Optimizing Transportation Cost Using Linear Programming: A Malaysian Case Study

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Abstract

This paper aims to review the linear programming methods used by Karsh R. Shah to solve the transportation problem faced by MITCO Labuan Company Limited (MLCL). In Karsh R. Shah's study, the Vogel's Approximation Method (VAM) and Modified Distribution (MODI) method were adopted to identify the best polymer materials distribution plan that gives the optimal shipping cost from four Petronas manufacturing plants to four demand destinations. The current study re-created the network representation, mathematical model, and spreadsheet models as comparisons. For determining the initial BSF, the North West Corner Method (NWCM) and VAM are used. Subsequently, the sensitivity analysis is used to analyze the impact of uncertainty of unit shipping costs on the company's total transportation costs. When compared to VAM, the shipping cost of the initial basic feasible solution through NWCM is the lowest, hence, it is tested for optimality using the simplex method (using Excel Solver). The optimum shipping cost obtained from this solution is RM120,000million. The sensitivity analysis developed in this study will further help MLCL in identifying the balance and range of production capacity, without affecting the optimal solution. Ultimately, this study has evidenced that the application of linear programming in an organization increases the effectiveness and efficiency of its operations, allowing organizations to maximize profits while minimizing costs.

Keywords: *Transportation problem, Optimal cost, Sensitivity analysis, Vogel's Approximation Method (VAM), North West Corner Method*

1. Introduction

Petronas Chemicals Marketing Sdn. Bhd. (PCM) formerly known as Malaysia International Trading Corporation Sdn. Bhd. (MITCO) was established in 1982 as a wholly-owned subsidiary of Petroliam Nasional Berhad (Petronas). Apart from directly supplying petrochemical materials to clients, PETRONAS also markets and trades petrochemical materials via PCM and its marketing arms in Labuan and India [1]. MITCO Labuan Company Limited (MLCL) is PCM's marketing branch in Labuan, and it has a subsidiary company in India called MITCO Labuan India Pvt Limited [1] [2].

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MLCL has emerged as a leading marketer of fertilizers, chemicals, and polymer materials throughout Southeast Asia. It has a vast network in over 30 countries, including China, Indonesia, India, the Philippines, Thailand, the United Arab Emirates, and Vietnam. It also takes advantage of the strategic development of two integrated petrochemical complexes (IPCs) along Peninsular Malaysia's east coast, as well as the plants' constant production of significant volumes of petrochemicals [1].

MLCL is one of the leading exporters of Poly Vinyl Chloride Polymer materials to the overseas market. This material is produced in Malaysia at four Petronas petrochemical plants and shipped to four main countries: China, the Middle East, Europe, and Southeast Asia [3]. In this regard, MLCL is confronted with the issue of high transportation costs. This could be due to business decisions about polymer materials distribution and shipping schedules that were made without critical and quantitative analysis of detailed cost implications [4].

The objectives of this study are:

i. to create a network representation, mathematical and spreadsheet model for a transportation problem, as well as to analyze it;
ii. to optimize the shipping cost and meet the demands while not surpassing the petrochemical plants' maximum capacity; and
iii. to determine the impact of uncertainty of unit shipping cost on the company's total transportation costs by performing the sensitivity analysis.

2. Transportation Problem

The transportation problem is a type of linear programming problem that appears frequently in the literature of Operation Research [5]. The focus of transportation problems is to obtain the optimal (best feasible) way for products from factories or plants (supply origins) to be transported to several warehouses (demand destinations). Its goal is to assist top management in determining how many units of a specific product should be delivered from each supply origin to each demand destination to satisfy the total prevailing demand while minimizing total transportation costs [6]. In general, the cost of shipping from origin to destination is proportional to the number of units shipped [7].

In the case of MLCL, the cost of shipping per unit varies due to variances in distance and currency exchange rates. Therefore, the company must allocate production capacities to various demand locations in the most optimal manner to optimize the total shipping costs [8]. For that purpose, the transportation problem model is used in this study. The optimal solution to a transportation problem can be determined in two phases. In the first phase, the initial basic feasible solution (BSF) must be determined, and in the second phase, the initial BSF must be optimized [9]. There are three approaches for determining the initial BSF, namely, North West Corner Method (NWCM), Vogel Approximation Method (VAM), and Least Cost Method while a simplex method using Excel Solver, MODI, LINDO, and other software can be used to find the optimal solution.

The following basic assumptions are made in this study:

i. Each petrochemical plant has a set of annual capacity that is distributed to customers.
ii. Each destination has a specific annual demand that must be met by the four (4) petrochemical plants.
iii. Every shipping route mentioned in this study is fully functioning.
iv. The polymer materials are shipped from their origin to their points of destination, not the other way around.

