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Neutrosophic Bicubic Bezier Surface Approximation Model for Uncertainty Data

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Abstract Surfaces and their descriptions are significant in design, physical science, geology, and other natural phenomena. This study introduces a neutrosophic Bézier surface approximation with a four-by-four control net for the bicubic situation. The neutrosophic notion defines the neutrosophic control net relation. The control net is mixed with the Bernstein basis function to generate a surface blending function and a neutrosophic bicubic Bézier surface. Finally, the neutrosophic bicubic Bézier surface is shown using an approximation approach and data points having neutrosophic properties.

Keywords Approximation; Control net relation; Fuzzy bicubic Bézier, Neutrosophic control net

Mathematics Subject Classification 03E72, 90C70, 65D17.

1 Introduction

An important consideration in curve and surface design is the amount of randomness present in the data-collecting phase. In the field of geometric modeling, this collection of data is also referred to as a control point for using approximate and interpolative methods [1]. The data set is a crucial part of the process of creating curves and surfaces, as well as an integral part of the process itself. This is because the data set will be used as the control points to produce curves and surfaces by utilizing approximation or interpolation methods. Uncertainty data, despite its impact on the curve and the resultant surface, is often ignored or thrown out. It is because of the insufficient information and not appropriate features of the uncertainty data. This will lead to insufficient scrutiny and analysis of the data unless a new definition of the curves and surfaces is introduced. Therefore, data sets with a certain amount of variability need to be filtered by using neutrosophic set theory before they can be used to build a surface and curve for any problem that must be analyzed. As a result, the dataset may be put to good use.

Florentin Smarandache [2] created Neutrosophy as a mathematical theory based on the idea that all things should be treated equally. Neutrophil sets may be distinguished by their membership, non-membership, or indeterminacy. Neutrosophic sets are used to describe the methods by which problems spanning many disciplines may be solved and represented. In neutrosophic set theory, a single element may have many types of membership at once, including real, fake, and ill-defined. This opens the door to describing more nuanced forms of doubt and ambiguity, such as when a statement might have contradictory truths. Some researchers have also tried using geometric modeling to implement neutrosophic set processes [3,4,16,17,18].

The data set is an essential characteristic that is an essential component in the visualization of curves or surfaces. Usually, uncertainty data will not be included in the data collection because it will either be removed or destroyed, regardless of the effect it has on the formation of surface curves. As a direct consequence of this, there will be deficiencies in both the analysis and the visualization. For instance, if a data collection has some degree of unpredictability, the data set ought to be filtered before it is utilized in the construction of curve and surface models. When there is ambiguity in the data being analyzed, developing geometric models using neutrosophic sets is an effective method for overcoming the challenge of visually representing the data. While geometric modeling may be used to generate geometric mathematical models, neutrosophic set theory is a representation that emphasizes ideas and methods for dealing with ambiguous situations. Geometric modeling is a method for developing geometric mathematical representations. However, there are scholars who conduct studies in the field of fuzzy geometry modeling [5,6,7,8,9,10].

The development of a geometric model that can deal with uncertain data is the focus of this study; specifically, the Neutrosophic Bézier surface approximation will be the model's primary focus. Before creating the neutrosophic Bézier surface, it is necessary to first determine the neutrosophic control point using the neutrosophic set theories and the properties it possesses. To construct neutrosophic Bézier surface models, which are subsequently visualized using an approximation approach, these control points are used in combination with the Bernstein basis function. The following describes how the format of this document should be used. The first section of this paper presented some background information about the subject. In Section 2, the neutrosophic principle and the Neutrosophic Control Point (NCP) are presented to the reader. In the third section, the method that may be used to approximate the Neutrosophic Bézier Surface (NBS) using NCP is discussed. In the fourth section, there is both a numerical example and a graphic representation of NBS. The curvature and surface characteristics, in addition to the method that was used to produce them, are both covered in this article. The investigation will be finished with the fifth and final portion.

2 Preliminaries

In fuzzy systems, the intuitionistic set may accept incomplete information; however, it cannot accommodate indeterminate or inconsistent information [11]. There are three membership functions for a neutrosophic set. With the extra parameter "indeterminacy" introduced to the Neutrosophic Set (NS) specification [11], there are three: a membership function, denoted by the letter, T; an indeterminacy membership function, denoted by the letter, I; and a non-membership function, denoted by the letter, F.

