MATEMATIKA, MJIM, Volume 39, Number 3, 263–279

Heat Transfer Enhancement of Convective Casson Nanofluid Flow by CNTs over Exponentially Accelerated Plate

Wan Nura'in Nabilah Noranuar, Ahmad Qushairi Mohamad*, Lim Yeou Jiann and Sharidan Shafie

Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia
81310 Johor Bahru, Johor, Malaysia

*Corresponding author: ahmadqushairi@utm.my

Article history
Received: 18 February 2023
Received in revised form: 5 December 2023
Accepted: 8 December 2023
Published on line: 31 December 2023

Abstract Carbon nanotubes (CNTs) nanofluids are gaining increased popularity among researchers due to their outstanding thermal properties, leading to numerous promising industrial applications. Analytical solutions discovered in the study of CNTs nanofluids, combined with a Casson-type fluid model, are extremely limited. Therefore, a study on the heat transfer analysis of an unsteady and incompressible Casson carbon nanofluid flow is conducted. Human blood-based single-walled carbon nanotubes (SWCNTs) and human blood-based multi-walled carbon nanotubes (MWCNTs) are considered as nanofluids that move beyond an exponentially accelerated vertical plate. A set of dimensional momentum and energy equations, along with their initial and exponentially accelerated boundary conditions, is employed to represent the problem. The transformation of these equations to the dimensionless expression is achieved by using suitable dimensionless variables. The resulting equations are then tackled using Laplace transformation to acquire the analytical solution for temperature and velocity. Figures and tables are produced for a further analysis of temperature and velocity characteristics. The study shows that an increase in nanoparticle volume fraction enhances nanofluid flow and heat transmission, proving highly beneficial for cancer treatment. However, the flow is retarded due to the increment of Casson parameter values, while an enhancement is observed with a superior accelerating parameter.

Keywords Casson nanofluid; Carbon nanotubes; Exponentially accelerated plate, Exact solution; Laplace transform.

Mathematics Subject Classification 35J05, 35K05, 80A20, 37N15, 35F05, 76R10.

1 Introduction

Studies on heat transfer and convective flow of nanofluids find relevance in several industrial applications, such as transportation, nuclear reactors, electronic, as well as food and biomedicine. Nanofluids, which are diluted liquid suspensions of nanoparticles with at least one of their principal dimensions smaller than 100nm, have emerged as potential candidates for designing heat transfer fluids. These nanoparticles are often made of metal, metal oxides, or carbon, and they have a significant boosting influence on the nanofluid's thermal conductivity, thus raising convection and conduction coefficients and enabling greater heat transmission. Their advancement promises improved thermal management solutions for the next generation of heat-dispersing electronic devices. In the space and defense sectors, factors of lightweight and small size become significant criteria for highly efficient cooling and heating systems to overcome the constraints of weight, space, and available energy in aircraft and space stations [1]. The novel study on nanofluids was firstly reported by Choi and Eastman [2]. Their experimental results inspired many researchers to work on nanofluid studies in various geometries and effects. Krishna *et al.* [3] conducted an analytical study on nanofluid flow affected by a free convection scenario and a plate with vertical motion. Nanofluids containing two types of metal oxide nanoparticles suspended respectively in water have been used with the consideration of radiation absorption effect. Considering the same types of oxide nanofluids, Babu *et al.* [4] analyzed the heat performance in a vertical cylinder. Madhura *et al.* [5] scrutinized the influence of nanoparticle shapes on the generalized formulation of natural convective flow with mass and heat transportation. Shekaramiz *et al.* [6] and Abderrahmane *et al.* [7] conducted a numerical study on two-dimensional boundary layer flow of MHD nanofluids in a wavy enclosure. The time-dependent flow of nanofluids with thermal radiation and magnetic field effects was carried out by Anwar *et al.* [8] by utilizing water-based copper and titanium oxide as their nanofluids. Anwar *et al.* [9] extend their previous work by imposing boundary conditions of ramped wall temperature and ramped wall velocity. Roy and Pop [10] examined the heat transfer on free convection flow of hybrid nanofluids by assuming the flow passed within two vertical plates. Hajizadeh *et al.* [11] considered a similar geometry to investigate flow characteristics of nanofluids under the influences of damped thermal flux and radiation.

Basically, the nanofluid thermal performance is highly dependent on the use of nanoparticles. Nanoparticles with a high thermal conductivity can be a good candidate for improving convective heat transfer. Carbon nanotubes (CNTs) are one of the nanoparticle developments, and they are often classified into two types; multi wall carbon nanotubes (MWCNTs) and single-wall carbon nanotubes (SWCNTs). They are very familiar with the outstanding thermal conductivity compared to other nanoparticles [12]. The impact of CNTs on free convection flow by choosing water as the base fluid was analyzed by Khalid *et al.* [13] and further continued by Khalid *et al.* [14] by replacing the base fluid with human blood. Both studies were solved for an oscillating boundary condition and the exact solutions were solved using method of Laplace transform. Next, Ebaid and Al Sharif [15] used a stretching surface with the imposition of a magnetic field effect to analyze the CNTs nanofluid flow and the study revealed that the temperature and velocity distributions ascended due to the inclusion of more nanoparticles into the base fluid. Similar work was done by Alkasasbeh *et al.* [16], considering both human blood and water as base fluids and their findings show that increasing values of the Casson nanofluid parameter result in a decreasing velocity profile and an increasing temperature profile. Saqib *et al.* [17] studied the flow of nanofluid with CNTs suspension in Carboxy-Methyl-Cellulose (CMC) between two vertical parallel plates. It was concluded that MWCNTs nanoparticles exhibited higher velocity compared to SWCNTs nanoparticles due to their low density. The identical effects of CNTs as Saqib *et al.* [17] were also revealed by Khan [18] in investigating the convective heat transfer for Brinkman fluid over an inclined plate. Then, some CNTs nanofluid related studies on heat transfer analysis were also conducted by Mosayebidorcheh and Hatami [19], Dawar *et al.* [20] and Noranuar *et al.* [21] but they concentrated on the rotating convective flow of CNTs nanofluid with radiation effect.

Researchers now place greater emphasis on the analysis of heat transportation because of its many industrial applications, such as bio-engineering processes, food processing industries, drilling operations, and metallurgy processes. Various types of fluids are employed as heat or energy carriers to enhance product quality in industries. Recently, most industrial applications have encountered non-Newtonian fluids. This type of fluid is found to be more complex than Newtonian fluid, and the simulation of fluid flow for non-Newtonian is significant for an industry to examine the heat transfer process [22]. This challenge encouraged researchers to take a particular interest in exploring the flow features of non-Newtonian fluids with nanoparticle effects. The Casson fluid is a non-Newtonian fluid model that describes the flow behavior of viscoplastic fluids, behaving as a shear thinning liquid when applied stress dominates the flow. Once the level of applied stress surpasses the amount of yield stress, liquid flow behavior is exhibited. Casson fluid now describes the rheological model for human blood because it consists of several substances, such as human red blood cells, globulin in aqueous solution, protein, fibrinogen, and base plasma [23]. Numerous studies of free convection on the Casson nanofluid model have been conducted, considering various aspects, including those of Alwawi *et al.* [24], Raza *et al.* [25], Ghadikolaei *et al.* [26] and Noranuar *et al.* [27]. Ullah *et al.* [28] considered the magnetic field and thermal radiation effects on the Casson nanofluid flow across a nonlinearly stretching sheet. Ali *et al.* [29] researched the time-dependent convective flow beyond an infinite vertical oscillating plate for MHD Casson nanofluid embedded in a porous material. A similar study of an oscillating plate was considered by Krishna [30] to scrutinize the thermal and convective flow features of Casson hybrid nanofluids in the presence of several effects. Siddique *et al.* [31] conducted a Casson nanofluid study by assuming that the radiative nanofluid is unsteadily flowing within a vertical channel.

