Malaysian Journal of Computing, 7 (2): 1082-1099, 2022

eISSN: 2600-8238 online

DETERMINING OPTIMAL TRANSPORTATION ALLOCATION USING LINEAR PROGRAMMING METHODS

Tengku Mohd Hazimi Tuan Hassan[1] and Maslin Masrom[2*]

[1,2]Razak Faculty of Technology and Informatics
Universiti Teknologi Malaysia
[1]tengkumohdhazimi@graduate.utm.my, [2]maslin.kl@utm.my

ABSTRACT

The transportation problem is a subset of the broader linear programming (L.P.) technique developed to assist managers in making decisions. It has been used in real-life applications to minimise total transportation costs by satisfying destination and source requirements. The study aimed to minimise the delivery of items from the plant to the dealer's location by determining the optimal allocation of inventory from suppliers to dealers in the Malaysian cities of Seremban, Johor Bahru, Kuala Lumpur/Selangor and Penang. Meanwhile, the specific objectives of the study were: (i) to formulate transportation problems using the L.P., (ii) to identify a basic feasible solution (BFS), and (iii) to do an optimality analysis. A case study of an operations research problem involving the optimisation of distribution costs and the effective delivery system of inventory by utilising transportation models, namely the North-West Corner Rule (NWCR), Simplex method, and Vogel's Approximation Method (VAM) are presented and discussed. The efficiency and effectiveness of the transportation problem algorithm are determined using the Excel Solver. The excel solution and sensitivity analysis provide information about the optimal distribution strategy for their products. The results indicated that the delivery cost is reduced when the transportation problem algorithm is used. It saved up to RM50,605.21, or 78.91 per cent of the cost, and it can be concluded that the profit margin is higher than with the previous strategies. Among all transportation models, VAM and Simplex methods are considered the best methods since they produce the ideal results, whereas the North-West Corner Rule is considered the simplest method but produces the worst results. In conclusion, this study could assist the distributors in reducing shipping costs and ensuring that the inventory is distributed efficiently.

Keywords: *Linear Programming, Optimality Test, Sensitivity Analysis, Transportation Problem*

Received for review: 01-06-2022; Accepted: 29-08-2022; Published: 01-10-2022
DOI: 10.24191/mjoc.v7i2.17600

1. Introduction

The XYZ company was mainly involved in the furniture business selling bedroom sets. There are three factories owned by this company in the state of Johor. The Sri Gading factory is the largest. This branch was chosen as a case study to solve the problem. There are only two divisions at XYZ company, that is, one for making, another for marketing, and one for running the business, as well as dealing with people and shipping, people in Seremban, Johor Bahru, Penang, and Kuala Lumpur/Selangor buy the most from the company. This company uses lorries to move its goods. At the very least, there will be six lorries each day that was used to deliver inventory. In addition, the company used a direct shipping manufacturer as part of its distribution plan. One of the networks the company used did not have any warehouses, and it did not have any direct contact with people. Dealers only work with people who buy from them.

In the XYZ Company (Pang & Chandrashekar, 2019), the unstable fuel price was very important because it played a big role in operating costs. The Federation of Malaysian Manufacturers (FMM) surveyed 15% of respondents who thought transportation costs would go up 25% to 30% because of a proposal to raise toll fees by concessionaires in the Klang Valley. This would lead to a rise in the prices of goods and raw materials, which would then raise manufacturers' operating costs.

In addition, a company does not know that the way they set up their inventory delivery system is not the best way to get things to people. Regarding physical distribution, Ahmed *et al.* (2016) stated that the transportation problem is a known method in operation research for its real-life application. The costs of a distribution and delivery inventory system may impact an organisation's performance and profitability. For example, in 2013, the cost of transportation for XYZ went up by 21.33 per cent, to RM 614,054.58 from RM506,095.44. This showed that XYZ Company did not have a good transportation system for delivering inventory. Thus, the primary objective of this study is to determine the optimal network model for XYZ company and the optimal transportation costs.

The different transportation models used in this study are the North-West Corner Rule (NWCR), Simplex, and Vogel's Approximation Method (VAM). Sensitivity analysis is used to determine the total cost of transportation's sensitivity to changes in the volume of goods and the cost of transportation from the origin to the destination. The Northwest Corner Rule, Vogel's Approximation method and Simplex Method have been identified to obtain the optimal feasible where the cost of transporting (*Cij*) product units from production plant *x* to the destination y (Fauziah & Siti Nurhafiza, 2015; Rafi & Islam, 2020).