3. Methodology

The data used in this study is taken from Shah (2020). The data includes information on production capacity, shipment quantity, and shipping cost from each petrochemical plant to each demand destination. Thus, the MLCL scenario befits the transportation problem model. Therefore, in this study, the network representation, mathematical model, and spreadsheet models are formulated. Next, using linear and spreadsheet programming, the optimal shipping cost of polymer materials from four supply origins to four demand destinations is identified. For determining the initial BSF, the NWCM and VAM are used. The initial BSF with the lowest value is then tested for optimality using the simplex method (using Excel Solver). Subsequently, the sensitivity analysis is used to analyze the impact of uncertainty of unit shipping costs on the company's total transportation costs.

3.1 Description of Data

In total, four Petronas petrochemical plants in Malaysia are identified as points of origin with four (4) countries: China, the Middle East, Europe, and Southeast Asia, as points of destination. Table 1 until Table 3 show the list of origins and destinations, production capacity, shipment quantity, and shipping cost of polymer materials from each origin to destination.

Table 1. List of Origins and Destinations

Notations	Origin Name	Notations	Destination Name
O_1	Plant 1	D_1	China
O_2	Plant 2	D_2	Middle East
O_3	Plant 3	D_3	South East Asia
O_4	Plant 4	D_4	Europe

Table 2. Production Capacity of Each Origin and Shipment Quantity of Each Destination

Origin	Production Capacity (thousands of tons/year)		Destination	Shipment Quantity (thousands of tons/year)
O_1	110		D_1	200
O_2	75		D_2	90
O_3	95		D_3	40
O_4	125		D_4	45

Table 3. The Shipping Costs of Polymer Materials from Each Origin to Destination

	Cost of Shipping (RM'000)			
Origin	D_1	D_2	D_3	D_4
O_1	200	300	100	600
O_2	400	350	150	650
O_3	300	250	150	600
O_4	500	400	200	700

3.2 Network Representation and Transportation Tableau

In general, the network representation is used as a basis in developing the transportation tableau (mathematical model). Assume 'O_i' as the origin and 'D_j' as the destination, each represented in Figure 1 by a node. The edge (*i, j*) which represents routes from sources (O_i) to destinations (D_j) contains two types of data: first, the shipping cost per unit (C_{ij}), and second, the number of polymer materials to be supplied, (X_{ij}) [10].

Next, the data is transformed in a tabular form called the transportation tableau (Table 4). Each cell signifies the shipping cost (C_{ij}) from the origin (O_i) to the destination (D_j).

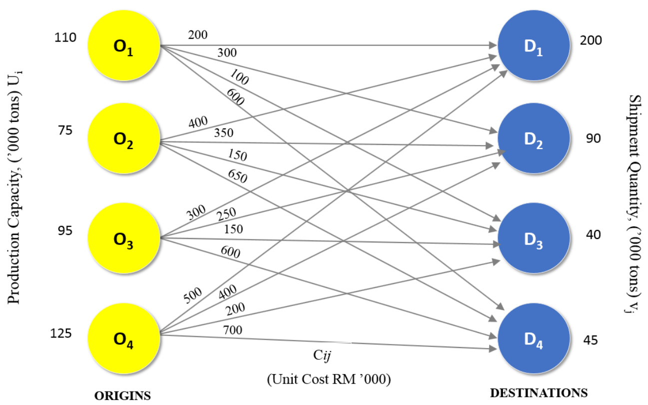

Figure 1. The Network Representation

Table 4. Transportation Tableau of Four Origin Points and Four Destination Points

	Destinations				
Origins	D_1	D_2	D_3	D_4	Production capacity ('000 tons) (U_i)
O_1	200	300	100	600	110
O_2	400	350	150	650	75
O_3	300	250	150	600	95
O_4	500	400	200	700	125
Shipment Quantity, ('000 tons) (V_j)	200	90	40	45	

The total production capacity (U_i) and the total shipment quantity (V_j) are 405,000 tons and 375,000 tons, respectively. It signifies that the total production capacity (supply) exceeds the total shipment quantity (demands). Thus, this is an unbalanced transportation problem. The unbalanced transportation problem is converted into the balanced one by adding dummy row or dummy column [11]. Consequently, a dummy destination, D_5, with demand equal to the excess supply of 30,000 tons is added (Table 5). The unit shipping cost for the dummy column and dummy row is set to zero because no actual shipment is made. With the addition of a dummy destination, D_5, the shipment quantity becomes equal to the production capacity, resulting in a balanced solution.

