrices with a large number of sub-function and alternative SPs, the work will be highly demanding and time-consuming. Besides, the designer may skip many of the feasible concepts.

Additionally, the work of Ullah et al. [30] also contains the generation of SV from the morphological matrix. They applied a morphological matrix concept generation approach to generate concepts for Space launching vehicle design, which was given an acronym SLV. In their study, the screening of SVs with incompatible adjacent SPs was done by observing and eliminating the infeasible combinations. The process is practicable for specific designs with catalogue-based SPs development. In furtherance to this effort, there is a need to develop a logical relation that can be generic for morphological matrix-based concept generation. This will pave the way for development of a conceptual design CAD system.

Another study on the compatibility check for SPs is that of Kang and Tang [16]. They generated matrices like FFM (function to function matrix), FCM (function to components matrix), and DSM (design structure matrix). Components that can connect to other components are selected while forming the morphological matrix. The screening is not automatic. Besides, the approach is more applicable to reverse engineering[16]. In furtherance to this, for automatic extraction of components from design catalogues, many authors developed some entities and numerical codes to represent functional elements [26] or physical effects[14] [15] [25]. Moreover, the characterisation entities were associated with the input and output functions of design problems. They used the characterisation entities to categorise physical effects. The characterisation entities were viewed as a set of features, properties, or elements of the individual physical effects.

Chen et al. [15] represented motion with three parameters. The type of motion was considered to as translatory, rotational or helical. Translatory motion was represented with T, rotational with R, while helical was considered as the addition of T and R (T+R). Orientation on the other hand, was considered based on the cartesian direction X, Y or Z. The third parameter is direction, which is -1 or +1. With these three parameters, the motional function is vectorially represented as [type, orientation, direction]. The representation [R, X, -1] implies that the function element is the rotation about the X axis in the anticlockwise direction. Furthermore, Chakrabarti and Bligh [14] described physical effects in three features which are ‘type’, ‘sense’ and ‘orientation’. They considered ‘type’ as the type of physical effect. For example, ‘type’ may be ‘force’, ‘velocity’ or ‘voltage’. Moreover, they used two signs (i.e., + and -) to represent ‘sense’ and direction coordinates (i, j, k) to represent ‘orientation’.

In addition, He et al. [25] developed a synthesis approach for eliciting mechanical components from a design catalogue. Each of the components in the catalogue is represented by its physical effects in terms of motion. Parameters of the physical effects are defined as the motion elements. As such, extraction of the components from the catalogue is based on the motion element it possesses. They considered four motion elements which are [Motion Type, Reciprocator, Constancy of Motion, and Direction of Motion]. The functional elements are given units of representation. Motion type is either rotational or translatory, represented as R and T, respectively. The direction of motion is represented as X, Y or Z. Furthermore, the constancy of motion is represented as 0 or 1 for constant or non-constant, respectively. Similarly, reciprocator is represented as 0 or 1 for reciprocating or non-reciprocating, respectively. As such, a spur gear can be represented as [R, 1, 0, X]. The motion element representation indicates that the spur gear has a rotational motion type. Besides, it does not reciprocate, its motion is constant and for the design, its orientation is in X direction. However, the direction of rotation is not represented as either clockwise or anticlockwise, as it is the case with the representation proposed by Chen et al. [15].

Moreover, He et al. [26], in their own effort, considered seven features [type, vector, direction, constancy, reciprocating properties, linearity and continuity] as characterisation entities for functions termed physical parameters. They assigned numerical values to each of the physical parameter. For instance, they considered six different entities for ‘type’ which are spatiotemporal, mechanical, thermal, electrical, magnetic, optical/acoustic. Numerical values 1 to 6 were assigned to each entity. For example, if the type is mechanical, the physical parameter is represented as 2; if it is electrical, it is represented as 4 and so on [26].

Equally, He and Hua [31] developed a feature-based function element model. They considered force and motion transformation as the two basic kinds of functions for mechanisms. Furthermore, they selected three fundamentals for the functional element to make the feature-based functional element model [kind, orientation and direction]. Kind is represented as FF=force; TF=torque; RM=rotary motion; TM=translator motion; HM=helical motion. Orientation is represented as i, j, k in X, Y and Z axis, respectively. Direction is represented as -1 for anticlockwise motion or motion in the negative direction, while +1 is for clockwise motion or motion in the positive direction.

All the physical parameters discussed in the previous works are functional requirements and physical effect based. They have mostly been used for categorisation of functional elements. The characterisation entities in this regard, can be effective in searching for solution structures and automatic eliciting of structures from a design catalogue. The approach needs to be made universal to enable the definition of other elements than motion. This is because not all the components of a machine are dynamic. Secondly, the representation needs to be in terms of factors that describe the SPs (structures).

With such characterisation, the designers may observe the boundary properties of the components that are to form a chain of SPs (SV). Therefore, the compatibility of all the adjacent SPs in an SV could be determined. Such compatibility determinants (CDs), when represented numerically, can be a basis for developing logical relations for testing the compatibility of SPs in an SV chain. This area is yet to be studied in the literature. Consequently, it forms the basis for the logical relation for determining the compatibility of SPs in an SV chain that the present study is built on.

## METHODOLOGY

The framework for generating concepts out of a morphological matrix is developed in this study. The framework entails a procedure for developing logical relations for screening the theoretical SVs elicited from a morphological matrix. Two sets of matrices are developed. The first is SPs-based morphological matrices, while the second is the SV matrix (concept matrix or G-Matrix). SPs matrices contain the characteristics that determine the compatibility of two adjacent SPs in an SV chain, termed compatibility determinants (CDs). CD matrices are developed from a morphological matrix. Besides, their number depends on the number of CDs identified in the design. Moreover, the elements of the SV matrix are the SVs. The framework for this study entails the development of a morphological matrix, determination of CDs for each of the SPs, development of elements of the (G-Matrix) and then establishing relationships between the elements of a morphological matrix and those of the G-Matrix. The analytical relationship between the two matrices was also established; the position of SV and its constituent SPs can be traced in both SPs and SVs matrices.

Furthermore, the feasible concepts are further evaluated, and selection for the preliminary design is based on optimality. Nonetheless, the evaluation method adopted in this study is SP based. Performance values are found for each of the SPs for each evaluation criterium (EC). The weighted factor of each SP for each EC is found. Thereafter, the weighted factors of the SVs are found by summation of the weighted factors of all the SPs that are contained in them. A case study was developed using the conceptual design of continuously variable transmission (CVT) to validate the framework. Besides, the CD generation process was tested on designers to determine the consistency and versatility of the approach.

### Development of Morphological Matrix

To build a morphological matrix, the sub-functions are placed in a column while corresponding solution concepts are placed in the cells of the rows of each function [10]. Figure 1 is representation of a morphological matrix. The sub-functions are represented as $F_1$ to $F_i$ to $F_n$. Furthermore, the solution principles are represented with $S_{nm}$. In $S_{nm}$, ‘n’ represents the respective sub-function for the SP number, while ‘m’ represents the column in which the SP is placed. As such, $S_{21}$, is an SP for sub-function $F_2$ located in row 1 of the SP columns [7]. Two SVs are represented by the combinatorial solution’s chains in Figure 1. The SVs are indicated as A and B. To ensure a flow of input to output functions in an SV chain, Summers [18] investigated the effect of function ordering in a morphological chart. He observed that arranging functions from the most important to the less important yielded an enhanced exploration of the design space.

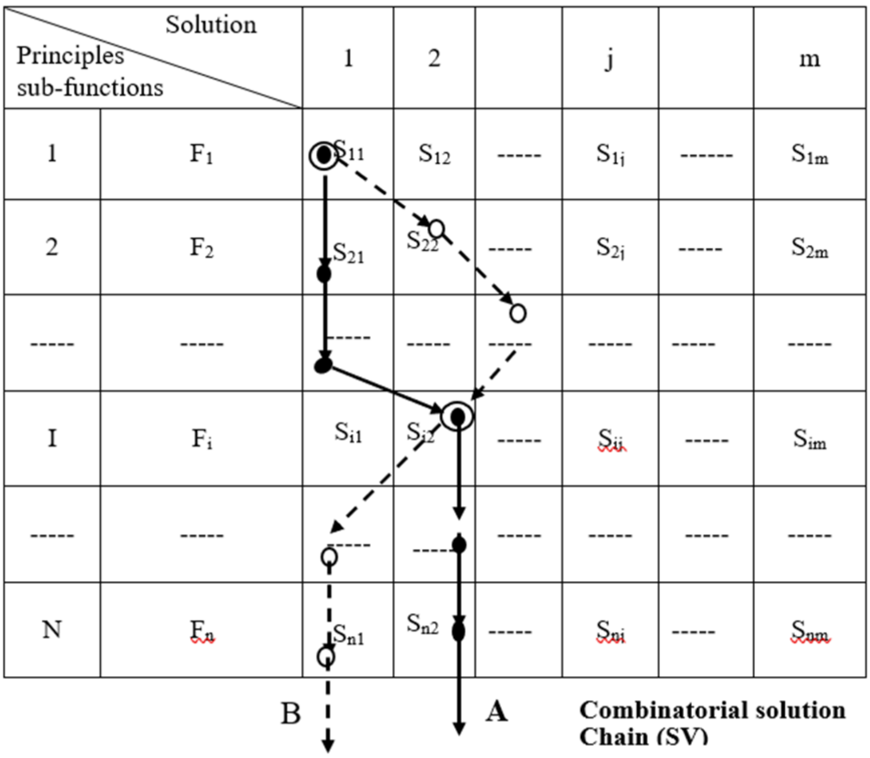


**Figure 1.** Morphological matrix

### Compatibility Determinant Matrices

Compatibility determinants (CDs) of an SP (structure) are the properties inherent in it (provided CD) or the properties required (required CD) of another SP (structures) for it to be connectible to them. The features on each SP that determine if any two contacting adjacent SPs in an SV chain are connectible with each other, are identified between SPs of adjacent rows in the morphological matrix. The morphological matrix for the conceptual design of the river cleaning machine shown in Figure 3 is used to explain the process. In the morphological matrix, four subfunctions are identified. Each of the subfunctions has five SPs, as shown.

To assign CD for a set of SPs in a morphological matrix, the procedure is enumerated in the flowchart in Figure 2. Starting from the first row of the morphological matrix, each of the SPs is observed to find the dissimilarities between their functional features. For example, the proposed SPs for sub-function ($F_1$) of river cleaning machine (in Figure 3) are observed to differ in their 'trash arresting areas'. $S_{13}$ is screened, while the remaining SPs in row 1 are open. As such, the first CD ($CD_1$) is the 'trash arresting area'. In the same manner, the second ($CD_2$) and the third CD ($CD_3$) 'mode of trash transfer' and 'trash discharge area' are derived from the SPs of $F_2$ and $F_3$, respectively. In this example, one CD has been derived from each set of SP. However, the number of CD that can be extracted from the SPs of each sub-function is not restricted. In some instances, more than one CD may be elicited from a set of SPs. Conversely, some set of SPs may yield no CD. Therefore, the flowchart in Figure 2 gives a condition of CD addition before moving to the next row.