Definition 1 [11] *Let $\hat{X}$ be the first world of conversation, with element in $\hat{X}$ denoted as $\hat{x}$. The Neutrosophic set is an object in the form;*

$$\hat{A} = \{\left\langle \hat{x} : T_{\hat{A}(\hat{x})}, I_{\hat{A}(\hat{x})}, F_{\hat{A}(\hat{x})} \right\rangle \hat{x} \in \hat{X}\}$$

where, the functions $T, I, F : \hat{X} \to]^{-}0, 1^{+}[$ define, respectively, the degree of truth membership, the degree of indeterminacy, and the degree of false membership of the element $\hat{x} \in \hat{X}$ to the set $\hat{A}$ with *the condition;*

$$0^- \leq T_{\hat{A}}(\hat{x}) + I_{\hat{A}}(\hat{x}) + F_{\hat{A}}(\hat{x}) \leq 3^+$$

There is no limit to the amount of $T_{\hat{A}}(\hat{x}), I_{\hat{A}}(\hat{x})$ *and* $F_{\hat{A}}(\hat{x})$

A value is chosen by NS from one of the real standard subsets or one of the non-standard subsets of $]^-0, 1^+[$. The actual value of the interval $[0, 1]$, on the other hand, $]^-0, 1^+[$ will be utilised in technical applications since its utilisation in real data such as the resolution of scientific challenges, will be physically impossible. As a direct consequence of this, membership value utilisation is increased. $\hat{A} = \{\left\langle \hat{x} : T_{\hat{A}(\hat{x})}, I_{\hat{A}(\hat{x})}, F_{\hat{A}(\hat{x})} \right\rangle \hat{x} \in \hat{X}\}$ and $T_{\hat{A}}(\hat{x}), I_{\hat{A}}(\hat{x}), F_{\hat{A}}(\hat{x}) \in [0, 1]$.

There is no restriction on the sum of $T_{\hat{A}}(\hat{x}), I_{\hat{A}}(\hat{x}), F_{\hat{A}}(\hat{x})$. Therefore,

$$0 \leq T_{\hat{A}}(\hat{x}) + I_{\hat{A}}(\hat{x}) + F_{\hat{A}}(\hat{x}) \leq 3.$$

Definition 2 [3,4] *Let* $\hat{A} = \{\left\langle \hat{x} : T_{\hat{A}(\hat{x})}, I_{\hat{A}(\hat{x})}, F_{\hat{A}(\hat{x})} \right\rangle \hat{x} \in \hat{A}\}$*and* $\hat{B} = \{\left\langle \hat{y} : T_{\hat{B}(\hat{y})}, I_{\hat{B}(\hat{y})}, F_{\hat{B}(\hat{y})} \right\rangle \hat{y} \in \hat{B}\}$ *be neutrosophic elements. Thus,* $\widehat{NR} = \{\left\langle (\hat{x}, \hat{y}) : T_{(\hat{x},\hat{y})}, I_{(\hat{x},\hat{y})}, F_{(\hat{x},\hat{y})} \right\rangle \hat{x} \in \hat{A}, \hat{y} \in \hat{B}\}$ *is a neutrosophic relation on* $\hat{A}$ *and* $\hat{B}$.

Definition 3 [3,4] *Neutrosophic set of* $\hat{A}$ *in space* $\hat{X}$ *is NP (Neutrosophic Point) and* $\hat{A} = \{\hat{A}_i\}$ *where* $i = 0, ..., n$ *is a set of NPs where there exists* $T_A : \hat{X} \to [0, 1]$ *as truth membership,* $I_A : \hat{X} \to [0, 1]$ *as indeterminacy membership and* $F_A : \hat{X} \to [0, 1]$ *as false membership with*

$$T_A(\hat{A}) = \begin{cases} 0 & if \quad \hat{A}_i \notin \hat{A} \\ a \in (0, 1) & if \quad \hat{A}_i \hat{\in} \hat{A} \\ 1 & if \quad \hat{A}_i \in \hat{A} \end{cases}$$

$$I_A(\hat{A}) = \begin{cases} 0 & if \quad \hat{A}_i \notin \hat{A} \\ b \in (0, 1) & if \quad \hat{A}_i \hat{\in} \hat{A} \\ 1 & if \quad \hat{A}_i \in \hat{A} \end{cases}$$

$$F_A(\hat{A}) = \begin{cases} 0 & if \quad \hat{A}_i \notin \hat{A} \\ c \in (0, 1) & if \quad \hat{A}_i \hat{\in} \hat{A} \\ 1 & if \quad \hat{A}_i \in \hat{A} \end{cases}$$