The study aims to determine the minimum cost of transporting the goods (bedroom sets) from the production plants (The Sri Gading factory) using lorries to the dealer. In this case study, two transport vehicles were identified (five tons lorries and ten tons lorries) and four dealers' locations (Johor Bahru, Seremban, Kuala Lumpur/Selangor, and Penang). The following (Table 1) is the summarised transportation cost per unit from each production plant to the warehouse, as well as the volume of supply for the production plant and demand by the warehouses.

Table 1. The XYZ Transportation Tableau

			Destination(s)				**Supply**
			1	**2**	**3**	**4**	
			Johor Bahru	**Seremban**	**Kuala Lumpur/ Selangor**	**Penang**	
Source(s)	**1**	**5 Tons lorries**	3.75	5.63	9.38	15.63	1710
	2	**10 Tons lorries**	4.69	7.5	12.5	20	3419
Demand			1539	513	2051	1026	

The formulation of the linear programming for the case study is as follows:

Objective function – minimise the total cost of transportation

$$Min\ Z = \Sigma\ C_{ij}\ X_{ij} \quad (1)$$

Variables:

X = Lorry capacity from production plant, whereby i = 1, 2.

Y = Dealers Location whereby j = 1,2,3 and 4

X_{ij} = Volume (in units) from the production plant X_i to dealers' location Y_j

C_{ij} = Transportation cost from the production plant X_i to dealers' location Y_j

Constraints:

$X_{11} + X_{12} + X_{13} + X_{14} \leq 1710$

$X_{21} + X_{22} + X_{23} + X_{24} \leq 3419$

$X_{11} + X_{21} \leq 1539$

$X_{12} + X_{22} \leq 513$

$X_{13} + X_{23} \leq 2051$

$X_{14} + X_{24} \leq 1026$

$X_{ij} \geq 0$

Assumptions:

As the number of units sent via a certain route increase, so does the transportation cost. The goal is to reduce overall transportation costs for the XYZ company, not only supply and dealers' locations (Aizemberg, Kramer, Pessoa & Uchoa, 2014). Prior to utilising any mode of transportation, the following fundamental assumptions are made:

i. The total supply from each source equals the total supply required by each destination.
ii. Dummy sources or dummy destinations are used if the originals do not match properly.
iii. In this case, the supply carries by lorries X_1 (5 tonnes) and X_2 (10 tons) is equal to the demand at four dealers' locations (J.B., Seremban, K.L., and Penang).
iv. Transportation costs from XYZ company to 4 dealers location are known and fixed.
v. The cost of transportation per unit is unaffected by the number of items shipped.
vi. The goal is to keep the overall transportation costs as low as possible.
vii. Transport problems can be formulated using the linear programming problem.

2. Methodology

The research used the transportation solution tableau, which has the same dimensions as the transportation expenses tableau, to compute an initial basic, workable solution. Each point *ij* is associated with the decision variable X_{ij}, which denotes the quantity of product transported from the origin X_i to the destination Y_i. The tableau of transportation solutions is in Table 2 below.

This research aimed to minimise the delivery of items from the plant to the dealer's location by using two lorries (5 tons and 10 tons) with supply (X) and demand (Y). The cost to deliver the goods from the factories using identified supply (lorries) 1, 2 to the dealers j (Johor Bahru, Seremban, Kuala Lumpur/Selangor, and Penang) is referred to as transportation cost. Based on the total calculation, it can be concluded that this was a balanced transportation problem. The desired outcome has been reached. Utilising the equation total supply = total demand. Total supply = 5129 unit (1710+3419) is equal to Total demand = 5129 unit (1539+513+2051+1026).

Table 2. The Transportation Solutions Tableau

			Destination(s) (*Y*)				Supply
			1	**2**	**3**	**4**	
			Johor Bahru	**Seremban**	**Kuala Lumpur/Selangor**	**Penang**	
Source(s) (*X*)	**1**	**5 Tons lorries**	3.75	5.63	9.38	15.63	1710
	2	**10 Tons lorries**	4.69	7.5	12.5	20	3419
Demand			1539	513	2051	1026	

There are three primary processes for resolving linear programming problems,

i. Development of the LPP transportation model
ii. Identify a Basic Feasible Solution (BFS); and
iii. Optimality examination