As indicated in Table 1, three CDs are derived from Figure 3. The first CD is derived from the set of SPs in row 1 (SP to sub-function 1). It is observed that the trash arresting areas are either opened or screened ($S_{13}$ is screened while the remaining SPs in row 1 are open). As such, the first CD is 'trash arresting area' which is a provided CD for SPs in row one. In addition, it was observed that based on CD derived from SPs in row one (i.e., 'trash arresting area'), the SPs in row two are also distinct in the required 'trash arresting area' for SPs in row one that they can connect or work with. So, 'trash arresting area' is a required CD for SPs in row two. This implies that SPs in row two can work with SPs in row one with either open, screened or both kinds of 'trash arresting area'.

Furthermore, the model of a CD for any design is shown in Figure 4. Each CD has as many elements as possible. Each of the elements is given a vectorial numerical value. If the elements of a CD are only three such that the first two are the differences in the CD while the third is universal, the first is given a vectorial numerical value of 1 while the second is given -1, then the universal element is made 0.

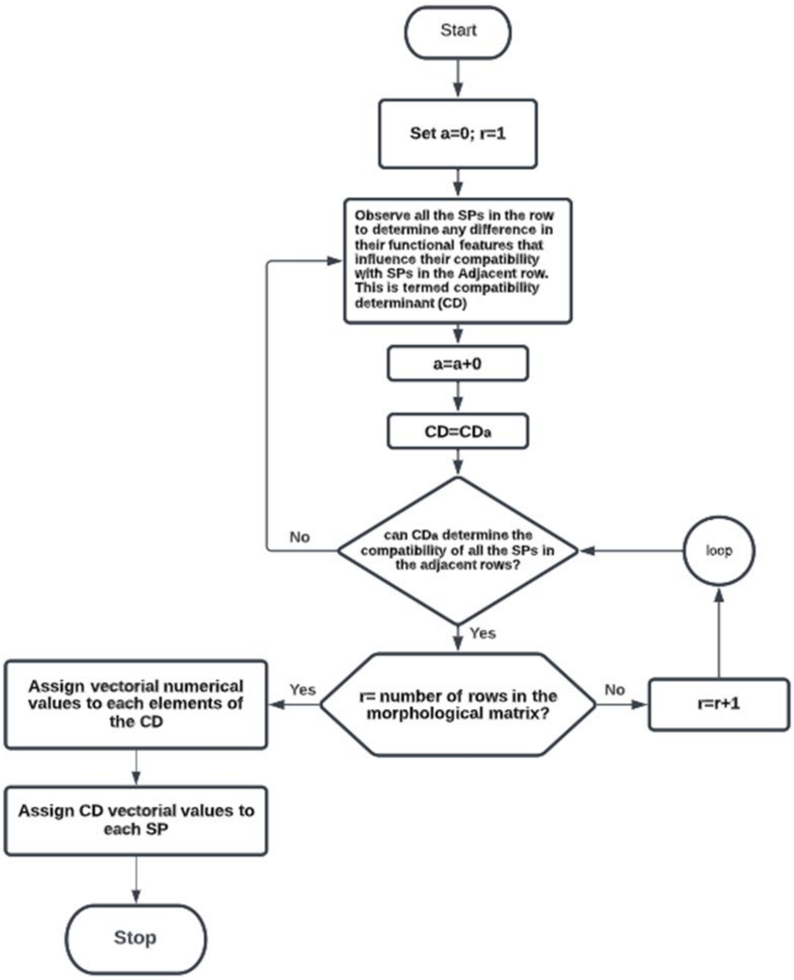


**Figure 2.** Procedure for finding compatibility determinants from a morphological matrix

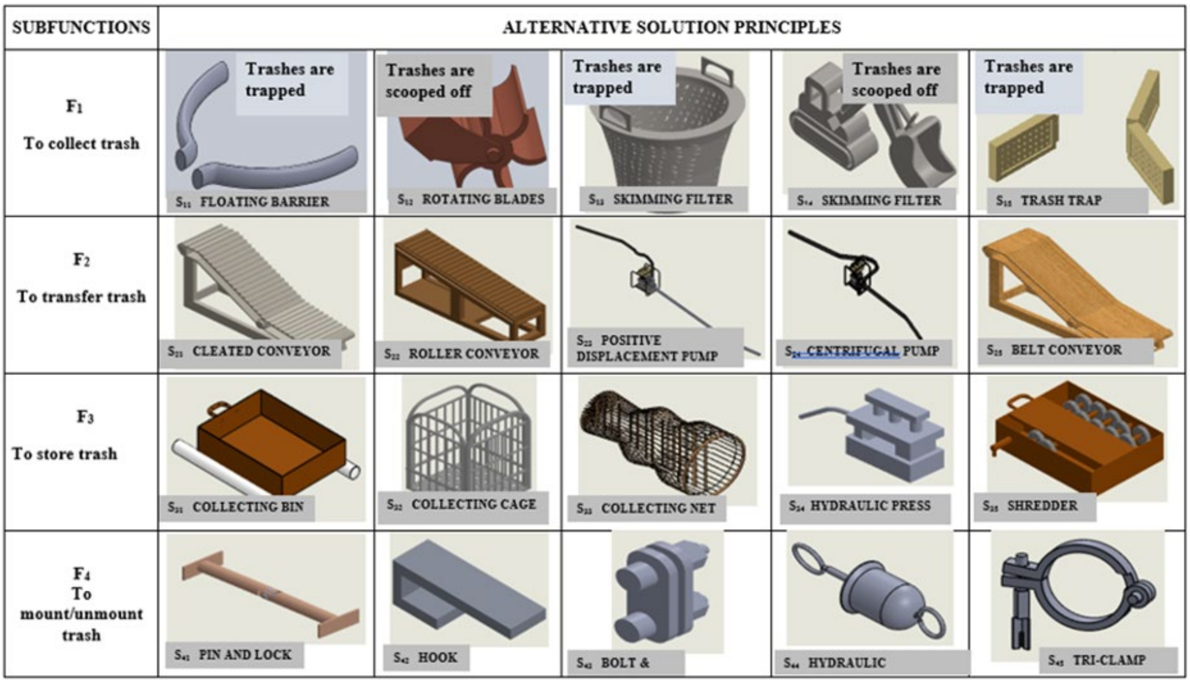


**Figure 3.** Morphological matrix for river cleaning machine design

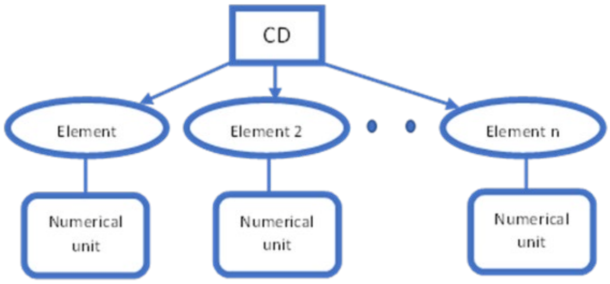


**Figure 4.** Model of compatibility determinant

For $CD_1$ in the river cleaning machine, the 'trash arresting area' for $S_{11}$, $S_{12}$, $S_{14}$ and $S_{15}$ is 'opened', while that of $S_{13}$ is 'screened'. Therefore, $CD_1$ is either 'opened' or 'screened'. Moreover, the 'mode of trash transfer' for $S_{11}$, $S_{12}$, and $S_{15}$ is 'flat conveying', while that of $S_{13}$ and $S_{14}$ is via 'suction'. Therefore, $CD_2$ is either 'flat conveying' or 'suction'. Similarly, for $CD_3$, the 'trash discharge area' may either be 'net/box screened' or 'solid opened'. The number of elements for any CD is not restricted. It may be two, three or more. For any of the CDs, an additional element, 'universal' is added. 'universal' is considered as the CD for SPs that exhibits all the Elements of CD. for instance, in $CD_1$, 'universal' implies both 'opened' and 'screened'. The CDs for the SPs in the morphological matrix of the river cleaning machine are shown in Table 1. Three CDs are identified. The elements of each CD and their vectorial numerical values are indicated.

**Table 1.** Generating values for compatibility determinants of solution principles for conceptual design of river cleaning machine

| Compatibility determinants | Equivalent values | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| ($CD_1$) Trash arresting area | Screened =1 | Opened =-1 | Universal=0 |
| ($CD_2$) mode of trash transfer | flat conveying=1 | Suction=-1 | Universal =0 |
| ($CD_3$) Trash discharge area | Net/box screened =1 | Solid open box =-1 | Universal=0 |

The CD vectorial numerical values are allocated to each SP. The CDs are considered as provided CD, for the set of SPs from which they are derived. Conversely, they are considered required CD for SPs in other rows of the morphological matrix. For instance, in the design of river cleaning machine, $CD_1$ ('trash arresting area') is a provided CD for set of SPs in row one. But $CD_1$ is a required CD for the SPs on other rows than the first row. This implies that based on CD derived from SPs in row one (i.e., 'trash arresting area'), the SPs in row two are also distinct in the required 'trash arresting area' for SP in row one that they can connect or work with. So, 'trash arresting area' is a required CD for SPs in row two.