2.1 Neutrosophic Point Relation (NPR)

NPR is built on the notion of neutrosophic set, which was addressed in the preceding section. If P, Q is a collection of Euclidean universal space points and $P, Q \in \mathbb{R}^2$ then NPR is defined as follows:

Definition 4 *Let* X, Y *be collection of universal space points with non-empty set and* $P, Q, I \in \mathbb{R} \times \mathbb{R} \times \mathbb{R}$, *then NPR is defined as*

$$\hat{R} = \left\{ \left\langle \left(p_i, q_j\right), T_R(p_i, q_j), I_R(p_i, q_j), F_R(p_i, q_j) \right\rangle T_R(p_i, q_j), I_R(p_i, q_j), F_R(p_i, q_j) \in I \right\}$$

Where $\left(p_i, q_j\right)$ *is an ordered pair of coordinates and* $\left(p_i, q_j\right) \in P \times Q$ *while* $T_R(p_i, q_j), I_R(p_i, q_j), F_R(p_i, q_j)$ *are the truth membership, indeterminacy membership and false membership that follows the condition of neutrosophic set which is* $0 \leq T_{\hat{A}}(\hat{x}) + I_{\hat{A}}(\hat{x}) + F_{\hat{A}}(\hat{x}) \leq 3$.

2.2 Neutrosophic Control Net Relation (NCNR)

A spline surface's geometry can only be defined by all the points that are required to create the surface. This is the meaning of the term "control net." In the process of developing smooth surfaces, as well as controlling and manufacturing them, the control net plays an essential role. In this portion, the neutrosophic control point relation (NCPR) is defined initially by utilizing the concept of the neutrosophic set from the research presented in [12,13,14] in the following manner:

Definition 5 *Let $\hat{R}$ be a NPR, then NCPR is defined as set of point $n+1$ that indicates the positions and coordinates of a location and is used to describe the curve and is denoted by*

$$\begin{aligned}\hat{B}_i^T &= \left\{\hat{b}_0^T, \hat{b}_1^T, ..., \hat{b}_n^T\right\}\\ \hat{B}_i^I &= \left\{\hat{b}_0^I, \hat{b}_1^I, ..., \hat{b}_n^I\right\}\\ \hat{B}_i^F &= \left\{\hat{b}_0^F, \hat{b}_1^F, ..., \hat{b}_n^F\right\}\end{aligned}$$

Where $\hat{B}_i^T$, $\hat{B}_i^I$ and $\hat{B}_i^F$ are neutrosophic control point for membership truth, indeterminacy and i is one less than n. Thus, the NCNR can be defined as follows.

Definition 6 *Let $\hat{B}$ be an NCPR, and then define an NCNR as points $n+1$ and $m+1$ for $\hat{B}$ in their direction, and it can be denoted by $\hat{B}_{i,j}$ that denotes the locations and the location of points to describe the surface, and it can be expressed as*

$$\hat{B}_{i,j} = \begin{bmatrix} \hat{B}_{0,0} & \hat{B}_{0,1} & \cdots & \hat{B}_{0,j} \\ \hat{B}_{1,0} & \hat{B}_{1,1} & \cdots & \hat{B}_{1,j} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{B}_{i,0} & \hat{B}_{i,1} & \cdots & \hat{B}_{i,j} \end{bmatrix},$$

where $\hat{B}_{i,j}$ are also the points that make up the control net for a polygon. As a result, the NCNR may be expressed as follows for the bicubic case:

$$\begin{bmatrix} \hat{B}_{0,0} & \hat{B}_{0,1} & \hat{B}_{0,2} & \hat{B}_{0,3} \\ \hat{B}_{1,0} & \hat{B}_{1,1} & \hat{B}_{1,2} & \hat{B}_{1,3} \\ \hat{B}_{2,0} & \hat{B}_{2,1} & \hat{B}_{2,2} & \hat{B}_{2,3} \\ \hat{B}_{3,0} & \hat{B}_{3,1} & \hat{B}_{3,2} & \hat{B}_{3,3} \end{bmatrix}$$