The use of exponential accelerated conditions in nanofluid flow is critical in several fields of contemporary research and technology. For instance, their practice in the medical field helps the delivery agents move and improves the travel of drugs to the target areas. Despite their practical recommendations, inadequate study has been found for exponentially accelerated conditions combined with the effect of nanofluid. However, several researchers are conducting Casson convective flow by taking another type of accelerated conditions, which is impulsive accelerated condition. Amongst, Shahrim *et al.* [32] and Reyaz *et al.* [33] discussed the time-dependent flow through an accelerated plate and generated the exact solutions using the Laplace transformation. Omar *et al.* [34] analyzed MHD Casson fluid flow beyond an accelerated plate under the influence of radiation effects. Deka [35] performed an identical study by considering mass and heat transfer and porosity effects. The investigation of Casson fluid flow across an accelerated plate is extended to an exponentially accelerated plate by Ramakrishna *et al.* [36], Kataria and Patel [37], and Pattnaik *et al.* [38]. The modification has been made to the boundary condition by expressing the condition in term of exponential. They applied the method of Laplace transform to solve the extended problem. Roa *et al.* [39] performed the same geometry of the problem, and taking the consequences of MHD, porosity, radiation absorption, and heat generation into account. Matta *et al.* [40] conducted a numerical study of the exponentially accelerated boundary condition for MHD Casson fluid with ramped surface concentration without nanoparticle effect.

Based on the above-mentioned literature, it is evident that the convective flow of Casson nanofluid affected by an exponentially accelerated plate has never been considered, particularly using the mixture of CNTs in Casson human blood as the nanofluid. Hence, it was an interesting attempt to fill this gap in the study and overcome the limitations in this field. Here, the motivation is to investigate the time-dependent free convection flow of Casson nanofluid with CNTs over an exponentially accelerated plate. The closed form solutions for temperature and velocity profiles with exponentially accelerated boundary condition are established by using the Laplace transform method. Since the present problem is considered unsteady, this transform is found to be the best tool to use as it involves the conversion of a real variable function (time domain) to a complex variable function (q-domain) and is able to solve the differential equation, which has many applications in science and engineering. Both profiles are further analyzed for their parameters' effects with the help of graphical results and a comprehensive discussion.

2 Mathematical Formulation

An incompressible Casson nanofluid containing carbon nanotubes, where its flow over an infinite vertical plate is unsteadily driven by a free convection phenomenon, is considered as clearly illustrated in Figure 1. The vertical plate is assumed to accelerate exponentially, with the upward direction along the plate defined as x-axis and the normal to the plate defined as y-axis, respectively. At first, when $t = 0$, both fluid and plate with a similar temperature of T_∞ are in a stationary condition. When $t > 0$, the plate temperature is raised to T_w and the plate is exponentially accelerated with a velocity $u = u_0 e^{at}$ in its own plane. Based on the above assumptions and using Boussinesq's approximation [8], the governing equations are defined as follows [20, 22]:

$$\rho_{nf}\frac{\partial u}{\partial t} = \mu_{nf}\left(1 + \frac{1}{\gamma}\right)\frac{\partial^2 u}{\partial y^2} + g(\rho\beta)_{nf}\left(T - T_\infty\right), \tag{1}$$

$$\left(\rho C_p\right)_{nf}\frac{\partial T}{\partial t} = k_{nf}\frac{\partial^2 T}{\partial y^2}, \tag{2}$$

with specified conditions for initial and boundary [36, 37]:

$$\begin{aligned} u\left(y,0\right) = 0, &\quad T\left(y,0\right) = T_\infty\ ; y > 0,\\ u\left(0,t\right) = u_0 e^{at}, &\quad T\left(0,t\right) = T_w; t > 0,\\ u\left(\infty,t\right) = 0, &\quad T\left(\infty,t\right) = T_\infty; t > 0, \end{aligned} \tag{3}$$

where u, T, γ, t, g, T_w, T_∞ are the velocity of nanofluid, temperature of nanofluid, Casson parameter, time, acceleration due to gravity, fluid temperature at the boundary layer, and free stream temperature far away from the plate. Meanwhile, the subscript nf in Eqs. (1) and (2) represents nanofluid with the term k_{nf}, $C_{p_{nf}}$, β_{nf}, μ_{nf}, ρ_{nf}, defined thermal conductivity, nanofluid specific heat at constant pressure, volumetric coefficient of thermal expansion, dynamic viscosity, and density. The expression of μ_{nf}, ρ_{nf}, $(\rho\beta)_{nf}$, $\left(\rho C_p\right)_{nf}$ and k_{nf} are [21]

$$\begin{gathered} \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}},\\ \rho_{nf} = (1-\phi)\,\rho_f + \phi\rho_{CNTs}, (\rho\beta)_{nf} = (1-\phi)\,(\rho\beta)_f + \phi(\rho\beta)_{CNTs},\\ \left(\rho C_p\right)_{nf} = (1-\phi)\left(\rho C_p\right)_f + \phi\left(\rho C_p\right)_{CNTs},\\ \frac{k_{nf}}{k_f} = \frac{1-\phi+2\phi\dfrac{k_{CNTs}}{k_{CNTs}-k_f} ln\, ln\, \dfrac{k_{CNTs}+k_f}{2k_f}}{1-\phi+2\phi\frac{k_f}{k_{CNTs}-k_f} ln\, ln\, \dfrac{k_{CNTs}+k_f}{2k_f}}, \end{gathered} \tag{4}$$

where ϕ is the nanoparticle volume fraction, and the subscripts f and $CNTs$ denoted to fluid and carbon nanotubes. The thermal features are assumed as constant as listed in Table 1.