In addition, CD matrices are composed by replacing each SP in the morphological matrix with the numerical value of the respective CD. the total number of CD matrices in a design is the number of CDs in it. The elements of the CD matrices, as shown in Figure 5, are the corresponding elements from the CDs vectors of each SP. Therefore, Figure 5 represents a set matrix for x number of CDs derived from a morphological matrix with r rows and c columns. For instance, in the design of river cleaning machine, for $CD_1$ matrix, $S_{11}(CD_1)$ and $S_{12}(CD_1)$ are -1 each, while $S_{13}(CD_1)$ is 1. Equally, for $CD_2$ matrix, $S_{11}(CD_2)$ is 0, $S_{12}(CD_2)$ is 1, while $S_{13}(CD_2)$ is -1. The CDs matrix for the river cleaning machine is shown in Figure 6.

$$CD_1 = \begin{pmatrix} S_{11}(CD_1) & S_{12}(CD_1) & - & S_{1c}(CD_1) \\ S_{21}(CD_1) & S_{22}(CD_1) & - & S_{2c}(CD_1) \\ ! & ! & - & ! \\ S_{r1}(CD_1) & S_{r2}(CD_1) & - & S_{rc}(CD_1) \end{pmatrix}$$

$$CD_2 = \begin{pmatrix} S_{11}(CD_2) & S_{12}(CD_2) & & S_{1c}(CD_2) \\ S_{21}(CD_2) & S_{22}(CD_2) & - & S_{2c}(CD_2) \\ ! & ! & - & ! \\ S_{r1}(CD_2) & S_{r2}(CD_2) & - & S_{rc}(CD_2) \end{pmatrix}$$

•

•

$$CD_x = \begin{pmatrix} S_{11}(CD_x) & S_{12}(CD_x) & & S_{1c}(CD_x) \\ S_{21}(CD_x) & S_{22}(CD_x) & - & S_{2c}(CD_x) \\ ! & ! & - & ! \\ S_{r1}(CD_x) & S_{r2}(CD_x) & - & S_{rc}(CD_x) \end{pmatrix}$$

**Figure 5.** Compatibility determinants matrices

Figure 6 contains three CD matrices. The vectorial numerical values of the CD elements that correspond to each SP in the morphological matrix are the elements of each of the matrices. The first, second, fourth and fifth elements of the first row of $CD_1$ matrix is -1, while that of the second element is 1. This implies that the trash arresting area of the first, second, fourth and fifth elements of the first row are open, while that of the third row is screened. The same is applicable to the remaining matrices.

$$CD_1 = \begin{bmatrix} -1 & -1 & 1 & -1 & -1 \\ -1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$CD_2 = \begin{bmatrix} 0 & 1 & -1 & 1 & 0 \\ 1 & 1 & -1 & -1 & 1 \\ 0 & -1 & -1 & -1 & 0 \\ 0 & 1 & 1 & 1 & -1 \end{bmatrix}$$

$$CD_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & 1 & -1 \\ -1 & -1 & -1 & -1 & 1 \end{bmatrix}$$

**Figure 6.** Compatibility determinants matrices for sps of river cleaning machine

## Formation of Solution Variants Matrix

The formation of SVs by the combination of SPs entails picking one SP from a row and combining the same with SPs from other rows. The SPs are identified as Snm such that n and m are the row and column numbers of the SP within the morphological matrix. Hence, an SV is a chain of SPs that can be represented as, [$S_{nm}$---$S_{(r-1)m}$--- $S_{rm}$]. The first SV is the chain of the first SP for each sub-function (i.e. [$S_{11}$, $S_{21}$,------$S_{(r-1)1}$, $S_{r1}$]). For instance, the first SV for the river cleaning machine is [$S_{11}$, $S_{21}$, $S_{31}$, $S_{41}$]. Subsequent SVs are formed by methodically substituting the SPs in the first SV with other SPs from the morphological matrix.

The number of SPs in an SV is the number of sub-functions in the morphological matrix. This implies that, for a morphological matrix with four sub-functions like the river cleaning machine, every SV contains four SPs. The first SP is from row one, the second SP is from row two, and the third is from row three and so on. Formation of SVs is an incremental substitution process in which SPs from the same row in the morphological matrix substitute each other in the SV chain. The substitution of SPs for formation of the subsequent SV is by picking the SPs in the same row serially in loops. The replacement is done among SPs of the same row, starting from the last row up to the first. The second SV is formed by replacing the first SP of the last row Sr1 with the second SP of the last row ($S_{r2}$) in the first SV chain. As such, the second SV is [$S_{11}$, $S_{21}$, $S_{31}$, $S_{41}$, ----- $S_{r2}$]. For the river cleaning machine, the second SV is [$S_{11}$, $S_{21}$, $S_{31}$, $S_{42}$].

Furthermore, the SPs of the last sub-function are serially replaced in the SV chain until the last SP of Fr ($S_{rc}$) is placed. At this stage, a total of 'c' (number of columns in the morphological matrix) SVs have been formed. The first 'c' SV formed are [$S_{11}$, $S_{21}$, $S_{31}$, $S_{41}$, ----- $S_{r1}$], [$S_{11}$, $S_{21}$, $S_{31}$, $S_{41}$, ----- $S_{r2}$], ------- [$S_{11}$, $S_{21}$, $S_{31}$, $S_{41}$, ----- $S_{r(c-1)}$] and [$S_{11}$, $S_{21}$, $S_{31}$,

$S_{41}$, ----- $S_{rc}$]. For example, in the design of river cleaning machine, c=5. Therefore, the first five SV formed for the design of river cleaning machine are [$S_{11}$, $S_{21}$, $S_{31}$, $S_{41}$], [$S_{11}$, $S_{21}$, $S_{31}$, $S_{42}$], [$S_{11}$, $S_{21}$, $S_{31}$, $S_{43}$], [$S_{11}$, $S_{21}$, $S_{31}$, $S_{44}$] and [$S_{11}$, $S_{21}$, $S_{31}$, $S_{45}$].

Afterwards, the SV chain is altered to form the next set of 'c' SVs by changing $S_{(r-1)1}$ (SPs for the sub-function that precedes the last sub-function $F_{(r-1)}$, into $S_{(r-1)2}$ (the second SP of the respective sub-function). Equally, the looping of the SP of Fr in the chain is executed by picking them serially from Sr1 to Src. Accordingly, the second set of 'c' SV is [$S_{11}$, $S_{21}$, $S_{31}$, $S_{41}$, ---- $S_{(r-1)2}$, $S_{r1}$], [$S_{11}$, $S_{21}$, $S_{31}$, $S_{41}$, ----- $S_{(r-1)2}$, $S_{r2}$], ------- [$S_{11}$, $S_{21}$, $S_{31}$, $S_{41}$, ----- $S_{(r-1)2}$, $S_{r(c-1)}$] and [$S_{11}$, $S_{21}$, $S_{31}$, $S_{41}$, ----- $S_{(r-1)2}$, $S_{rc}$]. The SPs of $F_{(r-1)}$ are serially replaced in the SV chain after every set of 'c' SVs are formed. This implies that the SPs of $F_{(r-1)}$ (i.e., $S_{(r-1)m}$) in the SV chain are replaced after every 'c' or simply c1 SVs are formed. In the same vein, the SP of $F_{(r-2)}$ (i.e., $S_{(r-2)m}$) in the SV chain are replaced after all the 'c' SPs of $F_{(r-1)}$ has been used in the SV chain. Therefore, the SPs of $F_{(r-2)}$ in the SV are serially replaced after every 'c' set of 'c' SVs have been formed. Simply put, the SPs of $F_{(r-2)}$ (i.e., $S_{(r-2)m}$) in the SV chain are replaced after c multiplied by c, or simply c2 SVs are formed. This is maintained till the SPs of all the sub-functions are changed.

On this note, it can be deduced that to extract SV from a morphological matrix with r rows and c columns; the first SV is a chain of the first SP ($S_{11}$, $S_{12}$, ------- $S_{(r-1)1}$, $S_{r1}$) of all sub-functions ($F_1$ to $F_r$). Subsequent SVs are formed by serially replacing the SPs ($S_{r1}$ to $S_{rc}$) of the last sub-function ($F_r$) or ($F_{(r-0)}$) when every $c^0$ SVs are formed until the cth SP of $F_{(r-0)}$ is used. Equally, the SPs of the sub-function $F_{(r-1)}$ are serially replaced when every $c^1$ SV is formed until the cth SP of $F_{(r-1)}$ is used. Similarly, the SPs of the sub-function $F_{(r-2)}$ is serially changed when every $c^2$ SVs are formed until the cth SP of $F_{(r-2)}$ is used. Likewise, the SPs of the sub-function $F_{(r-3)}$ is serially replaced when every $c^3$ SVs are formed until the cth SP of $F_{(r-3)}$ is used. Therefore, it can be concluded that the SPs of any sub-function $F_{(r-n)}$ are serially replaced when cn SVs are formed until the cth SP of $F_{(r-n)}$ is used.

Hence it can equally be concluded that each row of the G-matrix contains only one SP of the first sub-function ($F_1$) since each row of the G-matrix contains SVs formed by changing the SPs of all the sub-functions from the last sub-function Fr to the second sub-function F2. Moreover, as construed in the analysis above, the SPs of any sub-function $F_{(r-n)}$ is serially changed at every cn step, until the cth SP of $F_{(r-n)}$ is used. The SPs of the first sub-function ($F_1$), which is equal to $F_{(r-(r-1))}$, is serially changed when c(r-1) SV are formed until the cth SP of $F_{(r-(r-1))}$ is used. Based on this, it can be deduced that each row of the G-matrix contains $c^{(r-1)}$ SVs. Therefore, the G-matrix contains $c^{(r-1)}$ columns. Besides, only one SP of $F_1$ is contained in each row of the G-matrix. Therefore, the G-matrix contains c rows since there are 'c' $S_{1m}$ in the morphological matrix. Consequently, the total number of elements in the G-matrix is c^((r-1))×c^1=c^r.

For instance, the morphological matrix developed for the river cleaning machine, shown in Figure 3, contains four rows and five columns. Therefore, the G-matrix for the river cleaning machine contains five rows and $5^{(4-1)}$ [($5^3$), 125 columns, as indicated in Figure 7. Furthermore, a total of (54) theoretical SVs can be formed out of the morphological matrix.

$$\begin{pmatrix} G_{11} & G_{12} & G_{13} & G_{14} & G_{15} & G_{16} & G_{17} & G_{18} & G_{19} \text{---} G_{130} \text{------} & G_{1124} & G_{1125} \\ G_{21} & G_{22} & G_{23} & G_{24} & G_{25} & G_{26} & G_{27} & G_{28} & G_{29} \text{---} G_{230} \text{------} & G_{2124} & G_{2125} \\ G_{31} & G_{32} & G_{33} & G_{34} & G_{35} & G_{36} & G_{37} & G_{38} & G_{39} \text{---} G_{330} \text{------} & G_{3124} & G_{3125} \\ G_{41} & G_{42} & G_{43} & G_{44} & G_{45} & G_{46} & G_{47} & G_{48} & G_{49} \text{---} G_{430} \text{------} & G_{4124} & G_{4125} \\ G_{51} & G_{52} & G_{53} & G_{54} & G_{55} & G_{56} & G_{57} & G_{58} & G_{59} \text{---} G_{530} \text{------} & G_{5124} & G_{5125} \end{pmatrix}$$

**Figure 7.** Concept matrix (G-Matrix) and equivalent combinatorial solution

## Analysis of Solution Variants Identity in Morphological and G-Matrix

To enhance the tracing of SPs in SV chains from G-matrix to their respective roots in the morphological matrix, an analysis of the relationship between the morphological matrix and the G-matrix is necessary. Furthermore, the equivalent SV of any element of the G-matrix can be inferred. Let G($R_i$, $C_j$), be any element of a G-matrix that represents an SV. Also, let the total number of rows and columns of the G-matrix be represented as R and C, respectively. As established in the previous section, for a morphological matrix with r number of rows and c number of columns, the total number of theoretical SV in the G-matrix formed from it is $c^r$. Besides, the total number of rows and columns in the G-matrix are c and $c^{(r-1),}$ respectively.