Table 5. Transportation Tableau with Dummy Destination (D_5)

	destinations					
Origins	**D_1**	**D_2**	**D_3**	**D_4**	**D_5 (Dummy)**	**Production capacity ('000 tons) (U_i)**
O_1	200	300	100	600	0	110
O_2	400	350	150	650	0	75
O_3	300	250	150	600	0	95
O_4	500	400	200	700	0	125
Shipment Quantity, ('000 tons) (V_j)	200	90	40	45	30	405

3.3. Formulation of Mathematical Model

The purpose of the mathematical model for this case study is to determine the optimum allocation of production capacity in each origin (plant) to each destination (countries) to minimize the total cost of shipping. The following are the relevant decision variables:

a. C_{ij} = unit cost of shipping (RM'000/ton) from O*i* to D*j*
b. X_{ij} = the amount to be shipped (thousands of tons /year) from O*i* to D*j*
c. V_j = shipment Quantity (thousands of tons/year)
d. U_i = production capacity (thousands of tons /r year)

In total, there are four (4) demand destinations and four (4) petrochemical plants that are being considered. Thus, i = 1, 2, 3, 4 and j = 1, 2, 3, 4.

The objective function of this model is to identify the value of X_{ij} that will minimize the unit cost of shipping, C_{ij}, satisfying the shipment Quantity and production capacity constraints of the problem. The transportation problem can then be solved using linear programming as follows:

Min Z = minimize the cost of shipping (RM '000)

$$\begin{aligned}\text{Min } Z = {} & 200X_{11} + 300X_{12} + 100X_{13} + 600X_{14} + 400X_{21} + 350X_{22} + 150X_{23} + \\ & 650X_{24} + 300X_{31} + 250X_{32} + 150X_{33} + 600X_{34} + 500X_{41} + 400X_{42} + \\ & 200X_{43} + 700X_{44} \end{aligned} \tag{1}$$

subject to the constraints:

$$X_{11} + X_{12} + X_{13} + X_{14} = 110 \tag{2}$$
$$X_{21} + X_{22} + X_{23} + X_{24} = 75 \tag{3}$$
$$X_{31} + X_{32} + X_{33} + X_{34} = 95 \tag{4}$$
$$X_{41} + X_{42} + X_{43} + X_{44} = 125 \tag{5}$$
$$X_{11} + X_{21} + X_{31} + X_{41} = 200 \tag{6}$$
$$X_{12} + X_{22} + X_{32} + X_{42} = 90 \tag{7}$$
$$X_{13} + X_{23} + X_{33} + X_{43} = 40 \tag{8}$$
$$X_{14} + X_{24} + X_{34} + X_{44} = 45 \tag{9}$$

and

$X_{ij} \geq 0$ $(X_i = 1,2,3,4\ ;\ X_j = 1,2,3,4)$.

4. Results and Discussions

4.1 Initial Basic Feasible Solution

In this study, the initial basic feasible solution (BSF) is obtained using the NWCM and VAM. Figure 2 and 3 show the results obtained from both methods.

ORIGINS	Destination D_1	D_2	D_3	D_4	D_5	Production Capacity, Ui
O_1	200 (110)	300	100	600	0	110
O_2	400 (75)	350	150	650	0	75
O_3	300 (15)	250 (80)	150	600	0	95
O_4	500	400 (10)	200 (40)	700 (45)	0 (30)	125
Shipment Quantity, vj	200	90	40	45	30	Z= **RM120,000 million**

Figure 2. The Initial BSF Obtained Using NWCM Method

ORIGINS	Destination D_1	D_2	D_3	D_4	D_5	Production Capacity, Ui
O_1	200 (110)	300	100	600	0	110
O_2	400 (35)	350	150 (40)	650	0	75
O_3	300 (5)	250 (90)	150	600	0	95
O_4	500 (50)	400	200	700 (45)	0 (30)	125
Shipment Quantity, vj	200	90	40	45	30	Z= **RM122,500 million**

Figure 3. The Initial BSF Obtained Using VAM Method

Total Shipping Cost (initial BSF) obtained using the NWCM is
= 200x110+400x75+300x15+250x80+400x10+200x40+700x45+0x30
= RM120,000million

Total Shipping Cost (initial BSF) obtained using the VAM is

= 200x110+400x35+150x40+300x5+250x90+500x50+700x45+0x30
= RM122,500million

When the two findings are compared, it shows that NWCM provides the lowest initial BSF. Thus, it is used further for optimality tests.