Three strategies have been developed for obtaining the basic practicable solution in which the cost of carrying *Cij* product units from the manufacturing plant *X* to the destination *Y* is as follows:

i. North-West Corner Rule (NCWR)
ii. Vogel's Approximation Method (VAM) or Penalty Method; and
iii. Simplex Method

2.1. North-West Corner Rule

Many people use the North-West Corner Rule (NCWR) to compute and verify accuracy. First, this is a viable answer to a transportation issue. It has gained popularity because of its. It is easy to get started. The demand for a solution to this transportation issue must first be established. It is equal to the supply previously agreed upon. Second, in order to succeed, supply and demand must be balanced. If supply and demand are out of balance, dummy variables must be introduced to rectify the situation. The following are the specifics of the North-West Corner Rule's algorithm:

Refer to Table 3. Choosing a cell in the North-West corner, the transportation cost is 3.75. Then, compare that cell's needs to how much it has; that demand-supply is in red (1539, 1710).

Table 3. The Tableau of Alternative Modes of Transportation Utilising the North-West Corner Rule

			Destinations				Supply
			1	**2**	**3**	**4**	
			JB	**Seremban**	**KL**	**Penang**	
Source	**1**	**5 Tons lorries**	3.75	5.63	9.38	15.63	1710
	2	**10 Tons lorries**	4.69	7.5	12.5	20	3419
Demand			1539	513	2051	1026	

Refer to Table 4. Identify and assign to the lowest-valued cell 1539, the number given in this table. Then subtract the cell with the lowest value from the total. In this case, (1710–1539=171). Then, strike off the corresponding column or row to remove it. (In this column, the line indicates the location of the first destination).

Table 4. The Second Stage of Transportation Solutions Utilizing the North-West Corner

			Destinations				Supply
			1	2	3	4	
			JB	Seremban	KL	Penang	
Source	1	5 Tons lorries	[1539] 3.75	[171] 5.63	9.38	15.63	[1710 – 1539] = 171
	2	10 Tons lorries	4.69	7.5	12.5	20	3419
Demand			1539	513	2051	1026	

The same steps will continue with the rest of the cells if necessary. For example, the North-West (N-W) cell process will be repeated, as shown in Table 5.

Table 5. Third Stage in Resolving Transportation Issues by Utilizing the North-West Corner

			Destinations				Supply
			1	2	3	4	
			JB	Seremban	KL	Penang	
Source	1	5 Tons lorries	[1539] 3.75	[171] 5.63	9.38	[1026] 15.63	1710 – 1539 = 171
	2	10 Tons lorries	4.69	[342] 7.5	[2051] 12.5	20	3419-342 =3077 3077-2051= 1026
Demand			1539	513-171= 342	2051	1026	

Finally, the single cell will be assigned a value in either demand or supply because both demand and supply will be equal. Table 6 is the final table with all allotted cells and the first possible solution using the North-West Corner Rule method.

The initial basic feasible solution has the following values: X11 = 1539, X12 =171, X22 =342, X23 = 2051 and X24 = 1026. The remaining entries are non-basic and so equivalent to zero. Therefore, the initial basic feasible solution for the North-West Corner Rule:

Total cost (RM) = (1539 x 3.75) +(171 x 5.63) +(342 x 7.5) +(2051 x 12.5) + (1026 x 20) = RM55,456.48.

Table 6. The Initial Basic Feasible Solution Employing North-West Corner Rule

			Destinations				Supply
			1	2	3	4	
			JB	Seremban	KL	Penang	
Source	1	5 Tons lorries	1539 3.75	171 5.63	9.38	15.63	0
	2	10 Tons lorries	4.69	342 7.5	2051 12.5	1026 20	0
Demand			0	0	0	0	

An optimality test using the North-West Corner Rule was performed to determine whether the resulting answer was optimal. See Tables 7, 8, 9 and 10.

Table 7. The Initial Basic Feasible Solution Using North-West Corner Rule as a Point of Comparison

			Destinations				**Supply**
			1	**2**	**3**	**4**	
			JB	**Seremban**	**KL**	**Penang**	
Source	**1**	**5 Tons lorries**	1539 3.75	171 5.63	9.38	15.63	0
	2	**10 Tons lorries**	4.69	342 7.5	2051 12.5	1026 20	0
Demand			0	0	0	0	

Table 8. Optimality Test

	V_1	V_2	V_3	V_4	U_i
U_1	1539 3.75	171 5.63	9.38	15.63	0
U_2	4.69	342 7.5	2051 12.5	1026 20	171
V_j	1538	171	1880	855	