Furthermore, any SV in the G-matrix (i.e. an element of G-matrix designated as G($R_i$, $C_j$), can be represented as [$S_{1a}$, $S_{2m}$, ---------$S_{(r-1)m}$, $S_{rm}$]. In [$S_{1a}$, $S_{2m}$, ---------$S_{(r-1)m}$, $S_{rm}$], 'a' represents the column number of the first SP in the SV chain (i.e., the column number of the SP of $F_1$), while 'm' represent the column number of any other SPs in the SV chain. Moreover, every row of the G-matrix contains only one SP from the first row of the morphological matrix. Besides, the column number of the SP in $F_1$ from the morphological matrix is expected to be the same as the row number of the SV in which they are in the G-matrix. This is mathematically represented in Eq. (1).

$$a = R_i \tag{1}$$

Moreover, as analysed in the the previous section, it is established that the SP for a particular sub-function ($F_n$) is altered in an SV chain after the formation of every $c^{(r-n)}$ SVs. Equally, the occurrence of the SPs in the SV chain is interdependent. The column number of the SP in the morphological matrix (m) is a component of the column number of the SV in the G-matrix in which it is an element ($C_j$). Therefore, the morphological matrix column number ($m_2$) for SP of $F_2$ in SV can be found by dividing $C_j$ by $c^{(r-2)}$ in Eq. (2). The results of the division are considered in terms of the quotient ($Q_2$) and the remainder ($P_2$) (Eq. (2)). For the second SP in the SV chain, if $P_2$ is zero, m_2=Q_2. If $P_2$ is greater than zero, then $m_2=Q_2+1$. To determine 'm_n' for the subsequent SP in the SV chain, the remainder in the preceding division ($P_{(n-1)}$ ) is divided by the factor $c^{(r-n)}$ (Eq. 3). The value of 'm_n' is $Q_n$ if $P_n$ is equal to zero (Eq. (4)). Otherwise, if $P_n$ is greater than zero, '$m_n$' is equal to ($1+Q_n$). Furthermore, if $P_{(n-1)}$ is equal to zero, '$m_n$' is equal to c (Eq. (5)).

$$\frac{C_j}{c^{(r-2)}} = Q_2/P_2 \tag{2}$$

$$\frac{P_{n-1}}{c^{(r-n)}} = Q_n/P_n \tag{3}$$

$$m_n = Q_n \tag{4}$$

$$m_n = 1 + Q_n \tag{5}$$

$$m_n = c \tag{6}$$

Another important relationship is the expression for determining the elements of the G-matrix that represents a formulation of SV. For instance, given an SV [$S_{1a}$, $S_{2m}$, ---------$S_{(r-1)m}$, $S_{rm}$], based on Eq. (1), $R_i = a$, to find $C_j$, the SPs substitution index ($c^{(r-n)}$) for each SP in the SV chain from $n = 2$ to $n = r - 1$ is multiplied by $(m_n - 1)$. This sum is then added to $m_r$. The expression for determining $C_j$ is stated in Eq. (7).

$$C_j = \sum_{n=2}^{n=r-1} \left((m_n - 1) \times c^{r-n}\right) + m_r \tag{7}$$

Applying Eq. (1) and Eq. (7), the element of the G-matrix representing SV [$S_{13}$, $S_{25}$, $S_{35}$, $S_{41}$] of the river cleaning machine, can be determined. In this example, $R_i = 3$. Moreover, $C_j = (5-1) \times 5^{4-2} + (5-1) \times 5^{4-3} + 1 = 121$ (Eq. (7)). Therefore, SV [$S_{13}$, $S_{25}$, $S_{35}$, $S_{41}$] is represented in G-matrix as element G(3,121). The applicability of the expression was tested on many morphological matrices and was found valid.

## Logical Relation for Solution Variants (Concepts) Screening

The morphological matrix is represented as a rectangular matrix. As such, some of the concepts in the G-Matrix, especially those that contain SP of the empty cells in the morphological matrix, are unrealistic. Besides, not all the possible combinations contained in the G-Matrix are feasible concepts. To determine the feasibility of an SV, the CDs of all adjacent SPs in the combinatorial solution chain are examined. Then the feasibility of the SV is confirmed when the logical relation stated below are maintained.

For any SV of combinatorial solution chain [$S_{1a}$, $S_{2m}$, ---------$S_{(r-1)m}$, $S_{rm}$] to be a feasible concept, Eq. (8) to Eq. (13) must be fulfilled.

$$CD_1(S_{1a}) = CD_1(S_{2m}) \text{ or } CD_1(S_{1a}) = 0 \text{ or } CD_1(S_{2m}) = 0 \tag{8}$$

$$CD_2(S_{1a}) = CD_2(S_{2m}) \text{ or } CD_2(S_{1a}) = 0 \text{ or } CD_2(S_{2m}) = 0 \tag{9}$$

$$CD_i(S_{1a}) = CD_i(S_{2m}) \text{ or } CD_i(S_{1a}) = 0 \text{ or } CD_i(S_{2m}) = 0 \tag{10}$$

$$CD_1(S_{(r-1)a}) = CD_1(S_{rm}) \text{ or } CD_1(S_{(r-1)a}) = 0 \text{ or } CD_1(S_{rm}) = 0 \tag{11}$$

$$CD_2(S_{(r-1)a}) = CD_2(S_{rm}) \text{ or } CD_2(S_{(r-1)a}) = 0 \text{ or } CD_2(S_{rm}) = 0 \tag{12}$$

$$CD_i(S_{(r-1)a}) = CD_i(S_{2m}) \text{ or } CD_i(S_{(r-1)a}) = 0 \text{ or } CD_i(S_{2m}) = 0 \tag{13}$$

In Eq. (8) to Eq. (13), $CD_1$ to $CD_i$ are the various compatibility determinants from the first to the i^th^ CD. Moreover, $S_{1a}, S_{2m}, S_{(r-1)a}$, and $S_{rm}$, are the SPs in row one column a, row two column m, row $(r-1)$ column a and row $r$ column m respectively. The expressions can be verbalised as "for an SV to be feasible, the CDs of all the adjacent SPs in the combinatorial solution chain must be equal in magnitude and direction, or one or either of them must be zero.

Eq. (8) to Eq. (13) can be demonstrated using the SVs in Table 2. The CDs of the SVs [$S_{11}$, $S_{21}$, $S_{31}$, $S_{41}$] and [$S_{11}$, $S_{22}$, $S_{34}$, $S_{45}$] are shown in Table 2. It can be observed that for [$S_{11}$, $S_{21}$, $S_{31}$, $S_{41}$], $CD_1S_{11} = CD_1S_{2,1}$, $CD_2S_{11}$ and $CD_3S_{11}$ are both zero, also $CD_1S_{31}$ and $CD_2S_{31}$ are both zero and $CD_3S_{21} = CD_3S_{31}$ also $CD_1S_{31}$ and $CD_2S_{31}$ are both zero and $CD_3S_{31} = CD_3S_{41}$. Since the CDs of the SV fulfils the logical condition, it is considered as feasible. However, for SV [$S_{11}$, $S_{22}$, $S_{34}$, $S_{45}$], $CD_1S_{11} \neq CD_1S_{22}$ aslo $CD_2S_{11} \neq CD_2S_{22}$ though $CD_3S_{11} \neq 0$. With inequality of the CDs of any adjacent SP in the combinatorial chain, the SP [$S_{11}$, $S_{22}$, $S_{34}$, $S_{45}$] is infeasible.

**Table 2.** The compatibility determinants vectors for two solution variables

| | $CD_1$ | $CD_2$ | $CD_3$ | | $CD_1$ | $CD_2$ | $CD_3$ |
|---|---|---|---|---|---|---|---|
| $S_{11}$ | -1 | 0 | 0 | $S_{11}$ | 1 | -1 | 0 |
| $S_{21}$ | -1 | 1 | -1 | $S_{22}$ | -1 | 1 | -1 |
| $S_{31}$ | 0 | 0 | -1 | $S_{34}$ | 0 | -1 | 1 |
| $S_{41}$ | 0 | 1 | -1 | $S_{45}$ | 0 | -1 | 1 |

### Concept Evaluation and Selection

Concept evaluation for the framework of this study entails the determination of weighted factors for the evaluation criteria. This is similar to the weighted Pugh method [10] [32]. In this approach, the ECs are scaled at ratios based on the preference for the design. Each of the ECs is given a scale factor that sums up to 1 or 100%. The weight factor of an EC is the ratio it has compared to all other ECs in the design. For instance, safety, let efficiency, maintainability, material, reliability, and cost be six ECs used in a design. Moreso, each of these ECs may be allotted 0.16, 0.17, 0.16, 0.16, 0.18, 0.17, making 1. Each of these allotted scale values are termed scale factor. The factor used for evaluation in this method is the weighted factor. Depending on the number of EC considered in the design, the weight factor of the ECs are determined using Eq. (14) and Eq. (15).

$$\widehat{W}_u = \frac{F_u}{\sum_{v=1}^{t} F_v} \times 100\% \tag{14}$$

$$W_u = \frac{\widehat{W}_u}{100} \tag{15}$$

Where $\widehat{W}_u$, $W_u$, $F_u$, and $F_v$, are the percentage weight, weight factor, and scale factor of the EC, and the scale factor of the entire ECs respectively. Also, $v$ ranges from 1 to $t$, while $t$ is the last EC. Furthermore, the SPs are also rated with performance values. These performance values are equally converted into normalised weighted factors. Using Eq. (14) or Eq. (15). The choice of these Eq. (16) or Eq. (17) depends on the desirability of the EC. For EC whose higher values are desirable e.g. efficiency, safety and manufacturability, Eq. (17) is applied. Conversely, for those whose lower values are desirable, like material and cost, Eq. 16 is applied.

$$\boldsymbol{W_u S_{nm}} = W_u \times \frac{\min(X_{nm})}{X_{nm}} \tag{16}$$

$$\boldsymbol{W_u S_{nm}} = W_u \times \frac{X_{nm}}{\max(X_{nm})} \tag{17}$$

In Eq. (16) and Eq. (17), $W_uS_{nm}$ implies the weighted factor of SP in row 'n' and column 'm' of the morphological matrix, for EV 'u'. In addition, $W_u$ is the weight factor of the respective EC obtained from Eq. (15). Furthermore, $X_{nm}$ is the performance values of the SP in row 'n' and column 'm' of the morphological matrix. Finally, $\min(X_{nm})$ and $\max(X_{nm})$ are the minimum and maximum performance values of all the SP for the respective EC.

Furthermore, the weighted factor for each SV for each EC is determined by summing up the weighted factors of all the SPs in the combinatorial chain that makes the respective SV. Thereafter, the weighted factors for all the EC for each SV are summed up. The SV with the highest total weighted factor is selected for the preliminary design.