The suitable non-dimensional variables are

$$u^* = \frac{u}{u_0}, \quad t^* = \frac{t{u_0}^2}{\nu}, \quad y^* = \frac{yu_0}{\nu}, \quad T^* = \frac{T - T_\infty}{T_w - T_\infty}. \tag{5}$$

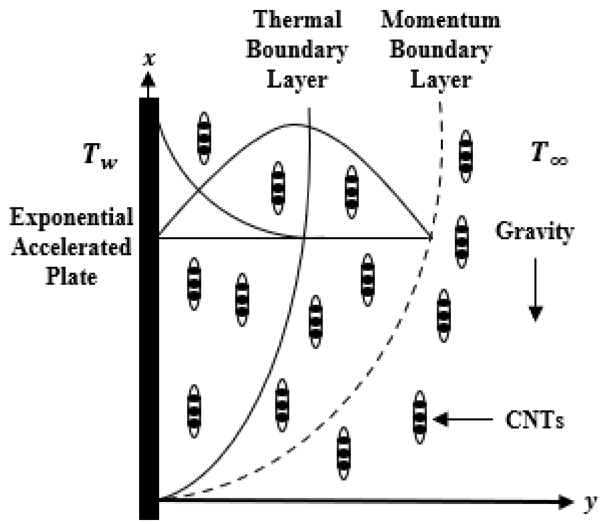

Figure 1: The physical model considered in this study

Table 1: Thermal features of MWCNTs, SWCNTs, and human blood

Properties/ Materials	$\rho\left(Kgm^{-3}\right)$	$k\left(Wm^{-1}K^{-1}\right)$	$C_p\left(JKg^{-1}K^{-1}\right)$	$\beta \times 10^{-5}\left(K^{-1}\right)$	$\sigma\left(Sm^{-1}\right)$
MWCNTs	1600	3000	796	44	1.9×10^{-4}
SWCNTs	2600	6600	425	27	10^6 - 10^7
Human blood	1053	0.492	3594	0.8	0.18

Imposing Eq. (5) in Eqs. (1) and (2) together with nanofluid constants (4) leads to

$$\frac{\partial u^*}{\partial t^*} = \frac{1}{\phi_1}\left(1 + \frac{1}{\gamma}\right)\frac{\partial^2 u^*}{\partial y^{*2}} + Gr\phi_2 T^*, \tag{6}$$

$$\frac{\partial T^*}{\partial t^*} = \frac{1}{a_1}\frac{\partial^2 T^*}{\partial y^{*2}}, \tag{7}$$

with associated conditions

$$\begin{aligned} &u^*(y,0) = 0, \quad T^*(y,0) = 0; \quad y > 0, \\ &u^*(0,t) = e^{a^* t}, \quad T^*(t,0) = 1; \quad t > 0, \\ &u^*(\infty,t) = 0, \quad T^*(\infty,0) = 0; \quad t > 0, \end{aligned} \tag{8}$$

where

$$Pr = \frac{\upsilon_f \left(\rho C_p\right)_f}{k_f}, \quad Gr = \frac{g\beta_f \left(T_w - T_\infty\right)}{{u_0}^3}, \quad a^* = \frac{a\upsilon}{{u_0}^2}, \tag{9}$$

are the dimensionless parameter of Prandtl number, Grashof number, and acceleration parameter. Meanwhile,

$$a_1 = \frac{\phi_3 Pr}{\lambda}, \quad \phi_1 = (1-\phi)^{2.5}\left((1-\phi) + \frac{\phi\rho_{CNTs}}{\rho_f}\right),$$
$$\phi_2 = \frac{(1-\phi) + \frac{\phi(\rho\beta)_{CNTs}}{(\rho\beta)_f}}{(1-\phi) + \frac{\phi\rho_{CNTs}}{\rho_f}}, \quad \phi_3 = (1-\phi) + \frac{\phi\left(\rho C_p\right)_{CNTs}}{\left(\rho C_p\right)_f} \tag{10}$$

are nanofluid constant parameters.

3 Solution of the Problem

The Laplace transform is applied to Eqs. (6) to (8), which gives (neglected *)

$$\frac{d^2}{dz^2}\overline{u}(y,q) - b_1 q\overline{u}(y,q) = -\phi_2 b_1 Gr\frac{1}{q}exp\left(-y\sqrt{a_1 q}\right), \tag{11}$$

$$\frac{d^2}{dz^2}\overline{T}(y,q) - a_1 q\overline{T}(y,q) = 0, \tag{12}$$

$$\overline{u}(0,q) = \frac{1}{q-a}, \ \ \overline{u}(\infty,q) = 0; \quad q > 0, \tag{13}$$

$$\overline{T}(0,q) = 0, \ \ \overline{T}(\infty,q) = 0; \quad q > 0. \tag{14}$$

Then, Eqs. (13) and (14) are used to solve the Eqs. (11) and (12), in which the Laplace solution turns into

$$\overline{u}(y,q) = \overline{u}_1(y,q) + \overline{u}_2(y,q) - \overline{u}_3(y,q), \tag{15}$$

$$\overline{T}(y,q) = \frac{1}{q}\exp\left(-y\sqrt{a_1 q}\right), \tag{16}$$

with

$$\overline{u}_1(y,q) = \frac{1}{q-a}exp\left(-y\sqrt{b_1 q}\right), \tag{17}$$

$$\overline{u}_2(y,q) = \frac{D_1}{q^2}exp\left(-y\sqrt{b_1 q}\right), \tag{18}$$

$$\overline{u}_3(y,q) = \frac{D_1}{q^2}exp\left(-y\sqrt{a_1 q}\right). \tag{19}$$

After that, Eqs. (15) and (16) are imposed with the inverse Laplace and the exact velocity and temperature solutions are acquired as follows

$$u(y,t) = u_1(y,t) + u_2(y,t) - u_3(y,t), \tag{20}$$

$$T(y,t) = erfc\left(\frac{y}{2}\sqrt{\frac{a_1}{t}}\right), \tag{21}$$

with

$$u_1(y,t) = \frac{1}{2}exp\left(at + y\sqrt{b_1 a}\right)erfc\left(\frac{y}{2}\sqrt{\frac{b_1}{t}} + \sqrt{at}\right) + \frac{1}{2}exp\left(at - y\sqrt{b_1 a}\right)erfc\left(\frac{y}{2}\sqrt{\frac{b_1}{t}} - \sqrt{at}\right),$$

$$u_2(y,t) = D_1\left[\left(\frac{\left(-y\sqrt{b_1}\right)^2}{2} + t\right)erfc\left(\frac{-y\sqrt{b_1}}{2\sqrt{t}}\right) - \left(-y\sqrt{b_1}\right)\sqrt{\frac{t}{\pi}}exp\left(\frac{-\left(-y\sqrt{b_1}\right)^2}{4t}\right)\right],$$

$$u_3(y,t) = D_1\left[\left(\frac{\left(-y\sqrt{a_1}\right)^2}{2} + t\right)erfc\left(\frac{-y\sqrt{a_1}}{2\sqrt{t}}\right) - \left(-y\sqrt{a_1}\right)\sqrt{\frac{t}{\pi}}exp\left(\frac{-\left(-y\sqrt{a_1}\right)^2}{4t}\right)\right],$$

where

$$b_0 = \frac{\gamma}{1+\gamma}, \quad b_1 = \phi_2 b_0, \quad b_2 = \phi_3 b_1, \quad c_1 = \frac{1}{a_1 - b_1}, \quad D_1 = b_2 c_1 Gr.$$