3 Approximation of Neutrosophic Bézier Surface (NBS)

Surface is vector value function with two parameters that define how the plane is projected into the three-dimensional frame of Euclidean geometry [15]. This mapping is referred to as the surface mapping. The tensor product technique is a method for constructing bidirectional curves that make use of basic functions as well as geometric coefficients. The NCNR and definition 1 are utilized in the construction of the NBS, which is followed by the incorporation of the Bézier blending function into a geometric model. After that, it delves into the quirks that are inherent to the NBC model. NBC, which stands for approximation approach, may be mathematically represented as follows:

Definition 7 *Let* $\hat{B}_i^T = \left\{\hat{b}_0^T, \hat{b}_1^T, ..., \hat{b}_n^T\right\}; \hat{B}_i^I = \left\{\hat{b}_0^I, \hat{b}_1^I, ..., \hat{b}_n^I\right\}; \hat{B}_i^F = \left\{\hat{b}_0^F, \hat{b}_1^F, ..., \hat{b}_n^F\right\}$ *where* $i = 0, 1, ..., n$ *is NCP. Cartesian Bézier surface is given by*

$$\widehat{BS}(u, w) = \sum_{i=0}^{n} \sum_{j=0}^{m} \hat{B}_{i,j}^T J_{n,i}(u) K_{m,j}(w)$$

$$\widehat{BS}(u, w) = \sum_{i=0}^{n} \sum_{j=0}^{m} \hat{B}_{i,j}^I J_{n,i}(u) K_{m,j}(w)$$

$$\widehat{BS}(u, w) = \sum_{i=0}^{n} \sum_{j=0}^{m} \hat{B}_{i,j}^F J_{n,i}(u) K_{m,j}(w)$$

where $J_{(n,i)}(u)$ *and* $K_{(n,i)}(w)$ *are the Bernstein function in the u and w parametric directions.*

$$J_{(n,i)}(u) = \begin{pmatrix} n \\ i \end{pmatrix} u^i (1-u)^{n-i} (0)^0 \equiv 1$$

$$K_{(m,j)}(w) = \begin{pmatrix} m \\ j \end{pmatrix} w^j (1-w)^{m-j} (0)^0 \equiv 1$$

with

$$\begin{pmatrix} n \\ i \end{pmatrix} = \frac{n!}{i!\,(n-1)!} (0)^0 \equiv 1$$

$$\begin{pmatrix} m \\ j \end{pmatrix} = \frac{n!}{j!\,(m-1)!} (0)^0 \equiv 1$$

3.1 Properties of Neutrosophic Bézier Surface (NBS)

As the surface-blending functions use the Bernstein basis, the Bézier surface has many of the same properties that are known for Bézier curves. The neutrosophic Bézier Surface (NBS) has the following fundamental characteristics:

i. The degree of NBS is always one less than the control net vertices in the direction that it is being measured in, regardless of the parametric direction.

ii. Control net vertices in a particular direction are two fewer than the continuity of the NBS in that direction.

iii. The NBS will, most of the time, conform to the control net of the shape.

iv. The only points that coincide between the control net and the NBS that it generates are the corner points.

v. The NBS is protected from outside interference by the control net's convex hull.

vi. The NBS does not show any signs of the quality known as variation declining. Undefined and unknown information exists on the variation-diminishing feature of bivariant NBS.

vii. The NBS does not change in any way when subjected to an affine transformation.

4 Numerical Example and Visualization

This section will describe the application and visualization of the Neutrosophic Bézier Surface (NBS). The examples only employ a numerical example at random and will use approximation method. A neutrosophic bicubic Bézier surface consisting of NCNR with a degree of polynomial is three $n = 3$ by will be shown.

4.1 Application of Neutrosophic Bézier Surface (NBS)

To illustrate the neutrosophic Bézier surface approximation, let consider a neutrosophic Bézier surface for 4×4 NCNR with the degree of truth membership, indeterminacy membership and false membership as follows in Table 5.1.