## TESTING THE RELIABILITY OF THE MATHEMATICAL RELATIONS BETWEEN THE SPS AND SVS

The mathematical relations developed to determine the SPs that make an SV are tested on different sizes of morphological matrices and respective G-matrices to verify the consistency of the mathematical relations. 20 random sample of morphological matrices sizes and their respective G-matrix elements were considered for verification. The results obtained are discussed in the discussion section.

The validity of the mathematical expressions in Eq. (2) to Eq. (6) was confirmed from many trials. Considering the river cleaning machine, r and c are 4 and 5 respectively. The total number of theoretical SVs that can be generated from it is $c^r$. Besides, the G-matrix for the design contains 5 (c) rows and $5^{4-1}$ (125) columns. For example, the formulation of the SV that is represented by the element G(3,121) from the G-matrix can be determined using the expression in Eq. (2) to Eq. 6.

Let the SV represented by G(3,121) in the G-matrix (i.e. $G_{3\ 121}$) be [$S_{1a}$, $S_{2m}$, ---------$S_{(r-1)m}$, $S_{rm}$]. Applying Eq. 1, $a = 3$. Furthermore, applying Eq. (2), $Q_2 = 4$ while $P_2 = 21$. Since $P_2 > 0$, $m_2 = 4 + 1 = 5$ (Eq. (5)). Moreover, from Eq. (3), $Q_3 = 4$ while $P_3 = 1$. Since $P_3 > 0$, $m_3 = 4 + 1 = 5$. Then $Q_4 = 1$ while $P_3 = 0$. Since $P_3 = 0$, $m_4 = 1$ (Eq. (4)). Therefore, the SV of the river cleaning machine represented by (3,121) is [$S_{13}$, $S_{25}$, $S_{35}$, $S_{41}$].

## CASE STUDY

Meanwhile, the conceptual design of CVT using the framework developed in this study is fully discussed here. Continuously variable transmission (CVT) is a transmission system used in automobiles and other applications for enabling the transmission of engine torque to the drive line at an infinitely variable velocity ratio. The main difference between an automatic step transmission system and a CVT system is the use of variator in CTV, in place of the step gears in the automatic step transmission system. Therefore, the conceptual design of CVT focuses on the design of variators. The conceptual design of CVT was done following the procedure laid down in this study.

### Development of Morphological Matrix for CVT

The main function of CVT is 'to transmit engine torque to the drive line at infinitely variable speed, based on the required driveline torque'. The main function was decomposed into four sub-functions. Furthermore, SPs were generated for each of the sub-functions. Thereafter, the morphological matrix was composed, as shown in Figure 8, to indicate the sub-functions and their respective SPs. Moreover, Table 3 represents the morphological matrix in letters. This is meant to facilitate the analysis process.

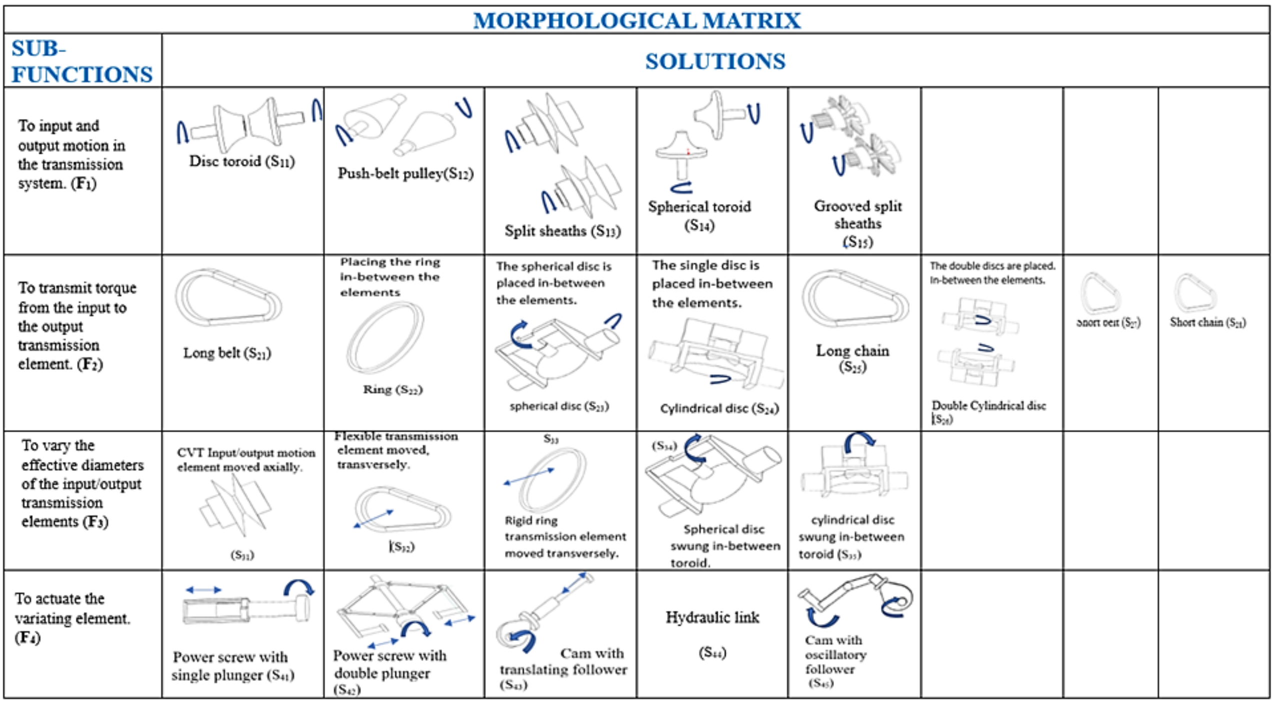


**Figure 8.** Morphological matrix for conceptual design of CVT

**Table 3.** Representation of morphological matrix of CVT

| Sub-functions | Solution principles | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $F_1$ | $S_{11}$ | $S_{12}$ | $S_{13}$ | $S_{14}$ | $S_{15}$ | - | - | - |
| $F_2$ | $S_{21}$ | $S_{22}$ | $S_{23}$ | $S_{24}$ | $S_{25}$ | $S_{26}$ | $S_{27}$ | $S_{28}$ |
| $F_3$ | $S_{31}$ | $S_{32}$ | $S_{33}$ | $S_{34}$ | $S_{35}$ | - | - | - |
| $F_4$ | $S_{41}$ | $S_{42}$ | $S_{43}$ | $S_{44}$ | $S_{45}$ | - | - | - |

### Identification of Compatibility Determinants and Performance Values for CVT

Four CDs are identified, as shown in Table 4. Observing the SPs of $F_1$, it is noticed that the orientation of shafts of the elements that input/output motion are different. Based on this, a CD is formulated, termed 'orientation of transmission shaft'. This is the first CD ($CD_1$). To $CD_1$ five elements are deduced. Some of the shafts are 'parallel with wide gap', some are 'parallel with narrow gap', some are 'inline' while some are 'perpendicular'. The fifth element is the 'universal' element. Therefore, the five elements of $CD_1$ ('orientation of transmission shaft') are ['universal', 'parallel with wide gap', 'parallel with narrow gap', 'inline', 'perpendicular'] respectively represented with units [0, 1, -1, 2, -2]. In the same vein, from the SPs of $F_2$, the transmission elements are noted to differ in their flexibility. Hence, the second CD ($CD_2$) is termed 'rigidity of linking element'. Three elements are deduced for $CD_2$, which are ['universal', 'rigid', 'flexible'] represented in units as [0, 1, -1] respectively.

Furthermore, the third CD ($CD_3$) is deduced from the SPs of $F_3$ as 'Rigidity of the variating element'. The elements deduced for $CD_3$ are respectively, ['universal', 'rigid', 'flexible'] represented in units as [0, 1, -1]. Lastly, the fourth CD ($CD_4$) is the motion type of the variating element. $CD_4$ equally have three elements which are ['universal', 'reciprocating', 'oscillating'] represented in units as [0, 1, -1] respectively. The various CDs and their elements are indicated in Table 4. The CDs are attached to each SP in the morphological matrix to form the CD matrices.

**Table 4.** Values for the compatibility determinant of CVT

| Compatibility determinant | Elements representation and units | | | | |
|---|---|---|---|---|---|
| $CD_1$ orientation of transmission shaft | Universal (0) | Parallel with wide gab (1) | Parallel with narrow gab (-1) | Inline (2) | Perpendicular (-2) |
| $CD_2$ rigidity of linking element | Universal (0) | Rigid (1) | Flexible (-1) | | |
| $CD_3$ rigidity of variating element | Universal (0) | Rigid (1) | Flexible (-1) | | |
| $CD_4$ motion type of variating element | Universal (0) | Reciprocating (1) | Oscillating (-1) | | |

The numerical representations for the CDs of each SPs are shown in Figure 9. The first matrix is that of $CD_1$. $S_{11}$, $S_{12}$, $S_{13}$, $S_{14}$ and $S_{15}$ have 2, -1, 1, -2 and 1, respectively. It implies that the shafts of $S_{11}$ are inline, those of $S_{12}$ are parallel with a narrow gap, those of $S_{14}$, are perpendicular to each other, while those of $S_{13}$ and $S_{15}$ are parallel with a wide gap. The empty cells in the first row are given the same CD numerical values. Their numerical values are higher than that of the established CD. This is to enable ease of manipulation of the matrices by the computer programme.

Furthermore, $CD_1$ for $S_{21}$, $S_{22}$, $S_{23}$, $S_{24}$, $S_{25}$, $S_{26}$, $S_{27}$and $S_{28}$ are 1, -1, -2, 2, 1, 2, -1, -1, respectively. Since $CD_1$ was not derived from $S_{21}$ to $S_{28}$, $CD_1$ is a required CD for them. SP $S_{21}$ and $S_{25}$ only work with SP $S_{11}$ to $S_{15}$ whose shafts are parallel with wide gap. Besides, $S_{22}$, $S_{27}$, and $S_{28}$ can work with only SP $S_{11}$ to $S_{15}$ whose shafts are parallel with narrow gap. Also, $S_{23}$ can work with only SP $S_{11}$ to $S_{15}$ whose shafts are perpendicular to each other. In the same vein, $S_{24}$ and $S_{26}$ can only work with SP $S_{11}$ to $S_{15}$ whose shafts are inline with each other. The CDs of the remaining SP were found in the same manner.

$$CD_1 = \begin{bmatrix} 2 & -1 & 1 & -2 & 1 & 3 & 3 & 3 \\ 1 & -1 & -2 & 2 & 1 & 2 & -1 & -1 \\ 1 & -1 & -1 & -2 & 2 & 4 & 4 & 4 \\ 0 & 0 & 0 & 0 & 0 & 5 & 5 & 5 \end{bmatrix}$$

$$CD_2 = \begin{bmatrix} 1 & 0 & -1 & 1 & -1 & 3 & 3 & 3 \\ -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 & 1 & 4 & 4 & 4 \\ 0 & -1 & 0 & 0 & 0 & 5 & 5 & 5 \end{bmatrix}$$

$$CD_3 = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 3 & 3 & 3 \\ 0 & 1 & 1 & 1 & 0 & 1 & -1 & -1 \\ 1 & -1 & 1 & 1 & 1 & 4 & 4 & 4 \\ 0 & 1 & 0 & 0 & 1 & 5 & 5 & 5 \end{bmatrix}$$

$$CD_4 = \begin{bmatrix} -1 & 1 & 1 & -1 & 1 & 3 & 3 & 3 \\ -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & 1 & -1 & -1 & 4 & 4 & 4 \\ 0 & 1 & 0 & 0 & -1 & 5 & 5 & 5 \end{bmatrix}$$

**Figure 9.** Representation of CDs in Morphological Matrix of CVT

### Concept Evaluation and Selection

Four ECs were chosen arbitrarily based on common needs for transmission systems from the researcher's experience. The ECs used are efficiency, manufacturability, maintainability, and cost. All the ECs were rated evenly, so their weight actors were all the same (0.25). Furthermore, performance values were allotted to each SP for each ECs. Using Eq. (16) and (17), the normalised weighted factors of each SP for each EC were determined. Moreover, the weighted factors of each SV for each EC were determined by summing up the weighted factors of the constituent SPs of the SV for the respective EC. Thereafter, the various weighted factors of each SV for each EC were summed up to obtain the total weighted factor for each SV. The SV with the highest value of the weighted factor was chosen for the preliminary design.