Table 9. Basic Variables

No.	Basic Cell	Basic $C_{ij} = U_i+V_j-C_j=0$ $U_i+V_j = C_{ij}$	Substitute
1	X_{11}	$U_1+V_1= 1539$	$U_1 = 0$ $V_1 = 1538$
2	X_{12}	$U_1+V_2= 171$	$U_1 = 0$ $V_2 = 171$
3	X_{22}	$U_2+V_2= 342$	$U_2 + (171) =342$ $U_2 = 171$
4	X_{23}	$U_2+V_3=2051$	$171+V_3 = 2051$ $V_3 = 1880$
5	X_{24}	$U_2+V_4=1026$	$171+V_4 = 1026$ $V_4 = 855$

Table 10. Non-Basic Variables

No.	Basic Cell	Basic $C_{ij} = U_i+V_j-C_j=0$ $U_i+V_j = C_{ij}$	Substitute
1	X_{13}	$U_1+V_3= C_{ij}$	$U_1 =0, V_3=1880$ $0+ 1880 = C_{ij}$ $C_{ij} =1880$
2	X_{14}	$U_1+V_4=C_{ij}$	$U_1=0, V_4=855$ $0+855 =C_{ij}$ $C_{ij}=855$
3	X_{21}	$U_2+V_1= C_{ij}$	$U_2 = 171, V_1=1538$ $171+1538$ $C_{ij}=1709$

All *Cij–Zij* values are non-negative, so the current solution is optimal. However, this solution may or may not be ideal for this problem. Therefore, a comparison of the Vogel Approximation and Simplex methods is implemented and described to determine which method produces a more accurate result at a lower total cost.

2.2. Vogel's Approximation Method

Since the initial basic viable solution obtained is always close to the ideal solution, Vogel's Approximation Method (VAM) is useful for solving transportation problems. However, the method of the North-West Corner Rule and VAM are opposed. For the North-West Corner Rule, the upper left-hand corner of the solution tableau was selected, but for VAM, the first step is to compute differences in each row and column before selecting a cell. Table 11 is the tableau of transportation solutions utilising VAM. The distinction between rows and columns is shown in Table 12.

Table 11. The Tableau of Transportation Solutions Utilizing VAM

			Destinations				Supply
			1	2	3	4	
			JB	Seremban	KL	Penang	
Source	1	5 Tons lorries	3.75	5.63	9.38	15.63	1710
	2	10 Tons lorries	4.69	7.5	12.5	20	3419
Demand			1539	513	2051	1026	

The following is the detailed algorithm for the VAM:

1) To begin, the differences in each row and column of the VAM between the two lowest costs are calculated. These are referred to as additional expenses or penalties.

Table 12. Penalties Calculation

			Destinations				Supply	Penalty
			1	2	3	4		
			JB	Seremban	KL	Penang		
Source	1	5 Tons lorries	3.75	5.63	9.38	15.63	1710	5.63-3.75 =1.88
	2	10 Tons lorries	4.69	7.5	12.5	20	3419	7.5-4.69 =2.81
Demand			1539	513	2051	1026		
Penalty			4.69-3.75 = 0.94	7.5-5.63 = 1.87	12.5-9.38 = 3.12	20-15.63 = 4.37		

1) Here, the penalties are (1.88, 2.81, 0.94, 1.87, 3.12, 4.37). Now, regardless of row or column, we will determine the maximum value of the penalty. [Maximum (Penalties) = 4.37 in this case]. See Table 13.
2) Now, the appropriate row or column is investigated. [In this case, the column (4)]. The cost with the lowest value. e.g., the lowest possible cost is chosen. [In this case, =1026].
   Appropriately eliminate the column or row by striking it out.
   Contrast the demand and supply for that cell [In this case, 684 and 2051].
   Assign the cell with the lowest value [in this case, 684].
3) Subtract the cell with the lowest value from the excluded cells. Specifically, the assigned cell value. [In this case, 2051 – 684 = 1367].
   Appropriately eliminate the column or row by striking it out. [Here is the column containing source 3 (indicated by a line).
   Throughout the process, total demand and supply have remained constant.
4) The procedure with the remaining cells will be repeated, and the process and locating the penalty will be repeated, following the same processes as previously described. Results are as in Table 14 below.