The following expression of skin friction, τ which quantifies the shear stress at the boundary and expression of Nusselt number, Nu are

$$\tau = \frac{1}{(1-\phi)^{2.5}}\left(1 + \frac{1}{\gamma}\right)\frac{\partial u}{\partial y}\bigg|_{y=0}, \tag{22}$$

$$\tau = \frac{1}{(1-\phi)^{2.5}}\left(1 + \frac{1}{\gamma}\right)\left[\begin{array}{c}\left(\exp(at)\sqrt{ab_1}\ erfc(at) - \exp(at)\sqrt{ab_1} - \sqrt{\frac{b_1}{\pi t}}\right) + \\ D_1\sqrt{\frac{b_1 t}{\pi}} - D_1\sqrt{\frac{a_1 t}{\pi}}\end{array}\right]$$

and

$$Nu = -\frac{k_{nf}}{k_f}\frac{\partial T}{\partial y}\bigg|_{y=0} = \lambda\sqrt{\frac{a_1 t}{\pi}}. \tag{23}$$

4 Results and Discussion

The influences of governing parameters on nanofluid flow and heat transfer are analyzed via graphical representation, as illustrated in Figures 2 to 9. Based on the study of Misra et al. [38] and Sinha *et al.* [39], the Prandtl number, Pr = 21 is considered for blood at human body temperature, $T = 310K$ and therefore, the analysis in this study is done by letting Prandtl number of base fluid (human blood) be Pr = 21.Moreover, it is worth noting that the value of Pr for blood is higher compared to water and other common base fluids, as their viscosity also plays a role in the value of Pr [40]. Two different nanofluids containing single-wall carbon nanotubes (SWCNTs) and multi-wall carbon nanotubes (MWCNTs) with blood as the base fluid are considered. The effect of nanoparticle volume fraction is analyzed in the range of $0.02 \leq \phi \leq 0.06$, because the sedimentation occurs when nanoparticle volume fraction exceeds 8% [41]. Moreover, assigning zero to the parameter of nanoparticle volume fraction reduces the present study to the case of regular fluid flow, where the nanofluid characteristics are eliminated. For validation of the present results, the temperature profiles of the current study (without nanoparticle volume fraction) are compared with those from previous research by Omar *et al.* [34] in Figure 2, and an excellent agreement is discovered between both studies. Besides, the accuracy of the present velocity profiles is also checked using the numerical Gaver-Stehfest algorithm [45, 46], and the results are satisfactory with a small difference as listed in Table 2.

Figure 3 exhibits the nanofluid velocity profiles for SWCNTs and MWCNTs under the impact of Grashof number Gr. As the Grashof number grows, both CNTs' velocities increase. The interaction between thermal buoyancy force and viscous hydrodynamic force is described by the Grashof number. A rise in the Grashof number, affected by free convection processes, leads to an increase in the buoyancy force, thereby accelerating the flow. Positive Gr values signify that the plate is cooled by free convection. As a result, the heat near the plate is transferred away to the fluid, raising its temperature and enhancing the buoyancy force. This finding holds significant implications for drug delivery, as particles moving through convection will be guided by fluid flow velocity to the cancerous area.

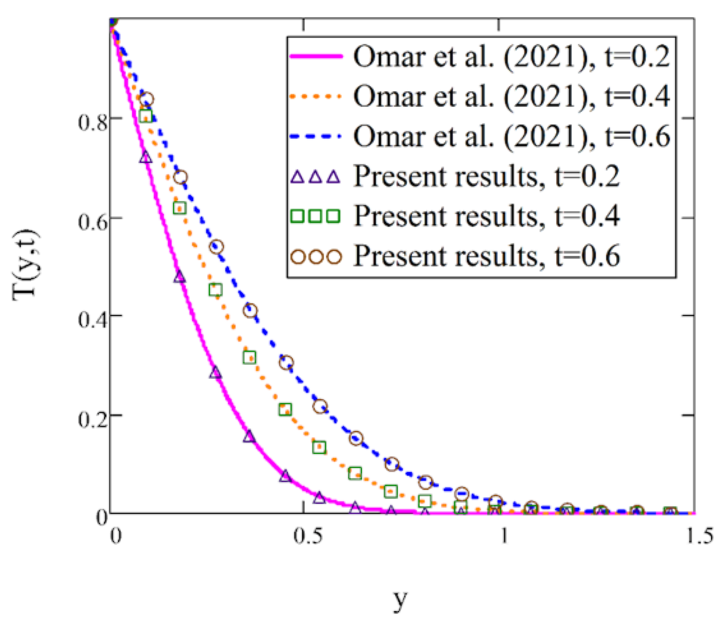

Figure 2: Comparison of $T(y,t)$ of the current study with Omar *et al.* [34]

Table 1: Accuracy of $u(y,t)$ between exact solution and numerical Gaver-Stehfest algorithm for SWCNTs and MWCNTs with $\gamma = 0.02,\ Gr = 0.5,\ \phi = 0.02,\ t = 0.2,\ \Pr = 21$

y	***SWCNTs***		***MWCNTs***	
	Exact Solution	**Gaver-Stehfest**	**Exact Solution**	**Gaver-Stehfest**
0	1.1052	1.1052	1.1052	1.1052
5	0.2854	0.2904	0.2904	0.2904
10	0.0290	0.0301	0.0306	0.0301
15	0.0010	0.0011	0.0011	0.0011

Figure 4 represents the differences in nanofluid velocity for both CNTs at distinct values of the acceleration parameter a. Both nanofluids show an augmented velocity profile when the acceleration parameter of the plate increases. In these figures, the boundary condition $u^*(0,t) = 1$ is satisfied by the curve of $a = 0$, representing the scenario of constant plate velocity, which results in a thinner boundary layer. Moreover, it is observed that the nanofluid velocity for an exponential velocity of the plate ($a = 0.2,\ 0.6,\ 1.0$) precedes the nanofluid velocity for a constant velocity of the plate ($a = 0$). When $a = 0$, the present case is reduced to a scenario similar to that in Anwar *et al.* [8] and Das and Jana [47]. In the medical field, the acceleration parameter may generate pressure, serving as the primary motivator for convective fluid flow through the blood vessels.

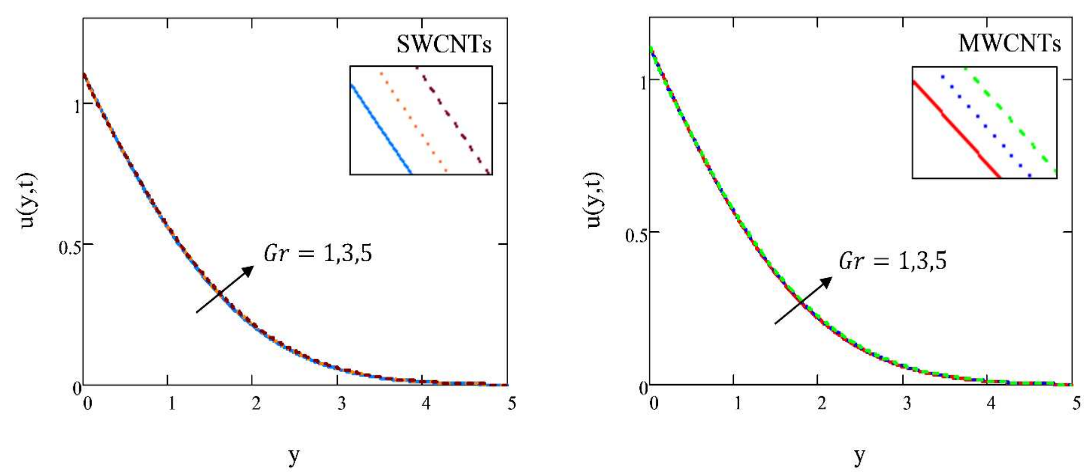

Figure 3: $u(y,t)$ of SWCNTs and MWCNTs for different Gr.