### Testing the Consistency of CD Generation on Designers

The second aspect of validation of the framework developed in this study is concerned with testing it on designers. The aim of the test is to examine the consistency and replicability of the CDs and the allocation of numerical values to represent the elements of each CDs for each SP. It is designed to check if the same results will be achieved if different designers allocate CDs to the SPs in the same morphological matrix. The test was conducted on the staff and Higher National Diploma students of Mechanical Engineering Technology at Federal Polytechnic Bauchi. The research was conducted on a total of twenty participants. The participants were divided into nine groups.

The design of a river cleaning machine (RCM) was used as an example to explain the new method to the participants. Furthermore, the morphological matrix of CVT developed in this study as a case study was used for the experiment. The participants were introduced to CVT. The main function and subfunctions of CVT, as indicated on the morphological matrix were explained. Furthermore, they were also shown pictures of existing CVTs like the pulley-based, toroidal, and cone CVT. The introduction section of the experiment took one hour and thirty minutes. Besides, the participants were provided with copies of the morphological matrix of CVT. Each group was given a period of thirty minutes to understudy the morphological matrix. Thereafter, they were given forty-five minutes to propose the appropriate CDs for the SPs. They were able to propose the CDs. Furthermore, each group was asked to assert the CDs to each of the SPs within thirty minutes. In all, three hours and fifteen minutes was expended on the experiment.

## RESULTS AND DISCUSSION

The test conducted to verify the reliability of the mathematical expressions between the SPs in an SV and their respective positions in both the morphological and the G-matrix indicates 100% accuracy on the specimen; Eq. (2) to Eq. (7) were applied. The mathematical relations were accurate in determining the SPs formulations of the SVs. Besides, by applying Eq. (7), the location of the SVs in the G-matrix was traced back and the values obtained were accurate.

Furthermore, the results obtained from the case study indicate total eliciting of the feasible concepts from the morphological matrix using the framework developed for this work. The morphological matrix of CVT built in this study contains four rows and eight columns, though not all the cells contain SPs. The total of theoretical concepts that can be extracted from the morphological matrix is $5 \times 8 \times 5 \times 5 = 1000$. Out of these 1000 theoretical concepts, 37 feasible concepts were obtained in Table 5. All 37 concepts are verified and found to be feasible based on compatibility of the SPs in the SV chains. Furthermore, the remaining SVs (non-feasible concepts) were also checked. A random sample of thirty SVs was picked and checked. They were all found to contain non-compatible SPs.

**Table 5.** Generated feasible concepts after 3D Matrix-based screening for CVT design

| S/N | Concept | S/N | Concept | S/N | Concept |
|---|---|---|---|---|---|
| 1 | $S_{11}$ $S_{24}$ $S_{35}$ $S_{41}$ | 14 | $S_{12}$ $S_{27}$ $S_{32}$ $S_{44}$ | 27 | $S_{14}$ $S_{23}$ $S_{34}$ $S_{43}$ |
| 2 | $S_{11}$ $S_{24}$ $S_{35}$ $S_{43}$ | 15 | $S_{12}$ $S_{28}$ $S_{32}$ $S_{41}$ | 28 | $S_{14}$ $S_{23}$ $S_{34}$ $S_{44}$ |
| 3 | $S_{11}$ $S_{24}$ $S_{35}$ $S_{44}$ | 16 | $S_{12}$ $S_{28}$ $S_{32}$ $S_{43}$ | 29 | $S_{14}$ $S_{23}$ $S_{34}$ $S_{45}$ |
| 4 | $S_{11}$ $S_{24}$ $S_{35}$ $S_{45}$ | 17 | $S_{12}$ $S_{28}$ $S_{32}$ $S_{44}$ | 30 | $S_{15}$ $S_{21}$ $S_{31}$ $S_{41}$ |
| 5 | $S_{11}$ $S_{26}$ $S_{35}$ $S_{41}$ | 18 | $S_{13}$ $S_{21}$ $S_{31}$ $S_{41}$ | 31 | $S_{15}$ $S_{21}$ $S_{31}$ $S_{42}$ |
| 6 | $S_{11}$ $S_{26}$ $S_{35}$ $S_{43}$ | 19 | $S_{13}$ $S_{21}$ $S_{31}$ $S_{42}$ | 32 | $S_{15}$ $S_{21}$ $S_{31}$ $S_{43}$ |
| 7 | $S_{11}$ $S_{26}$ $S_{35}$ $S_{44}$ | 20 | $S_{13}$ $S_{21}$ $S_{31}$ $S_{43}$ | 33 | $S_{15}$ $S_{21}$ $S_{31}$ $S_{44}$ |
| 8 | $S_{11}$ $S_{26}$ $S_{35}$ $S_{45}$ | 21 | $S_{13}$ $S_{21}$ $S_{31}$ $S_{44}$ | 34 | $S_{15}$ $S_{25}$ $S_{31}$ $S_{41}$ |
| 9 | $S_{12}$ $S_{22}$ $S_{33}$ $S_{41}$ | 22 | $S_{13}$ $S_{25}$ $S_{31}$ $S_{41}$ | 35 | $S_{15}$ $S_{25}$ $S_{31}$ $S_{42}$ |
| 10 | $S_{12}$ $S_{22}$ $S_{33}$ $S_{43}$ | 23 | $S_{13}$ $S_{25}$ $S_{31}$ $S_{42}$ | 36 | $S_{15}$ $S_{25}$ $S_{31}$ $S_{43}$ |
| 11 | $S_{12}$ $S_{22}$ $S_{33}$ $S_{44}$ | 24 | $S_{13}$ $S_{25}$ $S_{31}$ $S_{43}$ | 37 | $S_{15}$ $S_{25}$ $S_{31}$ $S_{44}$ |
| 12 | $S_{12}$ $S_{27}$ $S_{32}$ $S_{41}$ | 25 | $S_{13}$ $S_{25}$ $S_{31}$ $S_{44}$ | | |
| 13 | $S_{12}$ $S_{27}$ $S_{32}$ $S_{43}$ | 26 | $S_{14}$ $S_{23}$ $S_{34}$ $S_{41}$ | | |

In addition, the concept evaluation was carried out in Excel and the concept with the highest weighted factor and its respective weighted factors for each EC are indicated in Table 6. Moreover, the selected SV is indicated with a linking line in the morphological matrix in Figure 10.

**Table 6.** Selected concept for CVT design

| S/N | Concept | Efficiency | Manufacturability | Maintainability | Cost | Total score |
|---|---|---|---|---|---|---|
| 1 | $S_{15}S_{25}$ $S_{31}$ $S_{41}$ | 0.8000 | 0.6500 | 0.6000 | 0.4875 | 2.5375 |

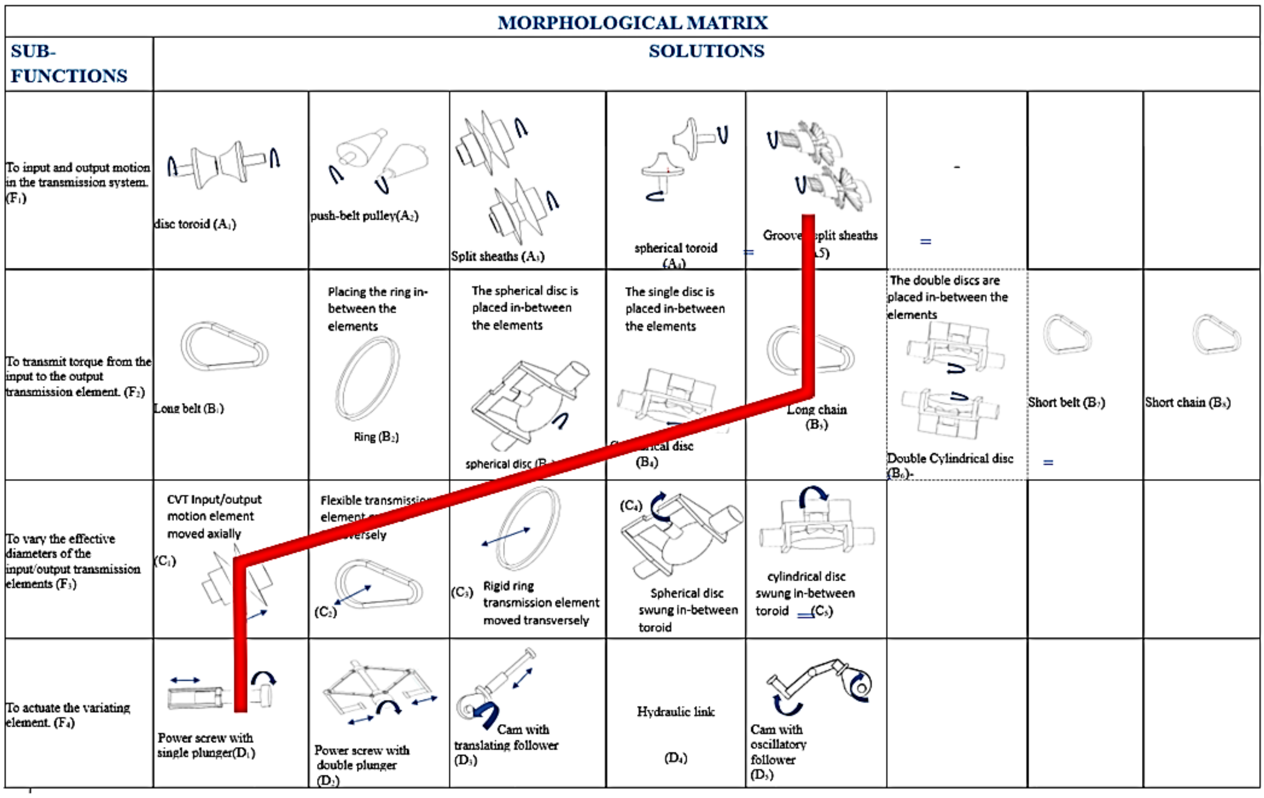


**Figure 10.** Solution chain of the selected SV

Finally, the results of the test conducted on the designers to determine the consistency of the CD generation process confirmed its consistency. The CDs proposed by each group of the participants in this experiment and the respective CDs matrices developed are as shown in Table 7 to Table 12 and Figure 11 to Figure 16, respectively.