Table 13. Second Step of VAM Calculation

			Destinations				Supply	Penalty
			1	2	3	4		
			JB	Seremban	KL	Penang		
Source	**1**	**5 Tons lorries**	3.75	5.63	[684] 9.38	[1026] 15.63	1710-1026 =~~684~~	1.88
	2	**10 Tons lorries**	[1539] 4.69	[513] 7.5	[1367] 12.5	20	3419-1367 = 2052 2052-1539 = ~~513~~	2.81
Demand			~~1539~~	~~513~~	2051-684 =~~1367~~	~~1026~~		
Penalty			0.94	1.87	3.12	4.37		

Table 14. Initial Feasible Solution for VAM

			Destinations				Supply
			1	2	3	4	
			JB	Seremban	KL	Penang	
Source	**1**	**5 Tons lorries**	3.75	5.63	[684] 9.38	[1026] 15.63	0
	2	**10 Tons lorries**	[1539] 4.69	[513] 7.5	[1367] 12.5	20	0
Demand			0	0	0	0	

Therefore, initial feasible solution for VAM:

Total cost = $X_{13} + X_{14} + X_{21} + X_{22} + X_{23}$

$9.38(684) + 15.63(1026) + 4.69(1539) + 7.5(513) + 12.5(1367)$

= RM50,605.21

2.3. Simplex Method

The Simplex method is a strategy for solving linear programming models manually by utilising slack variables, tableaus, and pivot variables to obtain the optimal solution to an optimisation problem (Khan, 2014). It begins at a corner point of the feasible area with all significant variables set to zero and proceeds incrementally to the next corner point, increasing the value of the objective function with each iteration. This process was repeated until an optimal solution was discovered. Excel Solver has been used to solve a linear programming model using the Simplex method. Excel Solver can easily and quickly tackle such large transportation problems with minimal effort (Nur Syafiqah, 2016). Using Excel Solver's, the optimal (maximum or minimum) solution to an optimisation model's objective function, subject to the model's restrictions on the values of other formula cells on a worksheet, could easily identify. See Tables 15, 16, 17 and 18. The total initial feasible solution for Simplex method is RM50,605.21.

Table 15. Original Data

	JB	**Seremban**	**KL**	**Penang**	**Supply**
5 Tons lorries (A)	3.75	5.63	9.38	15.63	1710
10 Tons lorries (B)	4.69	7.5	12.5	20.00	3419
Demand	1539	513	2051	1026	

Table 16. Volume Data Before Using Excel Solver

	J.B.	**Seremban**	**K.L.**	**Penang**	**Supply**	**Supply**
5 Tons lorries (A)					0	1710
10 Tons lorries (B)					0	3419
Location demand	0	0	0	0	0	
Demand	1539	513	2051	1026		5129
Total transportation cost (Cij x Xij)	0					

Table 17. Data After Using Excel Solver

Cost per unit						
	JB	**Seremban**	**KL**	**Penang**	**Supply**	
5 Tons lorries (A)	3.75	5.63	9.38	15.63	1710	
10 Tons lorries (B)	4.69	7.5	12.5	20	3419	
Demand	1539	513	2051	1026		
Volume						
	J.B.	Seremban	K.L.	Penang	Supply	Supply
5 Tons lorries (A)	0	0	684	1026	1710	1710
10 Tons lorries (B)	1539	513	1367	0	3419	3419
Location demand	1539	513	2051	1026	5129	
Demand	1539	513	2051	1026		5129
Total transportation cost (*Cij* x *Xij*)	50605.21					

Table 18. Objectives, Variables and Constraints Data After Using Excel Solver

Objective Cell (Min)				
Cell	**Name**	**Original Value**	**Final Value**	
$C$16	Total transportation cost (Cij x Xij) JB	0	50605.21	
Variable Cells				
Cell	**Name**	**Original Value**	**Final Value**	**Integer**
$C$11:$F$12				
$C$11	5 Tons lorries (A) J.B.	0	0	Contin
$D$11	5 Tons lorries (A) Seremban	0	0	Contin
$E$11	5 Tons lorries (A) K.L.	0	684	Contin
$F$11	5 Tons lorries (A) Penang	0	1026	Contin
$C$12	10 Tons lorries (B) J.B.	0	1539	Contin
$D$12	10 Tons lorries (B) Seremban	0	513	Contin
$E$12	10 Tons lorries (B) K.L.	0	1367	Contin
$F$12	10 Tons lorries (B) Penang	0	0	Contin
Constraints				
Cell	**Name**	**Cell Value**	**Formula**	**Status**
$C$13	Location demand JB	1539	$C$13=$C$14	Binding
$D$13	Location demand Seremban	513	$D$13=$D$14	Binding
$E$13	Location demand K.L.	2051	$E$13=$E$14	Binding
$F$13	Location demand Penang	1026	$F$13=$F$14	Binding
$G$11	5 Tons lorries (A) Supply	1710	$G$11<=$H$11	Binding
$G$12	10 Tons lorries (B) Supply	3419	$G$12<=$H$12	Binding