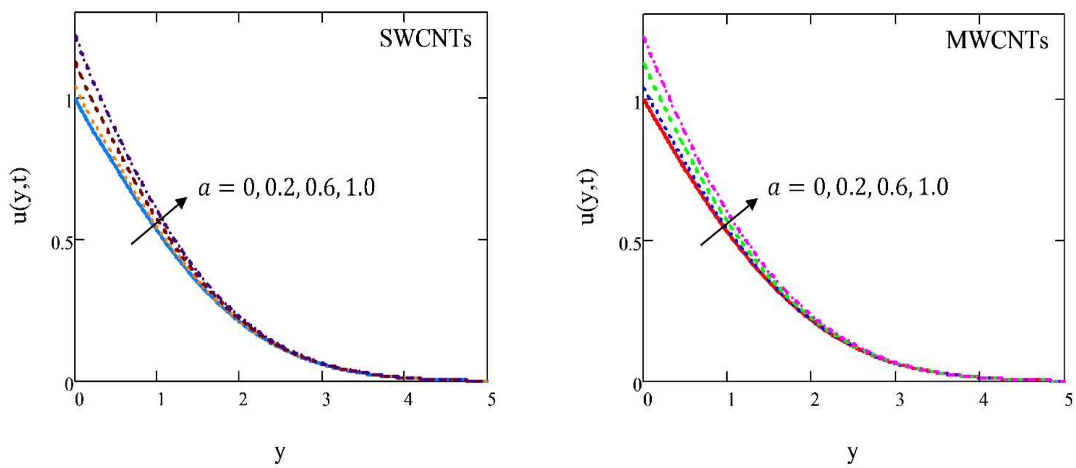

Figure 4: $u(y,t)$ of SWCNTs and MWCNTs for different a.

The velocity of both nanofluids under the influence of Casson parameter γ is depicted in Figure 5. It is noticed that as the Casson parameter value ascends, the boundary layer thickness and the velocity profiles of SWCNTs and MWCNTs reduce. This effect is supported by the fact that an increase in the Casson parameter is equivalent to an increment of fluid plasticity, causing the fluid motion to retard. From the graph, it shows that as the value of Casson parameter rises and approaches infinity, the non-Newtonian behavior disappears and the fluid behaves as a Newtonian fluid. When γ approach infinity ($\gamma \rightarrow \infty$), the modelling of fluid flow in this present study reduces to the study of Vemula *et al.* [48]. To sustain the rheological of blood flow, the results of Casson parameter are responsible for the non-Newtonian behavior of blood. According to Chaturani and Palanisamy [49], this result can be applied to cure the blood clots in a coronary artery.

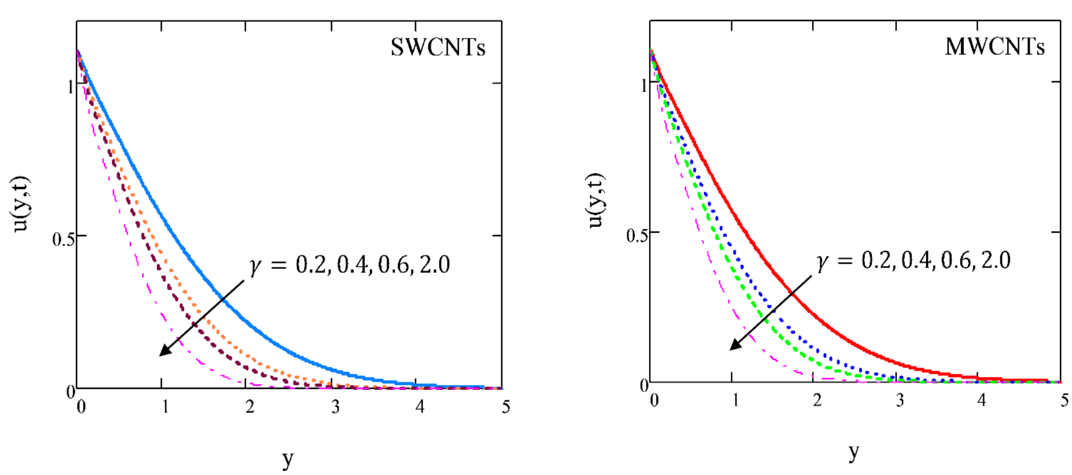

Figure 5: $u(y,t)$ of SWCNTs and MWCNTs for different γ.

Figures 6 and 7 highlight the propagation of velocity and temperature profiles for both types of CNTs nanofluid which are affected by CNTs volume fraction ϕ. The results reveal that both profiles improve significantly by raising the volume fraction of CNTs. This enhancement is in response to the upsurge of nanofluid thermal conductivity due to the addition of CNTs in human blood, which in turn enhances thermal diffusion and causes a rise in the thickness of the thermal boundary layer. These findings highlight the significance of nanoparticles in cancer treatment, where their use can improve the delivery of therapeutic agents to the targeted cells.

Further analysis on the impacts of SWCNTs and MWCNTs nanoparticles is represented in Figures 8 and 9. From these two figures, assigning $\phi = 0$ indicates to the neglection of nanoparticle effect and the fluid behaves as regular fluid (Casson fluid) [37, 43]. Figure 8 shows that the velocity under the influence of nanoparticles (Casson nanofluid) is greater than the fluid without nanoparticles (Casson fluid). In Figure 9, it can be seen that the addition of SWCNTs and MWCNTs to regular fluid (Casson fluid) gives a significant enhancement in term of convective heat transfer. The reason behind this enhancement is that CNTs have exceptionally high thermal conductivity (6000 W/mK for single-wall CNTs and 3000 W/mK for multi-wall CNTs), which makes them a great candidate for improving heat transmission. Further, due to SWCNTs have higher thermal conductivity compared to MWCNTs, it is noticed that SWCNTs significantly improves the transmission of heat.

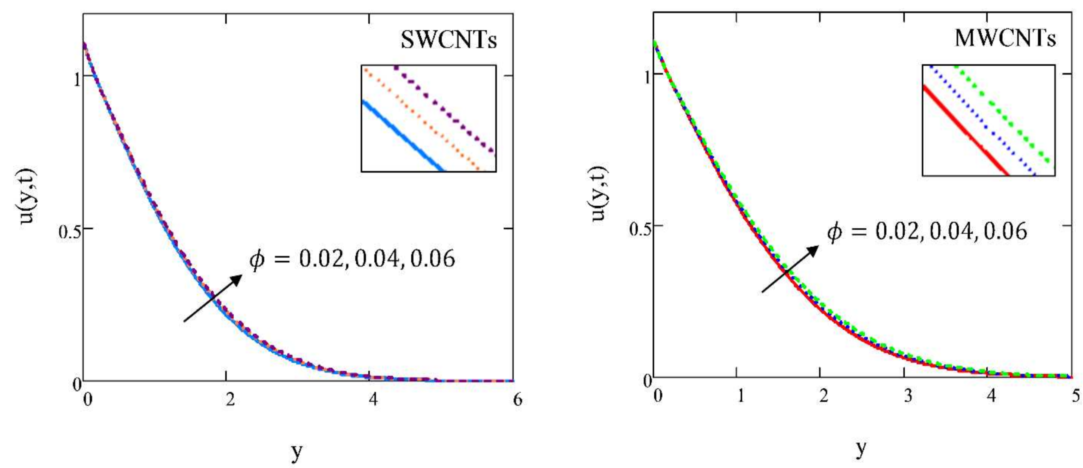

Figure 6: $u(y,t)$ of SWCNTs and MWCNTs for different ϕ.