**Table 7.** CDs Proposed by Group A

| Compatibility Determinants | Equivalent values | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| (CD1) Shaft orientation | In-line = 1 | Parallel = -1 | Perpendicular = 2 | Universal = 0 |
| (CD2) Freedom of transversal motion of the liking element | Free =1 | Not free = -1 | Universal = 0 | |
| (CD3) Centre distance between driver and driven shaft | Narrow = 1 | Wide = -1 | Universal = 0 | |
| (CD4) Motion type combination of actuator | Reciprocating and rotary = 1 | Oscillatory and rotary = -1 | Universal = 0 | |

$$CD_1 = \begin{bmatrix} 1 & -1 & -1 & 2 & -1 & 3 & 3 & 3 \\ -1 & -1 & 2 & 1 & -1 & 1 & -1 & -1 \\ -1 & -1 & -1 & 2 & 1 & 4 & 4 & 4 \\ 0 & -1 & 0 & 0 & 0 & 5 & 5 & 5 \end{bmatrix}$$

$$CD_2 = \begin{bmatrix} 1 & 1 & -1 & 1 & -1 & 3 & 3 & 3 \\ -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 & 1 & 4 & 4 & 4 \\ 0 & 0 & 0 & 0 & 0 & 5 & 5 & 5 \end{bmatrix}$$

$$CD_3 = \begin{bmatrix} 1 & 0 & -1 & 1 & -1 & 3 & 3 & 3 \\ -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 & 1 & 4 & 4 & 4 \\ 0 & 0 & 0 & 0 & 0 & 5 & 5 & 5 \end{bmatrix}$$

$$CD_4 = \begin{bmatrix} -1 & 1 & 1 & -1 & 1 & 3 & 3 & 3 \\ 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & -1 & -1 & 4 & 4 & 4 \\ 1 & 1 & 1 & 0 & -1 & 5 & 5 & 5 \end{bmatrix}$$

**Figure 11.** CD matrices generated by Group A

**Table 8.** CDs proposed by Group B

| Compatibility determinants | Equivalent values | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| (CD1) Position of driving shaft relative to driven shaft | In-line = 1 | Parallel = -1 | Perpendicular = 2 | Universal = 0 |
| (CD2) Nature of transmitting element | Flexible = 1 | Rigid = -1 | Universal = 0 | |
| (CD3) Nature of variating element | Flexible = 1 | Rigid = -1 | Universal = 0 | |
| (CD4) Motion type of variating element | Translatory = 1 | Oscillatory = -1 | Universal = 0 | |

$$CD_1 = \begin{bmatrix} 1 & -1 & -1 & 2 & -1 & 3 & 3 & 3 \\ -1 & -1 & 2 & 1 & -1 & 1 & -1 & -1 \\ -1 & -1 & -1 & 2 & 1 & 4 & 4 & 4 \\ 0 & 0 & 0 & 0 & 0 & 5 & 5 & 5 \end{bmatrix}$$

$$CD_2 = \begin{bmatrix} -1 & 0 & 1 & -1 & 1 & 3 & 3 & 3 \\ 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & -1 & 4 & 4 & 4 \\ 0 & 0 & 0 & 0 & 0 & 5 & 5 & 5 \end{bmatrix}$$

$$CD_3 = \begin{bmatrix} -1 & 0 & -1 & -1 & -1 & 3 & 3 & 3 \\ -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & -1 & -1 & 4 & 4 & 4 \\ 0 & 0 & 0 & 0 & 0 & 5 & 5 & 5 \end{bmatrix}$$

$$CD_4 = \begin{bmatrix} -1 & 1 & 1 & -1 & 1 & 3 & 3 & 3 \\ 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & -1 & -1 & 4 & 4 & 4 \\ 0 & 1 & 0 & 0 & -1 & 5 & 5 & 5 \end{bmatrix}$$

**Figure 12.** CD matrices generated by Group B

**Table 9.** CDs proposed by Group C

| Compatibility Determinants | Equivalent values | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| (CD1) Relationship between input and output shaft | In-line = 1 | Parallel = -1 | Perpendicular = 2 | Universal = 0 |
| (CD2) Direction of motion of the input and output shaft | Similar = 1 | Opposite = -1 | Universal = 0 | |
| (CD3) Distance between motion input and output element | Short = 1 | Long = -1 | Universal = 0 | |
| (CD4) Motion type of variating element | Translatory = 1 | Oscillatory = -1 | Universal = 0 | |

$$CD_1 = \begin{bmatrix} 1 & -1 & -1 & 2 & -1 & 3 & 3 & 3 \\ -1 & -1 & 2 & 1 & -1 & 1 & -1 & -1 \\ -1 & -1 & -1 & 2 & 1 & 4 & 4 & 4 \\ 0 & 0 & 0 & 0 & 0 & 5 & 5 & 5 \end{bmatrix}$$

$$CD_2 = \begin{bmatrix} -1 & 0 & 1 & -1 & 1 & 3 & 3 & 3 \\ 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & -1 & 4 & 4 & 4 \\ 0 & 0 & 0 & 0 & 0 & 5 & 5 & 5 \end{bmatrix}$$

$$CD_3 = \begin{bmatrix} 1 & 1 & -1 & 1 & -1 & 3 & 3 & 3 \\ -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 & 1 & 4 & 4 & 4 \\ 0 & 1 & 0 & 0 & 0 & 5 & 5 & 5 \end{bmatrix}$$

$$CD_4 = \begin{bmatrix} -1 & 1 & 1 & -1 & 1 & 3 & 3 & 3 \\ 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & -1 & -1 & 4 & 4 & 4 \\ 1 & 1 & 1 & 0 & -1 & 5 & 5 & 5 \end{bmatrix}$$

**Figure 13.** CD matrices generated by Group C

**Table 10.** CDs proposed by Group D

| Compatibility determinants | Equivalent values | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| (CD1) Relationship between axis of the motion input and output elements | Co-axial = 1 | Parallel = -1 | Perpendicular = 2 | Universal = 0 |
| (CD2) Vertical height of the motion input/output element | Short = 1 | Long = -1 | Universal = 0 | |
| (CD3) Texture of the variating material | flexible = 1 | Rigid = -1 | Universal = 0 | |
| (CD4) Output motion type of the actuating element | Reciprocating = 1 | Oscillatory = -1 | Universal = 0 | |

$$CD_1 = \begin{bmatrix} 1 & -1 & -1 & 2 & -1 & 3 & 3 & 3 \\ -1 & -1 & 2 & 1 & -1 & 1 & -1 & -1 \\ -1 & -1 & -1 & 2 & 1 & 4 & 4 & 4 \\ 0 & -1 & 0 & 0 & 0 & 5 & 5 & 5 \end{bmatrix}$$

$$CD_2 = \begin{bmatrix} 1 & -1 & 1 & 1 & 1 & 3 & 3 & 3 \\ 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & 4 & 4 & 4 \\ 0 & 1 & 0 & 0 & 0 & 5 & 5 & 5 \end{bmatrix}$$

$$CD_3 = \begin{bmatrix} -1 & 0 & 1 & -1 & 1 & 3 & 3 & 3 \\ -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & -1 & -1 & 4 & 4 & 4 \\ 0 & 0 & 0 & 0 & 0 & 5 & 5 & 5 \end{bmatrix}$$

$$CD_4 = \begin{bmatrix} -1 & 1 & 1 & -1 & 1 & 3 & 3 & 3 \\ 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & -1 & -1 & 4 & 4 & 4 \\ 0 & 1 & 0 & 0 & -1 & 5 & 5 & 5 \end{bmatrix}$$

**Figure 14.** CD matrices generated by Group D

**Table 11.** CDs proposed by Group E

| Compatibility determinants | Equivalent values | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| (CD1) Relationship between the longitudinal axis of the input and output shaft | Co-axial = 1 | Parallel = -1 | Perpendicular = 2 | Universal = 0 |
| (CD2) Distance between input and output element | Short = 1 | Long = -1 | Universal = 0 | |
| (CD3) Texture of the motion transmission element | flexible = 1 | Rigid = -1 | Universal = 0 | |
| (CD4) Output motion of variating element | linear = 1 | Oscillatory = -1 | Universal = 0 | |

$$CD_1 = \begin{bmatrix} 1 & -1 & -1 & 2 & -1 & 3 & 3 & 3 \\ -1 & -1 & 2 & 1 & -1 & 1 & -1 & -1 \\ -1 & -1 & -1 & 2 & 1 & 4 & 4 & 4 \\ 0 & -1 & 0 & 0 & 0 & 5 & 5 & 5 \end{bmatrix}$$

$$CD_2 = \begin{bmatrix} 1 & 1 & -1 & 1 & -1 & 3 & 3 & 3 \\ -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 & 1 & 4 & 4 & 4 \\ 0 & -1 & 0 & 0 & 0 & 5 & 5 & 5 \end{bmatrix}$$

$$CD_3 = \begin{bmatrix} -1 & 0 & 1 & -1 & 1 & 3 & 3 & 3 \\ 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & -1 & 4 & 4 & 4 \\ 0 & 0 & 0 & 0 & 0 & 5 & 5 & 5 \end{bmatrix}$$

$$CD_4 = \begin{bmatrix} -1 & 1 & 1 & -1 & 1 & 3 & 3 & 3 \\ 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & -1 & -1 & 4 & 4 & 4 \\ 0 & 1 & 0 & 0 & -1 & 5 & 5 & 5 \end{bmatrix}$$