2.4. Description of Data

The transportation problem concerns the transit of any product from its origins, X_1..., X_m, to its final destinations, Y_1..., Y_n (Reeb & Leavengood, 2002) where:

i. Origin X_i possesses an inventory of *i* units, where *i* equals 1..., *m*.
ii. The geographical location Y_j requires the delivery of b_j units from the sources, where $j=1..., n$.
iii. Where $i=1..., m$ and $j=1..., n$, c_{ij} signifies the cost per unit spread between the origin X_i and the destination Y_j.

iv. The preceding problem can be mathematically expressed as selecting a collection of x_{ij}'s with i=1..., m and j=1..., n that satisfy supply and demand constraints.
v. The objective is to minimise the entire cost of distribution.
vi. A linear model has been used:

$$Min\ z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}X_{ij} \quad (2)$$

Subject to

$$\sum_{j=1}^{n} X_{ij} \leq ai,\ i=1,\ ....,\ m$$

$$\sum_{i=1}^{m} X_{ij} \geq b_j,\ j=1,\ ....,\ n$$

$$Xij \geq 0, \quad i = 1,\ ......,\ m, \quad j=1......,\ n$$

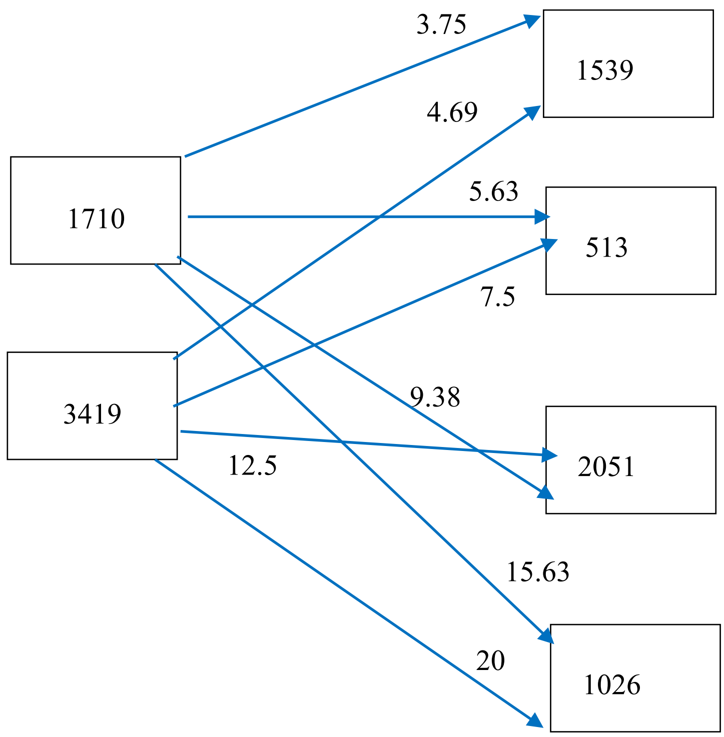

Figure 1. Supply and Demand Path

Xij: The quantity of bedsheets that will be distributed via lorries to the sites Y_j, where i=1,2, and j=1,2,3.

$$Min\ Z = 3.75X_{11} + 5.63X_{12} + 9.38X_{13} + 15.63X_{14} + 4.69X_{21} + 7.5X_{22} + 12.5X_{23} + 20X_{24}$$

Subject to:

$X_{11} + X_{12} + X_{13} + X_{14}$ = 1710

$X_{21} + X_{22} + X_{23} + X_{24}$ = 3419

$X_{11} + X_{21}$ = 1539

$X_{12} + X_{22} = 513$
$X_{13} + X_{23} = 2051$
$X_{14} + X_{24} = 1026$

$X_{11}, X_{12}, X_{13}, X_{14}, X_{21}, X_{22}, X_{23}, X_{24} \geq 0$

The pertinent data for each transportation problem has already been described in a matrix format using a tableau called transportation expenses tableau. See Tables 19 and 20.