4.2. Optimality Test Using Simplex Method

In a typical transportation problem with m origins and n destinations, the optimality test of any feasible solution requires the number of allocations (N) equals $m + n - 1$ in independent cells [12]. In this study, $m + n - 1 = 4 + 5 - 1 = 8$ is equivalent to the number of allocations (N). Therefore, the conditions are satisfied and degeneration does not exist. So, an optimality test can be performed using the simplex method.

Unit Cost (RM '000)

	Destinations				
Origin	D_1	D_2	D_3	D_4	D_5
O_1	200	300	100	600	0
O_2	400	350	150	650	0
O_3	300	250	150	600	0
O_4	500	400	200	700	0

shipment quantity ('000 tons)

	Destinations							
Origin	D_1	D_2	D_3	D_4	D_5	Production		Capacity
O_1	110	0	0	0	0	110	≤	110
O_2	0	30	0	45	0	75	≤	75
O_3	90	5	0	0	0	95	≤	95
O_4	0	55	40	0	0	95	≤	125
Total	200	90	40	45	30			
	=	=	=	=				
Demand	200	90	40	45	30			

Total Shipping Cost = (RM '000) 120,000

Figure 4. Optimality Test Result Using Excel Solver

Figure 4 above shows the spreadsheet formulation and optimality test results achieved using the Excel Solver. The constraints of production capacity and shipping quantity are in Capacity Cell (K31: K34) and Demand Cell (D37: H37), respectively. The Unit Cost cell (D24: H27) contains the unit shipping cost from four (4) petrochemical plants to four (4) destinations. The shipping quantity variable cell (D31: H34) indicates the optimal polymer shipping quantity while the optimal total cost is in Cell K37. The optimum shipping cost obtained from this solution is RM120,000million and the Table 6 below is the optimal distribution and shipping plan.

Table 6. The Optimal Distribution and Shipping Plan

Origin	Destination	Cost (RM'000)	Allocation ('000)	Total Cost (RM'000)
Plant 1 (O_1)	China (D_1)	200	110	22,000
Plant 2 (O_2)	Middle East (D_2)	350	30	10,500
Plant 2 (O_2)	Europe (D_4)	650	45	29,250
Plant 3 (O_3)	China (D_1)	300	90	27,000
Plant 3 (O_3)	Middle East (D_2)	250	5	1,250
Plant 4 (O_4)	Middle East (D_2)	400	55	22,000
Plant 4 (O_4)	South East Asia (D_3)	200	40	8,000
Plant 4 (O_4)	Dummy (D_5)	0	30	0
			Optimum Cost	120,000

4.3. Sensitivity Analysis

Sensitivity analysis is a technique for determining how the values of independent variables will impact a specific dependent variable under a set of assumptions. It identifies the range of feasibility, range of optimality and shadow prices which are vital in decision making [13]. This study uses sensitivity analysis to explore the impact of uncertainty of unit shipping cost from each origin to each demand destination on total transportation costs. The Sensitivity Report in Figure 5 shows the range of optimality that describes how changes in the objective function's coefficients (unit shipping cost) affect the optimal solution. The three columns labeled "Objective Coefficient," "Allowable Increase," and "Allowable Decrease" under the heading "Variable Cells" offer us the conditions under which the solution stays optimal [14].

For instance, the allowable increase for point O_3D_1 (from Plant 3 to China) is 0 while the allowable decrease is 150. So, the upper limit for O_3D_1 is 300 (300+0) while the lower limit is 150 (300-150). This means the unit shipping cost from Plant 3 to China Market can be adjusted from RM150,000 up to RM300,000 without changing the optimal shipping cost. So, if the MLCL company decides to reduce the cost of shipping units from Plant 3 to the China market to RM200,000, the company will still benefit from optimal shipping costs but the overall cost will be lowered to:

= 200 x 110+350 x 75 +200 x 90 +250 x 5 +400 x 10 +200 x 40 +700 x 45 +0 x 30
= 111,000 (R111,000million)

Microsoft Excel 16.0 Sensitivity Report
Worksheet: [PMA_Solver_ support document.xlsx]Sheet1
Report Created: 1/23/2022 10:35:14 AM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
D31	O1 D1	110	0	200	150	1E+30
E31	O1 D2	0	150	300	1E+30	150
F31	O1 D3	0	150	100	1E+30	150
G31	O1 D4	0	150	600	1E+30	150
H31	O1 D5	0	250	0	1E+30	250
D32	O2 D1	0	0	400	1E+30	0
E32	O2 D2	30	0	350	0	0
F32	O2 D3	0	0	150	1E+30	0
G32	O2 D4	45	0	650	0	1E+30
H32	O2 D5	0	50	0	1E+30	50
D33	O3 D1	90	0	300	0	150
E33	O3 D2	5	0	250	50	0
F33	O3 D3	0	100	150	1E+30	100
G33	O3 D4	0	50	600	1E+30	50
H33	O3 D5	0	150	0	1E+30	150
D34	O4 D1	0	50	500	1E+30	50
E34	O4 D2	55	0	400	0	0
F34	O4 D3	40	0	200	0	1E+30
G34	O4 D4	0	0	700	1E+30	0
H34	O4 D5	0	0	0	1E+30	0