Table 5.1 Neutrosophic Control Point Relation (NCPR) with its respective degrees

NCP $\hat{B}_{i,j}$	Truth Membership $\hat{B}^T_{i,j}$	False Membership $\hat{B}^F_{i,j}$	Indeterminacy Membership $\hat{B}^I_{i,j}$
$\hat{B}_{0,0} = (-15, 15)$	0.4	0.7	0.2
$\hat{B}_{0,1} = (-15, 5)$	0.9	0.3	0.1
$\hat{B}_{0,2} = (-15, -5)$	0.4	0.4	0.5
$\hat{B}_{0,3} = (-15, -15)$	0.6	0.5	0.2
$\hat{B}_{1,0} = (-5, 15)$	0.6	0.4	0.3
$\hat{B}_{1,1} = (-5, 5)$	0.8	0.2	0.3
$\hat{B}_{1,2} = (-5, -5)$	0.5	0.5	0.3
$\hat{B}_{1,3} = (-5, -15)$	0.7	0.4	0.2
$\hat{B}_{2,0} = (5, 15)$	0.6	0.2	0.5
$\hat{B}_{2,1} = (5, 5)$	0.8	0.4	0.1
$\hat{B}_{2,2} = (5, -5)$	0.5	0.7	0.1
$\hat{B}_{2,3} = (5, -15)$	0.5	0.3	0.5
$\hat{B}_{3,0} = (15, 15)$	0.7	0.3	0.3
$\hat{B}_{3,1} = (15, 5)$	0.4	0.6	0.3
$\hat{B}_{3,2} = (15, -5)$	0.4	0.6	0.3
$\hat{B}_{3,3} = (15, -15)$	0.7	0.4	0.2

Table 5.1 is also can be represented in matrix form, as seen below:

$$\hat{B}_{3,3} = \begin{bmatrix} \langle(-15,15);0.4,0.7,0.2\rangle & \langle(-15,5);0.9,0.3,0.1\rangle & \langle(-15,-5);0.4,0.4,0.5\rangle & \langle(-15,-15);0.6,0.5,0.2\rangle \\ \langle(-5,15);0.6,0.4,0.3\rangle & \langle(-5,5);0.8,0.2,0.3\rangle & \langle(-5,-5);0.5,0.5,0.3\rangle & \langle(-5,-15);0.7,0.4,0.2\rangle \\ \langle(5,15);0.6,0.2,0.5\rangle & \langle(5,5);0.8,0.4,0.1\rangle & \langle(5,-5);0.5,0.7,0.1\rangle & \langle(5,-15);0.5,0.3,0.5\rangle \\ \langle(15,15);0.7,0.3,0.3\rangle & \langle(15,5);0.4,0.6,0.3\rangle & \langle(15,-5);0.4,0.6,0.3\rangle & \langle(15,-15);0.7,0.4,0.2\rangle \end{bmatrix}$$

The NCNR 4×4control net for the neutrosophic bicubic of Bézier surface approximation may be seen in Figure 5.1