Table 19. Transportation Expenses Matrix Tableau

	Y_1	Y_2	...	Y_n	Supply
X_1	C_{11}	C_{12}	...	C_{1n}	a_1
X_2	C_{21}	C_{22}	....	C_{2n}	a_2
.	.	.	…	.	.
X_m	C_{m1}	C_{m2}	...	C_{mn}	
Demand	b_1	b_2	...	b_n	

Table 20. Results of Transportation Expenses Tableau

	Destinations				**Supply**
	Y_1 (JB)	**Y_2 (Seremban)**	**Y_3 (KL)**	**Y_4 (Penang)**	
X_1 (Lorries 5 tons)	3.75	5.63	9.38	15.63	1710
X_2 (Lorries 10 tons)	4.69	7.5	12.5	20	3419
Demand	1539	513	2051	1026	

Transportation models are utilised to tackle the problems of minimal cost and generalised minimum cost flow problems, respectively (Muhammad, 2012; Latunde, Richard, Esan & Dare, 2020). The entire scenario can be modelled as a network, and extremely efficient methods are available for solving the optimisation problem in this network model (Shaiful Bakri & Dayangku Farahwaheda, 2019). The Simplex method is a strategy for solving linear programming problems manually by using slack variables, tableaus, and pivot variables to obtain the optimal solution to an optimisation problem (Khan, 2014).

Transportation management of the XYZ company has been analysed using the transportation or network model, and optimum transportation cost. Four key distribution centres for items from the XYZ corporation are examined in this case study, including Seremban and Johor Bahru; Kuala Lumpur/Selangor; and Penang. Three methods were used: North-West Corner Rule, Vogel's Approximation Method (VAM) and Simplex method. The solution may help the company improve its performance and increase profit while minimising resource waste with network model approaches. Furthermore, by using VAM and the Simplex method in their delivery inventory system, XYZ company can save money on transportation.

3. Results and Discussion

Each transportation method produces unique results, starting with the extreme upper-left corner. First, the units are distributed as many as possible under the North-West Corner Rule, up to their maximum capacity. Then, the processes are repeated horizontally and downwardly until supply and demand balance. Meanwhile, Vogel's Approximation method accounts for transportation costs when allocating units to each origin-destination pair. Meanwhile, the Simplex method makes efficient use of the Excel Solver.

Table 21. Optimal Solution Comparison

Origin → Destination	North-West Corner Rule	Vogel's Approximation	Simplex Method
5-ton lorries→ JB	1539	0	0
5-ton lorries→ Seremban	171	0	0
5-ton lorries→ K.L.	0	684	684
5-ton lorries→ Penang	0	1026	1026
10-ton lorries→ J.B.	0	1539	1539
10-ton lorries→ Seremban	342	513	513
10-ton lorries→ K.L.	2051	1367	1367
10-ton lorries→ Penang	1026	0	0
Total Transportation Cost	**RM55,456.48**	**RM50,605.21**	**RM50,605.21**

As shown in Table 21, each strategy produces distinct solutions. VAM and the Simplex Method resulted in the lowest total transportation cost of RM50,605.21. The per cent decrease in the value cost is computed by dividing the original cost (generated by multiplying the demand from each warehouse by the largest unit of supply) by the optimised cost. Using Table 22, we can summarise the reduction in the cost of value achieved by each method.

Table 22. Percentage Comparison Between Three Methods

Method(s)	Reduced Cost Value (Original Cost – Feasible Solution)	Percentage (%)
North-West Corner Rule	240,000- 55,456.48 = RM184,543.50	76.89
Vogel's Approximation Method	240,000- 50,605.21 = RM189,394.00	78.91
Simplex Method	240,000- 50,605.21 = RM189,394.00	78.91

It is observed that Vogel's Approximation Method and the Simplex Method offered the highest cost savings (RM189,394.00) with a reduced cost of value per cent of 78.91%.

Table 23. Sensitivity Analysis Using Excel Solver.