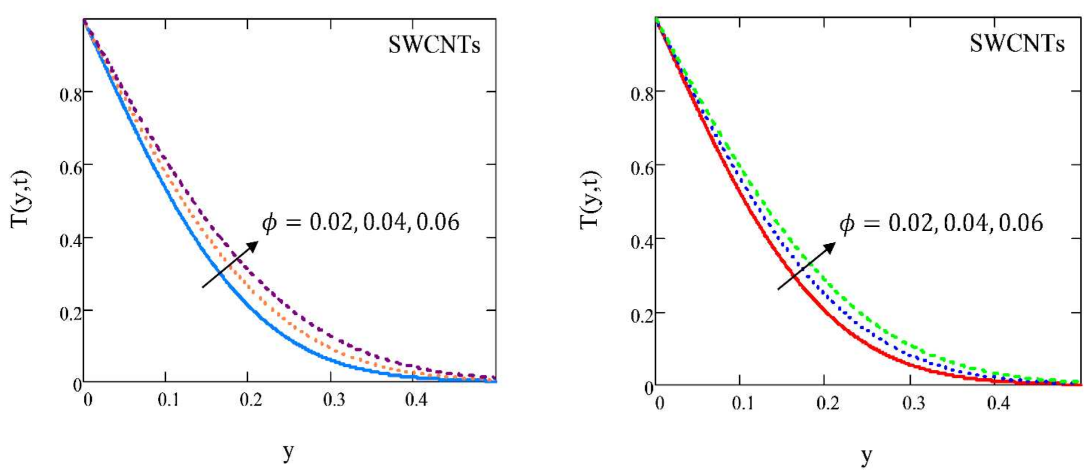

Figure 7: $T(y,t)$ of SWCNTs and MWCNTs for different ϕ.

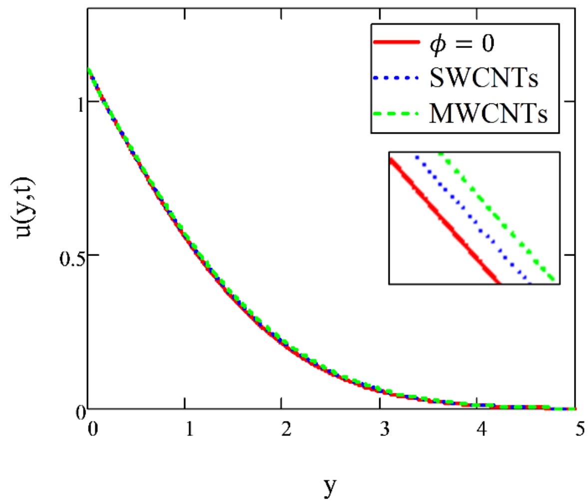

Figure 8: Comparison of $u(y,t)$ for SWCNTs, MWCNTs and without nanoparticles.

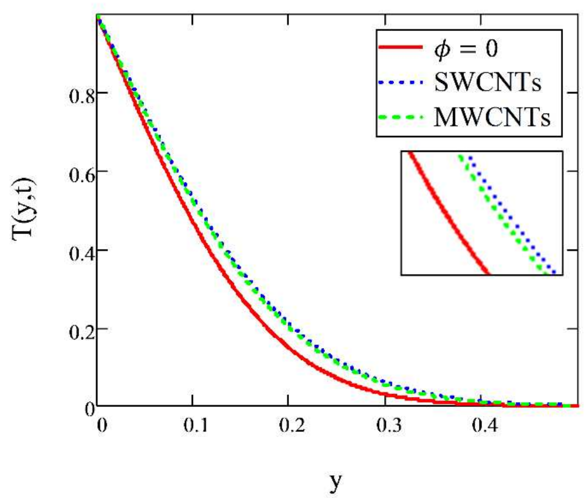

Figure 9: Comparison of $T(y,t)$ for SWCNTs, MWCNTs and without nanoparticles.

Besides that, Tables 3 and 4 provide the numerical values of skin friction and Nusselt number. These physical quantities are evaluated to describe the flow pattern at the wall, and they are essential for flow and thermal stability. Clearly understanding the meaning of skin friction, the viscous drag at the surface of an object is the factor that influences the level of skin friction. According to Table 3, all the parameters affect the skin friction oppositely to the velocity profile. The skin friction weakens by increasing the acceleration parameter a, Grashof number Gr, and nanoparticle volume fraction ϕ. While for the Casson parameter γ, it is vice versa. Thus, it may conclude that an increase of fluid flow with increasing values of a, ϕ, and Gr is because of the viscous drag at the boundary layer is getting weaker. Based on Table 4, an increment of ϕ values rise the Nusselt number. Thus, it can be concluded that a high thermal conductivity fluid promotes a higher heat transmission rate near the surface.

Table 3: Variation of Skin friction

γ	a	ϕ	Gr	τ	
				SWCNTs	***MWCNTs***
0.2	0.5	0.02	2	-4.0292	-3.9889
0.4	0.5	0.02	2	-3.1025	-3.0714
0.6	0.5	0.02	2	-2.7210	-2.6937
0.2	0.6	0.02	2	-4.1782	-4.1364
0.2	0.8	0.02	2	-4.4881	-4.4434
0.2	0.5	0.04	2	-4.2123	-4.1306
0.2	0.5	0.06	2	-4.4026	-4.2781
0.2	0.5	0.02	4	-4.1499	-4.1075
0.2	0.5	0.02	5	-4.2103	-4.1669

Table 4: Variation of Nusselt number

ϕ	Nu	
	SWCNTs	***MWCNTs***
0.02	6.7671	6.7625
0.04	7.6782	7.6956
0.06	8.5400	8.5991

5 Conclusion

In this paper, a comprehensive study on unsteady Casson nanofluid flow past through an exponential accelerated plate has been carried out. An analytical solution using the Laplace transformation has been achieved to study the fluid flow and thermal features of MWCNTs and SWCNTs nanofluids with a Casson fluid model. The preceding discussion points out the following

(i) Plate with a constant velocity (Figure 4, curve $a = 0$).

(ii) Unsteady flow of Newtonian nanofluid (Figure 5, curve $\gamma = 2 / \gamma \to \infty$).

(iii) Flow in the absence of nanoparticle volume fraction effect (Figure 8, curve $\phi = 0$).