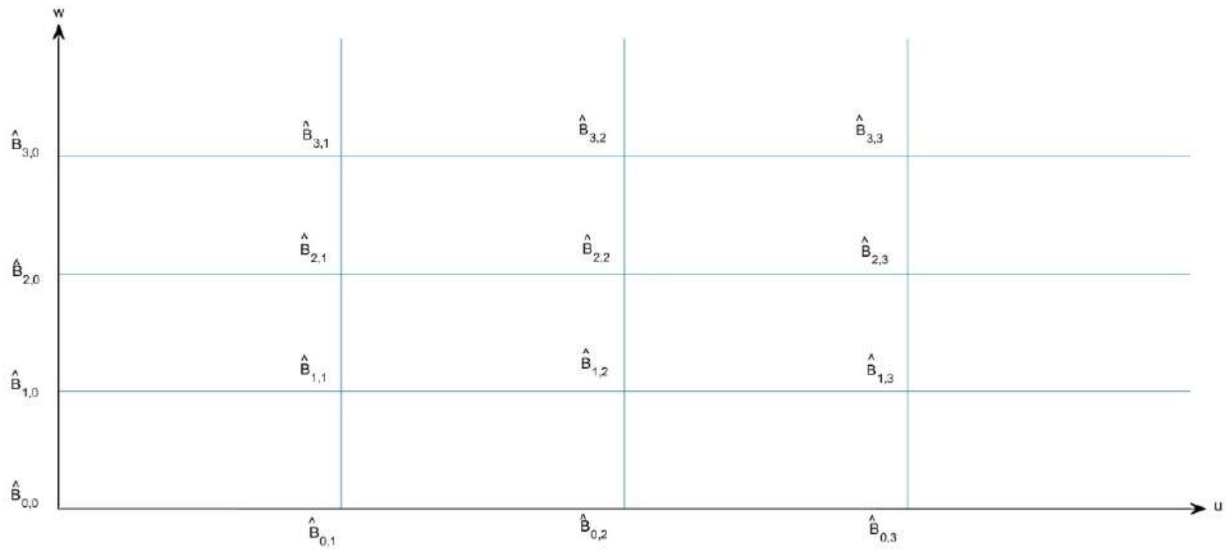

Figure 5.1 NCNR $4 \times 4$for neutrosophic bicubic Bézier surface

Thus, by using Definition 7, the respective surface is visualized in Figure 5.2 until Figure 5.7.

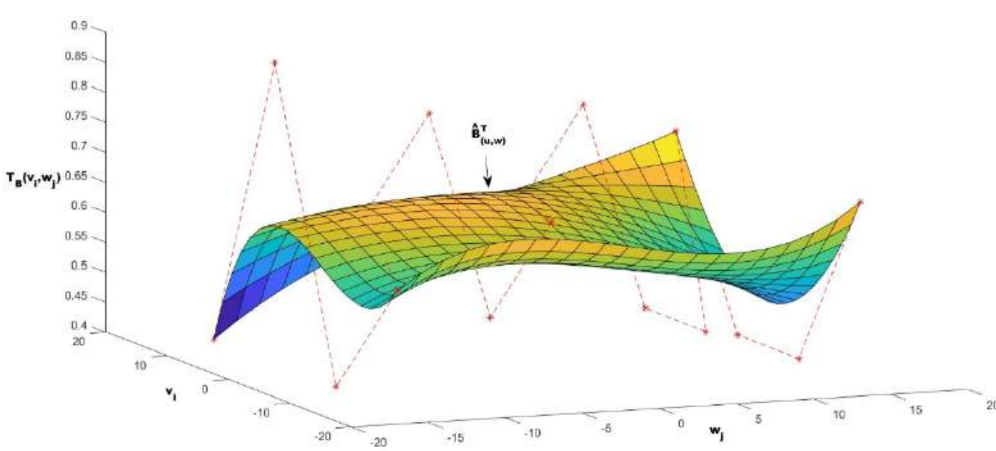

Figure 5.2 Truth Membership Bézier Surface with respective control point and control polygon

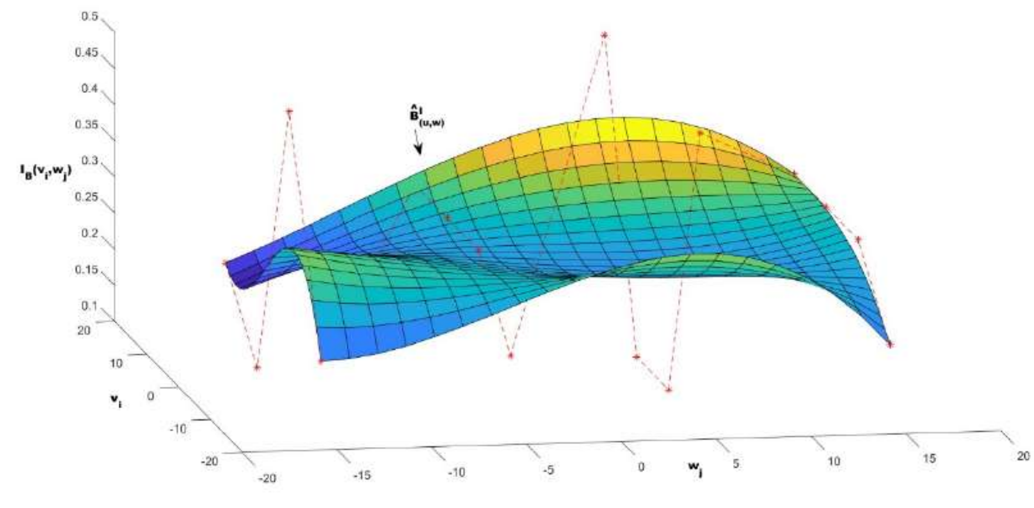

Figure 5.3 Indeterminacy Membership Bézier Surface with respective control points and control polygon.