**Figure 15.** CD matrices generated by Group E

**Table 12.** CDs proposed by Group F

| Compatibility Determinants | Equivalent values | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| (CD1) I input and output axis | In-line = 1 | Parallel = -1 | Perpendicular = 2 | Universal = 0 |
| (CD2) Centre distance of shaft | Short = 1 | Long = -1 | Universal =0 | |
| (CD3) flexibility of transmission element | Flexible = 1 | Rigid = -1 | Universal =0 | |
| (CD4) Motion type of variating element | Linear = 1 | Oscillating = -1 | Universal=0 | |

$$CD_1 = \begin{bmatrix} 1 & -1 & -1 & 2 & -1 & 3 & 3 & 3 \\ -1 & -1 & 2 & 1 & -1 & 1 & -1 & -1 \\ -1 & -1 & -1 & 2 & 1 & 4 & 4 & 4 \\ 0 & 0 & 0 & 0 & 0 & 5 & 5 & 5 \end{bmatrix}$$

$$CD_2 = \begin{bmatrix} 1 & 1 & -1 & 1 & -1 & 3 & 3 & 3 \\ -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 & 1 & 4 & 4 & 4 \\ 0 & -1 & 0 & 0 & 0 & 5 & 5 & 5 \end{bmatrix}$$

$$CD_3 = \begin{bmatrix} 1 & 0 & -1 & 1 & -1 & 3 & 3 & 3 \\ -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 & 1 & 4 & 4 & 4 \\ 0 & -1 & 0 & 0 & 0 & 5 & 5 & 5 \end{bmatrix}$$

$$CD_4 = \begin{bmatrix} -1 & 1 & 1 & -1 & 1 & 3 & 3 & 3 \\ 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & -1 & -1 & 4 & 4 & 4 \\ 0 & 1 & 0 & 0 & -1 & 5 & 5 & 5 \end{bmatrix}$$

**Figure 16.** CD matrices generated by Group F

It can be observed that the CDs are different from each other in nomenclature. Moreover, the CDs generated by each group are not derived from the same set of SPs. Group A, for example, had $CD_1$ and $CD_2$ as 'Shaft orientation' and 'Freedom of transversal motion of the liking element', respectively, which are both derived from the SPs of $F_1$. Similarly, their $CD_3$ ('material of transversal motion liking element') and $CD_4$ ('Motion type combination of actuator') are derived from the SPs of $F_2$ and SPs of $F_4$, respectively. Therefore, the SPs from which the CDs were derived by group A can be described as [$2F_1$, $1F_2$, $1F_3$]. This implies that two CDs were derived from $F_1$, one from $F_2$ and one from $F_3$. In the same vein, the CDs generated by groups B, C, D, E, F, G, H, and I are derived from the SPs of [$1F_1$, $1F_2$, $2F_3$], [$3F_1$, $1F_3$], [$2F_1$, $1F_3$, $1F_4$], [$2F_1$, $1F_2$, $1F_3$], [$2F_1$, $1F_2$, $1F_3$], [$1F_1$, $1F_2$, $2F_3$], [$2F_1$, $1F_2$, $1F_4$] and [$1F_1$, $3F_3$] respectively. This implies the versatility of the approach, its replicability, and its flexibility.

Besides observing the CD matrix and applying each for screening the CVT morphological matrix, the same 37 sets of SVs were obtained as the feasible concept. This is a confirmation of the validity of the framework. Moreover, since one of the metrics for measuring a conceptual design method is quantity, the framework stands out as an approach that is capable of complete exploration of the morphological matrix. Also, with the framework, the combinatorial explosion of the morphological matrix-based concept generation method [26] is significantly minimised.

## CONCLUSIONS

The study delved into the development of SVs matrix (G-matrix) and establishing mathematical relations for analysing both the morphological matrix and the G-matrix. Besides, logical relations for screening feasible concepts out of the theoretical concepts generated from a morphological matrix was also established. The mathematical and logical relations established were found to be replicable, versatile, and reliable. Therefore, it can be unequivocally stated that the objective of the research is attained. The G-matrix can be manipulated to conduct all forms of analysis on the constituent SVs. By so doing, complete extraction of the feasible concepts from the morphological matrix with a combinatorial explosion is possible using the framework developed in this study. Further work for this study is around concept evaluation and selection. Being morphological matrix based, such concept evaluation and selection criteria need to be rooted in the SPs and combined in the same manner to obtain evaluation variables G-matrices.

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## REFERENCES

[1] B. Helms and K. Shea, "Computational synthesis of product architectures based on object-oriented graph grammars," *Journal of Mechanical Design, Transactions of the ASME*, vol. 134, no. 2, pp.1-14, 2012.

[2] S. Pugh, *Total Design: Integrated Methods for Successful Product Engineering*, First. England: Addison-westley Publishing Company Inc, 1991.

[3] N. V. Hernandez, L. C. Schmidt, and G. E. Okudan, "Systematic ideation effectiveness study of TRIZ," *Journal of Mechanical Design, Transactions of the ASME*, vol. 135, no. 10, pp.1-19, 2013.

[4] K. Fu, J. Murphy, M. Yang, K. Otto, D. Jensen, and K. Wood, "Design-by-analogy: experimental evaluation of a functional analogy search methodology for concept generation improvement," *Research in Engineering Design*, vol. 26, pp. 77–95, 2015.

[5] S. R. Daly, C. M. Seifert, S. Yilmaz, and R. Gonzalez, "Comparing ideation techniques for beginning designers," *Journal of Mechanical Design, Transactions of the ASME*, vol. 138, no. 10, pp.1-12, 2016.

[6] S. Narsale, Y. Chen, M. Mohan, and J. J. Shah, "Design ideator: A conceptual design toolbox," *Journal of Computing & Information Science in Engineering*, vol. 19, no. 4, pp.1-12, 2019.

[7] G. Pahl, W. Beitz, J. Feldhusen, and K. H. Grote, *Engineering Design a Systematic Approach*, 3rd Englis. London: Springer-velarg London Limited, 2007.

[8] A. Chawla and J. D. Summers, "Function ordering within morphological charts: An experimental study," In Proceedings of the ASME 2018 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference IDETC/CIE 2018 August 26-29, 2018, Quebec City, Quebec, Canada, pp. 1–12.

[9] R. Duran-Novoa, J. Lozoya-Santos, R. Ramírez-Mendoza, F. Torres-Benoni, and A. Vargas-Martínez, "Influence of the method used in the generation of valid engineering concepts," *International Journal on Interactive Design and Manufacturing*, vol. 13, no. 3, pp. 1073–1088, 2019.

[10] D. G. Ullman, *The Mechanical Design Process*, 4th ed. New York: McGrawHill, 2010.

[11] L. Yuan, Y. Liu, Y. Lin, and J. Zhao, "An automated functional decomposition method based on morphological changes of material flows," *Journal of Engineering Design*, vol. 28, no. 1, pp. 47–75, 2017.

[12] M. Sallaou and G. M. Fadel, "Energy based functional decomposition in preliminary design," *Journal of Mechanical Design*, vol. 133, no. May 2011, pp. 1–10, 2018.

[13] L. Fiorineschi, F. S. Frillici, and F. Rotini, "Supporting systematic conceptual design with Triz," In *International Design Conference - DESIGN 2018*, 2018, pp. 1091–1102.

[14] A. Chakrabarti and T. P. Bligh, "A scheme for functional reasoning in conceptual design," *Design Studies*, vol. 22, no. 6, pp. 493–517, 2001.

[15] Y. Chen, P. Feng, B. He, Z. Lin, and Y. Xie, "Automated conceptual design of mechanisms using improved morphological matrix," *Journal of Mechanical Design, Transactions of the ASME*, vol. 128, no. 3, pp. 516–526, 2006.

[16] Y. Kang and D. Tang, "Matrix-based computational conceptual design with ant colony optimisation," *Journal of Engineering Design*, vol. 4828, 2013.

[17] F. Zwicky, "The morphological approach to discovery, invention, research and construction," in *New Methods of Thought and Procedure*, F. Zwicky, A. G. Wilson, eds., Springer, Berlin, pp. 273–297., 1967.

[18] J. D. Summers, "How function ordering within morphological charts influence exploration," *Journal of Mechanical Design,* vol. 141, pp. 1–7, 2019.

[19] H. Ma, X. Chu, D. Xue, and D. Chen, "A systematic decision making approach for product conceptual design based on fuzzy morphological matrix," *Expert Systems with Applications*, vol. 81, pp. 444–456, 2017.

[20] R. G. Weber and S. S. Condoor, "Conceptual design using a synergistically compatible morphological matrix," in FIE '98. 28th Annual Frontiers in Education Conference. Moving from "Teacher-Centered" to "Learner-Centered" Education. Conference Proceedings, 1998, vol. 1, pp. 171–176.

[21] M. P. Weiss and Y. Gilboa, "More on synthesis of concepts as an optimal combination of solution principles," In *International Design Conference*, 2004, pp. 1–9.

[22] C. R. B. Arnold, R. B. Stone, and D. A. Mcadams, "MEMIC : An interactive morphological matrix tool for automated concept generation," 2008, p. 2008.

[23] J. Ölvander, B. Lundén, and H. Gavel, "A computerised optimisation framework for the morphological matrix applied to aircraft conceptual design," *Computer-Aided Design*, vol. 41, no. 3, pp. 187–196, 2009.

[24] R. F. Bo, "Genetic algorithm based approach to concept solving for mechanical product in conceptual design," In Fifth International Conference on Natural Computation (ICNC 2009), vol. 4, pp. 254–258, 2009.

[25] B. He, P. Zhang, and J. Wang, "Automated synthesis of mechanisms with consideration of mechanical efficiency," *Journal of Engineering Design*, vol. 25, no. 4–6, pp. 213–237, 2014.

[26] B. He, W. Song, and Y. Wang, "Computational conceptual design using space matrix," *ASME Journal of Computing and Information Science in Engineering,* vol. 15, no. 1, pp. 1–7, 2015.

[27] N. Angie, E. M. Tokit, N. A. Rahman, F. A. Zahrah, M. Saat *et al.*, "A preliminary conceptual design approach of food waste composter design," vol. 08, no. 02, pp. 397-407, 2021.

[28] M. Cardin, R. De Neufville, and D. M. Geltner, "Design catalogs : A systematic approach to design and value flexibility in engineering systems," vol.18, no. January 2016, pp. 453–47, 2015.

[29] D. Engida Woldemichael and F. Mohd Hashim, "A framework for function-based conceptual design support system," *Journal of Engineering, Design and Technology*, vol. 9, no. 3, pp. 250–272, 2011.

[30] R. Ullah, D. Q. Zhou, P. Zhou, M. Hussain, and M. Amjad Sohail, “An approach for space launch vehicle conceptual design and multi-attribute evaluation,” *Aerospace Science and Technology*, vol. 25, no. 1, pp. 65–74, 2013.

[31] B. He and Y. Hua, “Synthesis of mechanisms integrated with motion and force transformation,” *International Journal of Precision Engineering and Manufacturing*, vol. 17, no. 12, pp. 1643–1649, 2016.

[32] N. Rashid, M. Nuri, K. Hudha, and S. A. Mazlan, “Design and simulation of a new single actuator double acting electro-mechanical continuously variable transmission,” *International Journal of Mechanical Engineering and Robotics Research*, vol. 8, no. 1, pp. 114–120, 2019.