Some useful conclusions are made as follows

- The nanofluid velocity profiles enhance with superior a, ϕ, and Gr.
- While an ascend of γ values reduce the nanofluid velocity profiles.
- An increment of ϕ values rise the temperature profiles of nanofluid.
- An increase of a, ϕ, and Gr diminishes the values of skin friction. While it is vice versa for γ.

- The values of Nusselt number increase when ϕ increases.
- The velocity profiles for MWCNTs precede the velocity profiles of SWCNTs.
- SWCNTs give a prominent enhancement in convective heat transfer compared to MWCNTs.

The novel of Casson nanofluid flow with SWCNTs and MWCNTs as the nanoparticles has a significant application in many fields of aerospace, nuclear energy, marine or biomedical engineering. The nanofluid with extremely high heat fluxes can provide the necessary cooling or heating rates in biomedical applications, particularly in drug delivery, as well as in military, defence, and space systems, which may include submarines, military vehicles, and even high-power lasers. So, it is necessary to have more works on the nanofluid as it may contribute to the development of space and defense sectors or medical field.

Acknowledgments

This research was funded by Universiti Teknologi Malaysia under Others Grant Scheme (R.J130000.7354.4B748) and Matching Grant Scheme (Q.J130000.3054.03M77).

References

[1] Sofiyev, A. H. and Kuruoglu, N. Buckling analysis of shear deformable composite conical shells reinforced by CNTs subjected to combined loading on the two-parameter elastic foundation. *Defence Technology*. 2022. 18(2): 205-218.

[2] Choi, S. U. and Eastman, J. A. Enhancing thermal conductivity of fluids with nanoparticles. Argonne National Lab.(ANL), Argonne, IL (United States). 1995.

[3] Krishna, M. V., Ahamad, A. N. and Chamkha, A. J. Radiation absorption on MHD convective flow of nanofluids through vertically travelling absorbent plate. *Ain Shams Engineering Journal*. 2021. 12(3): 3043-3056.

[4] Babu, S. R., Pradeep, K. P., Basha, S. A. and Maruthi, R. M. Experimental investigation on free convective heat transfer performance of oxide nanofluids along a vertical cylinder. *Ecological Engineering & Environmental Technology*. 2023. 24(5): 185-194.

[5] Madhura, K. R., Atiwali, B. and Iyengar, S. S. Influence of nanoparticle shapes on natural convection flow with heat and mass transfer rates of nanofluids with fractional derivative. *Mathematical Methods in the Applied Sciences*. 2021. 46(7): 8089-8105.

[6] Shekaramiz, M., Fathi, S., Ataabadi, H. A., Varnamkhasti, H. K. and Toghraie, D. MHD nanofluid free convection inside the wavy triangular cavity considering periodic temperature boundary condition and velocity slip mechanisms. *International Journal of Thermal Sciences.* 2021. 170.

[7] Abderrahmane, A., Manoongam, A., Alizadeh, A., Younis, O., Zekri, H., Isa, S. S. P. M., Baghaei, S., Jamshed, W. and Guedri, K. Investigation of the free convection of nanofluid flow in a wavy porous enclosure subjected to a magnetic field using the Galerkin finite element method. *Journal of Magnetism and Magnetic Materials*. 2023. 569: 170446.

[8] Anwar, T., Kumam, P., Shah, Z., Watthayu, W. and Thounthong, P. Unsteady Radiative Natural Convective MHD Nanofluid Flow Past a Porous Moving Vertical Plate with Heat Source/Sink. *Molecules*. 2020. 25(4).

[9] Anwar, T., Kumam, P. and Watthayu, W. An exact analysis of unsteady MHD free convection flow of some nanofluids with ramped wall velocity and ramped wall temperature accounting heat radiation and injection/consumption. *Scientific Reports*. 2020. 10(1): 17830.

[10] Roy, N.C. and Pop, I. Analytical investigation of transient free convection and heat transfer of a hybrid nanofluid between two vertical parallel plates. *Physics of Fluids*. 2022. 34(7): 072005.

[11] Hajizadeh, A., Shah, N. A., Shah, S. I. A., Animasaun, I. L., Gorji, M. R. and Alarifi, I. M. Free convection flow of nanofluids between two vertical plates with damped thermal flux. *Journal of Molecular Liquids*. 2019. 289.

[12] Sharma, S., Tiwari, S.K. and Shakya, S. Mechanical properties and thermal conductivity of pristine and functionalized carbon nanotube reinforced metallic glass composites: A molecular dynamics approach. *Defence Technology*. 2021. 17(1): 234-244.

[13] Khalid, A., Jiann, L. Y., Khan, I. and Shafie, S. Exact solutions for unsteady free convection flow of carbon nanotubes over an oscillating vertical plate. *AIP Conference Proceeding.* 2017. 1830(1): 020054.

[14] Khalid, A., Khan, I., Khan, A. and Shafie, S. Case study of MHD blood flow in a porous medium with CNTS and thermal analysis. *Case Studies in Thermal Engineering*. 2018. 12: 374-380.

[15] Ebaid, A. and Al Sharif, M. A. Application of Laplace Transform for the Exact Effect of a Magnetic Field on Heat Transfer of Carbon Nanotubes-Suspended Nanofluids. *Zeitschrift für Naturforschung A*. 2015. 70(6): 471-475.

[16] Alkasasbeh, H. T., Swalmeh, M. Z., Saeed, H. G. B., Al-Faqih, F. M. and Talafha, A. G. Investigation on Cnts-Water and Human Blood Based Casson Nanofluid Flow over a Stretching Sheet under Impact of Magnetic Field. *Frontiers in Heat and Mass Transfer*. 2020. 14.

[17] Saqib, M., Khan, I. and Shafie, S. Application of Atangana–Baleanu fractional derivative to MHD channel flow of CMC-based-CNT's nanofluid through a porous medium. Chaos, *Solitons & Fractals*. 2018. 116: 79-85.

[18] Khan, I. Ramped heating in CNTS fractional nanofluids. *Case Studies in Thermal Engineering*. 2023. 45: 102836.

[19] Mosayebidorcheh, S. and Hatami, M. Heat transfer analysis in carbon nanotube-water between rotating disks under thermal radiation conditions. *Journal of Molecular Liquids*. 2017. 240: 258-267.

[20] Dawar, A., Shah, Z., Islam, S., Idress, M. and Khan, W. Magnetohydrodynamic CNTs Casson Nanofluid and Radiative heat transfer in a Rotating Channels. *Journal of Physics Research and Applications*. 2018. 1(1): 017-032.

[21] Noranuar, W. N. N. N., Mohamad, A. Q., Shafie, S. Khan, I., Jiann, L. Y. and Ilias, M. R. Non-coaxial rotation flow of MHD Casson nanofluid carbon nanotubes past a moving disk with porosity effect. *Ain Shams Engineering Journal*. 2021. 12(4): 4099-4110.

[22] Aneja, M., Chandra, A. and Sharma, S. Natural convection in a partially heated porous cavity to Casson fluid. *International Communications in Heat and Mass Transfer*. 2020. 114.

[23] Anwar, T., Kumam, P. and Watthayu, W. Unsteady MHD natural convection flow of Casson fluid incorporating thermal radiative flux and heat injection/suction mechanism under variable wall conditions. *Scietific Reports*. 2021. 11(1): 4275.