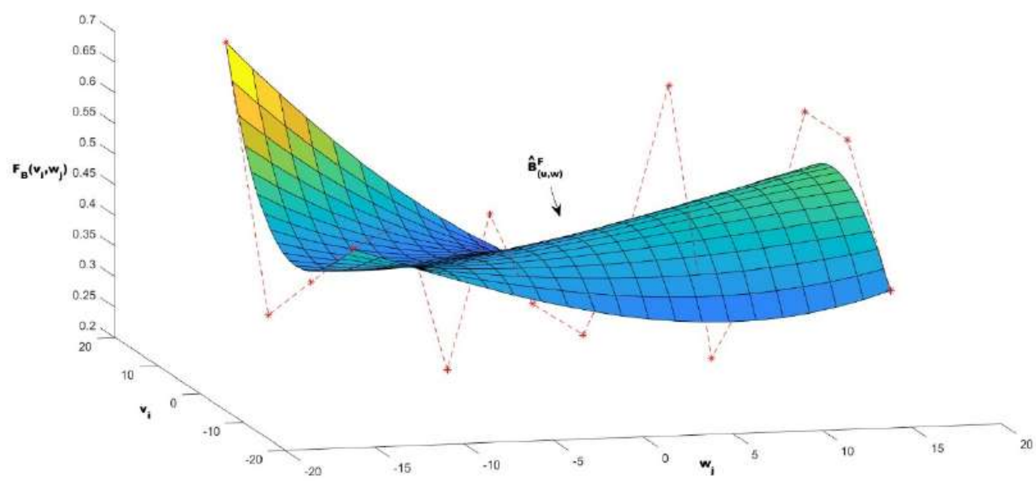

Figure 5.4 False Membership Bézier Surface with respective control points and control polygon.