Figure 5. The Range of Optimality

In addition, if the objective coefficients have an infinity allowable decrease or increase, it means that the coefficient value can be increased or decreased without bound. The analysis further illustrates that the reduced cost of all basic variables is zero because the objective function does not need to be changed in any way to make the variable positive.

On the other hand, the range of feasibility in Figure 6 describes how changes in the Right-Hand Side (RHS) value of any constraint affect the optimal solution. Theoretically, the size of the feasible region changes when the RHS value of a constraint is changed. It means, when the RHS value of the constraint is increased with positive coefficients, the border matching the constraint moves up whereas, if the RHS value is decreased, the border matching the constraint moves down.

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D35	Total D1	200	450	200	5	55
E35	Total D2	90	400	90	30	55
F35	Total D3	40	200	40	30	40
G35	Total D4	45	700	45	30	45
H35	Total D5	30	0	30	0	1E+30
I31	O1 Production	110	-250	110	55	5
I32	O2 Production	75	-50	75	55	30
I33	O3 Production	95	-150	95	55	5
I34	O4 Production	95	0	125	1E+30	30

Figure 6. The Range of Feasibility

The shadow price reveals how the objective function will change when the RHS value is adjusted. For example, the shadow price for the demand constraint (D_1) is 450. This indicates that if the unit cost is decreased by 1 (from 200 to 199), the corresponding optimal shipping cost will decrease by RM450,000. The shadow price will only be valid as long as all changes in the RHS values are within the range of “Allowable Increases" and "Allowable Decreases". So, if the demand constraint (D_1) is decreased by 60 (which is not within the permitted range), the shadow price is no longer valid and a new LP needs to run to get new shadow prices.

Moreover, all capacity and demand constraints except Plant 4 are binding (zero slack or surplus), implying that, the polymer production capacity in Plant 1, 2, and 3 is fully utilized in the final solution. These binding constraints are considered to be the bottleneck or the main limiting factor in an optimal solution. Therefore, changing these constraints (i.e., RHS values) could result in a different solution. Meanwhile, all the materials assigned to the dummy destination represent excess capacity. Therefore, the O_4 production constraint is not binding with 30 units of slack. This means there are 30,000 tons of polymer materials produced in Plant 4 that is not being used to produce the optimal solution

5. Conclusion

This study has demonstrated the use of linear programming in solving the transportation problem encountered by MITCO Labuan Company Limited (MLCL). A detailed network representation, mathematical, and spreadsheet model are formulated and analyzed to determine the optimal shipping cost of Vinyl Chloride Polymer materials from four Petronas petrochemical plants in Malaysia to four demand destinations. Although various heuristic approaches for obtaining initial BSF have been explored in many kinds of literature [15], this case study used the traditional methods, the NWCM and VAM to identify the initial BSF [16,17].

When compared to VAM, the shipping cost of the initial BSF through NWCM is the lowest, which gives a value closer to the optimal solution. Hence, it is tested for optimality using the simplex method (Excel Solver). The optimum shipping cost obtained from this solution is RM120,000million. According to the results of the optimality test, MLCL should adopt the following polymer materials distribution plan to achieve the optimal shipping cost:

a. 200,000 tons/year supply for China from Plant 1 (110,000 tons/year) and Plant 3 (90,000 tons/year);
b. 90,000 tons/year supply for Middle East from three sources namely Plant 2 (30,000 tons/year), Plant 3 (5,000 tons/year) and Plant 4 (55,000 tons/year);
c. 40,000 tons/year supply for South East Asia from Plant 4;
d. 45,000 tons/year supply for Europe from Plant 4.

The sensitivity analysis developed in this study will further help MLCL in identifying the balance and range of production capacity, without affecting the optimal solution. This will also enable MLCL to conduct a cost-benefit analysis related to the production capacity and distribution of polymer products to international markets. This will not only allow MLCL to find a better distribution plan and shipping schedule but also enables Petronas, the manufacturing company, to organize effective production strategies in its petrochemical plants. Ultimately, this study has evidenced that the application of linear programming in an organization increases the effectiveness and efficiency of its operations, allowing organizations to maximize profits while minimizing costs.

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