[24] Alwawi, F.A., Alkasasbeh, H. T., Rashad, A. M. and Idris, R. MHD natural convection of Sodium Alginate Casson nanofluid over a solid sphere. *Results in Physics*. 2020. 16.

[25] Raza, A., Khan. S. U., Farid, S., Khan, M. I., Sun, T. C., Abbasi, A., Khan, M. I. and Malik, M. Y. Thermal activity of conventional Casson nanoparticles with ramped temperature due to an infinite vertical plate via fractional derivative approach. *Case Studies in Thermal Engineering.* 2021. 27.

[26] Ghadikolaei, S.S., Hossienzadeh, K., Ganji, D. D. and Jafari, B. Nonlinear thermal radiation effect on magneto Casson nanofluid flow with Joule heating effect over an inclined porous stretching sheet. *Case Studies in Thermal Engineering*. 2018. 12: 176-187.

[27] Noranuar, W. N. N., Mohamad, A. Q., Shafie, S. and Jiann, L. Y. Heat and Mass Transfer on Magnetohydrodynamics Casson Carbon Nanotubes Nanofluid Flow in an Asymmetrical Channel via Porous Medium. *Symmetry*. 2023. 15(4): 946.

[28] Ullah, I., Khan, I. and Shafie, S. MHD Natural Convection Flow of Casson Nanofluid over Nonlinearly Stretching Sheet Through Porous Medium with Chemical Reaction and Thermal Radiation. *Nanoscale Research Letters.* 2016. 11(1): 527.

[29] Ali, F., Arif, M., Khan, I., Sheikh, N. A. and Saqib, M. Natural convection in polyethylene glycol based molybdenum disulfide nanofluid with thermal radiation, chemical reaction and ramped wall temperature. *International Journal of Heat and Technology*. 2018. 36(2): 619-631.

[30] Krishna, M. V. Chemical reaction, heat absorption and Newtonian heating on MHD free convective Casson hybrid nanofluids past an infinite oscillating vertical porous plate. *International Communications in Heat and Mass Transfer*. 2022. 138: 106327.

[31] Siddique, I., Sadiq, K., Jaradat, M. M. M., Ali, R. and Jarad, F. Engine oil based MoS2 Casson nanofluid flow with ramped boundary conditions and thermal radiation through a channel. *Case Studies in Thermal Engineering*. 2022. 35.

[32] Shahrim, M. N., Mohamad, A. Q., Jiann, L. Y., Zakaria, M. N., Shafie, S. Ismail, Z. and Kasim, A. R. M. Exact Solution of Fractional Convective Casson Fluid Through an Accelerated Plate. *CFD Letters*. 2021. 13(6): 15-25.

[33] Reyaz, R., Mohamad, A. Q., Jiann, L. Y. and Shafie, S. Analytical Treatment for Accelerated Riga Plate on Fractional Caputo-Fabrizio Casson Fluid. *CFD Letters*. 2023. 15(4): 114.

[34] Omar, N. F. M., Osman, H. I., Mohamad, A. Q., Jusoh, R. and Ismail. Z. Effects of radiation and magnetohydrodynamic on unsteady Casson fluid over accelerated plate. *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences*. 2021. 85(1): 93-100.

[35] Deka, A. K., In presence of thermal radiation through porous medium unsteady MHD Casson fluid flow past an accelerated vertical plate. *International Journal of Statistics and Applied Mathematics*. 2020. 5(4): 213-228.

[36] Ramakrishna, S.B., Ramesh, N. L. and Thavada, S. K. Effects of chemical reaction, Soret and Lorentz force on Casson fluid flow past an exponentially accelerated vertical plate: A comprehensive analysis. *Heat Transfer*. 2022. 51(2): 2237-2257.

[37] Kataria, H. R. and Patel, H. R. Effects of chemical reaction and heat generation/absorption on magnetohydrodynamic (MHD) Casson fluid flow over an exponentially accelerated vertical plate embedded in porous medium with ramped wall temperature and ramped surface concentration. *Propulsion and Power Research*. 2019. 8(1): 35-46.

[38] Misra, J. C., Shit, G. C., Chandra, S. and Kundu, P. K. Hydromagnetic flow and heat transfer of a second-grade viscoelastic fluid in a channel with oscillatory stretching walls: application to the dynamics of blood flow. *Journal of Engineering Mathematics*. 2010, 69(1): 91-100.

[39] Sinha, A., Mondal, A., Shit, G. C. and Kundu, P. K. Effect of heat transfer on rotating electroosmotic flow through a micro-vessel: haemodynamical applications. *Heat and Mass Transfer*. 2015, 52(8): 1549-1557.

[40] Saeed, A., Alsubie, A., Kumam, P., Nasir, S., Gul, T. and Kumam, W. Blood based hybrid nanofluid flow together with electromagnetic field and couple stresses. *Scientific Reports*. 2021, 11(1): 12865.

[41] Khan, A., Khan, D., Khan, I., Ali, F., Karim, F. U. and Imran, M. MHD flow of sodium alginate-based Casson type nanofluid passing through a porous medium with Newtonian heating. *Scientific Reports*. 2018, 8(1), 8645.

[42] Pattnaik, J. R., Dash, G. C. and Singh, S. Radiation and mass transfer effects on MHD flow through porous medium past an exponentially accelerated inclined plate with variable temperature. *Ain Shams Engineering Journal*. 2017. 8(1): 67-75.

[43] Rao, S. R., Vidyasagar, G. and Deekshitulu, G. V. S. R. Unsteady MHD free convection Casson fluid flow past an exponentially accelerated infinite vertical porous plate through porous medium in the presence of radiation absorption with heat generation/absorption. *Materials Today: Proceedings*. 2021. 42: 1608-1616.

[44] Matta, S., Malga, B. S., Appidi, L. and Kumar, P. P. Finite element analysis of heat generation/absorption of viscous dissipation effects on MHD Casson fluid flow over exponentially accelerated temperature with ramped surface concentration, in International Conference on Multifunctional Materials (Icmm-2019). 2020.

[45] Villinger, H., Solving cylindrical geothermal problems using the Gaver-Stehfest inverse Laplace transform. *Geophysics*. 1985. 50(10): 1581-1587.

[46] Stehfest, H., Algorithm 368: Numerical inversion of Laplace transforms [D5]. *Communications of the ACM*. 1970. 13(1): 47-49.

[47] Das, S. and Jana, R.N. Natural convective magneto-nanofluid flow and radiative heat transfer past a moving vertical plate. *Alexandria Engineering Journal*. 2015. 54(1): 55-64.

[48] Vemula, R., Debnath, L. and Chakrala, S. Unsteady MHD Free Convection Flow of Nanofluid Past an Accelerated Vertical Plate with Variable Temperature and Thermal Radiation. *International Journal of Applied and Computational Mathematics*. 2016. 3(2): 1271-1287.

[49] Chaturani, P. and Palanisamy, V. Casson fluid model for pulsatile flow of blood under periodic body acceleration. *Biorheology*. 1990. 27(5): 619-630.