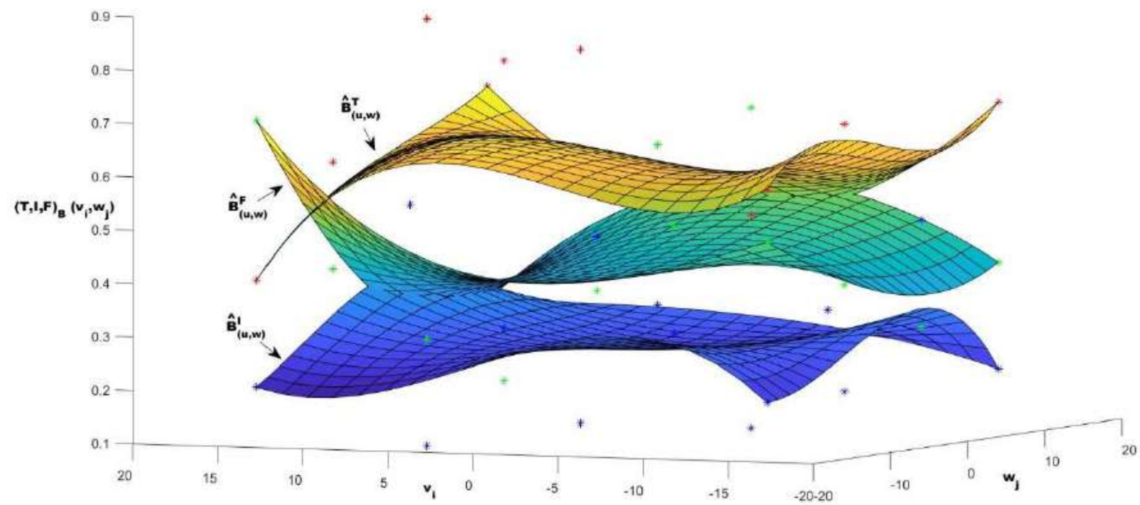

Figure 5.5 Neutrosophic Bicubic Bézier Surface Approximation

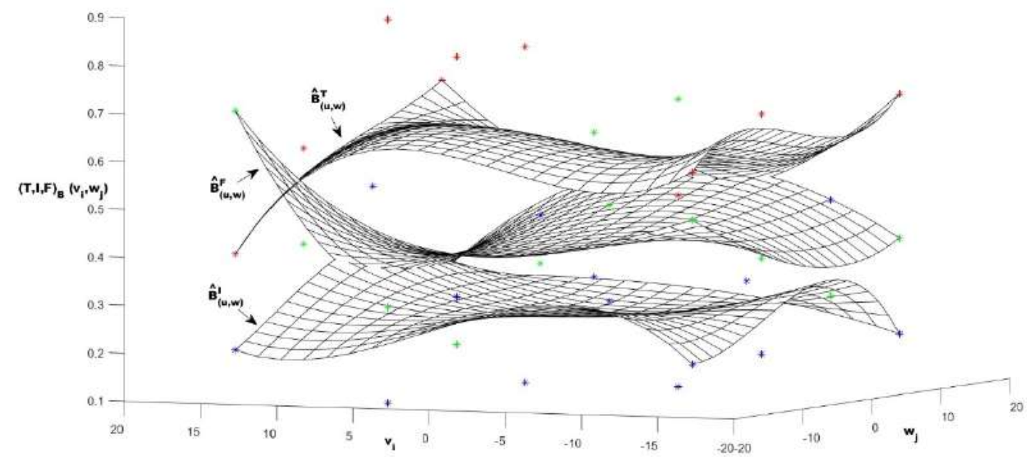

Figure 5.6 Mesh (colorless) Neutrosophic Surface Approximation via Bicubic Bézier

A neutrosophic bicubic Bézier surface's border curves take the form of Bézier curves. Figure 5.1 shows that the tangent vectors for the four-vertex corners of the NCNR patch $\left(\hat{B}_{0,0}, \hat{B}_{0,3}, \hat{B}_{3,0}, \hat{B}_{3,3}\right)$ are oriented and have different intensities depending on the positions of the neighboring points along the border. This is the case regardless of whether the patch has three or four vertices. To provide just one example, the NCNR vertices $\hat{B}_{2,0}$ and $\hat{B}_{3,1}$ control the tangent vectors in both the and directions for $\hat{B}_{3,0}$. In a similar manner, the NCNR vertices $\left(\hat{B}_{0,1}, \hat{B}_{1,0}\right)$, $\left(\hat{B}_{0,2}, \hat{B}_{1,3}\right)$ followed by $\left(\hat{B}_{2,3}, \hat{B}_{3,2}\right)$, govern the same corners $\hat{B}_{0,0}$, $\hat{B}_{0,3}$ followed by $\hat{B}_{3,3}$. In addition, the four internal vertices of the NCNR $\left(\hat{B}_{1,1}, \hat{B}_{1,2}, \hat{B}_{2,1}, \hat{B}_{2,2}\right)$ depend on the magnitude and direction of the twist vectors that are present at the patch corners. Therefore, the neutrosophic bicubic Bézier surface is profoundly influenced by NCNR, and it assumes the form of NCNR from Figure 5.2 to Figure 5.6. Additionally, the neutrosophic bicubic Bézier surface may be reshaped by adjusting and manipulating the value of NCNR. In the next section will briefly summarize this paper.

5 Conclusion

Neutrosophic bicubic Bézier surface approximation was introduced in this work by means of NPR and NCNR. This model's strengths lie in its simplicity and readability, as well as its ability to transform the neutrosophic data in a neutrosophic bicubic for Bézier surface. Expanding this model to include interpolation of Bézier surface or B-spline and approximation and interpolation of non-uniform rational B-spline (NURBS) surface will provide more accurate and seamless results. A neutrosophic bicubic Bézier surface will be used to filter and analyze all the data, and the results surface that NCNR provides will include all of the relevant details. With its neutrosophic bicubic Bézier surface features, the NCNR can provide a thorough analysis and description of a modelling issue, with each surface visually including truth membership, false membership, and indeterminacy membership, each of which has its own meaning and explanation. Fuzzy modelling, and especially its application to the visualisation of neutrosophic data issues, may benefit from the provided NPR, NCNR, and neutrosophic bicubic Bézier surface. The proposed method can be applied to a wide variety of surface visualisation problems, such as those involving the modelling of spatial regions with ambiguous borders in geographic data systems (GIS), satellite imagery, object rebuilding from an aerial laser scanning device, visualisation of bathymetric statistics, and many others.

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