	Variable Cells		Final	Reduced	Objective	Allowable	Allowable
	Cell	Name	Value	Cost	Coefficient	Increase	Decrease
	$C$11: $F$12						
1	$C$11	5 Tons lorries (A) J.B.	0	2.18	3.75	1E+30	2.18
2	$D$11	5 Tons lorries (A) Seremban	0	1.25	5.63	1E+30	1.25
3	$E$11	5 Tons lorries (A) KL	684	0	9.38	1.25	1.25
4	$F$11	5 Tons lorries (A) Penang	1026	0	15.63	1.25	1E+30
5	$C$12	10 Tons lorries (B) JB	1539	0	4.69	2.18	1E+30
6	$D$12	10 Tons lorries (B) Seremban	513	0	7.5	1.25	1E+30
7	$E$12	10 Tons lorries (B) KL	1367	0	12.5	1.25	1.25
8	$F$12	10 Tons lorries (B) Penang	0	1.25	20	1E+30	1.25
	Constraints						
			Final	Shadow	Constraint	Allowable	Allowable
	Cell	Name	Value	Price	R.H. Side	Increase	Decrease
1.	$C$13	Location demand JB	1539	4.69	1539	0	1539
2.	$D$13	Location demand Seremban	513	7.5	513	0	513
3.	$E$13	Location demand K.L.	2051	12.5	2051	0	1367
4.	$F$13	Location demand Penang	1026	18.75	1026	0	1026
5.	$G$11	5 Tons lorries (A) Supply	1710	-3.12	1710	1367	0
6.	$G$12	10 Tons lorries (B) Supply	3419	0	3419	1E+30	0

3.1 Sensitivity Analysis

Sensitivity analysis clarifies how the ideal solution changes as the model's coefficients are altered. The purpose of sensitivity analysis is to determine the effect on the overall cost of transportation by modifying the volume and cost of transportation from each supply point to the respective destinations (Ghazali, Abd Majid & Mohd Shazwani, 2012). The relative effect on overall transportation expenses of varying the cargo volume and cost of each route is determined using a non-probabilistic technique (Jayaraman, 1998; Juman & Nawarathne, 2019). The per cent decrease in the value cost is computed by dividing the original cost (generated by multiplying the demand from each warehouse by the largest unit of supply) by the optimised cost. Using Table 23, the reduction in the cost of value achieved by each method is summarised. The ideal solution obtained using the Simplex method by Excel Solver results in a minimum cost of transportation of RM50,605.21, shown in Table 21.

The active constraint is satisfied when the values in the 'Final Value' column equal those in the ''Constraint R.H. Side. The values under 'Final Value' equal those under 'Constraint R.H. Side' for all locations, indicating that the solutions are considered active and binding with no slack. This is true if the combined supply from the two lorries equals the combined demand from the four locations, which totals 5129 units.

The 'Shadow Price' is the difference in the objective function's ideal value for each unit change in the constraint's right-hand side. If the marginal change is inside the permitted range, the optimal value of the issue is conserved. For instance, it is noted that Constraint 6 (10 Tons lorries (B)) 'Shadow Price' was zero. Therefore, the initial optimal solution consumes 3419 of the 3419 units supplied by the 10 Tons lorries (B) (which allows for a maximum decrease of 0 units and a maximum increase of *1E+30* or infinity). Due to the passive nature of Constraint 6, any change in the interval on the RHS of the constraint will leave the optimal value unaffected by the initial solution.

While this occurs, the 'Shadow Price' for constraint 1 is 4.69 units. There is no room for growth now that the highest limit of 1539 has been reached. According to the Shadow Price of the restriction, if the volume is reduced from 1539 to 1538, the ideal value is reduced by 4.69 units. It is calculated as follows: V(P) + (Shadow Price) = RM50,605.21– [(1539-1538) x 4.69] = RM50,600.52.

In the reduced cost column, the ideal value of the goal function is shown because of each unit change in the objective coefficients. Notably, the reduced cost for variable cells 3,4,5 and 6 is zero because the capacity has been entirely utilised. If the cost of transportation between the 5-ton lorries and the K.L. location is reduced by one, the optimal value decreases by 1.25. V(P) + (Reduced Cost) = RM50,605.21 + [(684-683) x 1.25] = RM50,606.46.

4. Conclusion

Three methods under the transportation model were used in this case study to reduce overall transportation costs between the production site and each specified warehouse. After analysing the results, the most optimal solution with the lowest cost could be determined. According to the results, Vogel's Approximation Method (VAM) and the Simplex Method resulted in the greatest cost savings, with an overall transportation cost of RM50,605.21 and a cost savings of 78.91 per cent. Cost savings would result in a boost to the company's profitability. Additionally, the sensitivity report provided by the Excel Solver saves time by providing sufficient information if the coefficients and parameters are adjusted without requiring the Excel Solver to be re-optimised numerous times. Additionally, it enables us to obtain a better grasp of the best solution's structure, which enables us to conduct additional research and analysis to enhance the problem and solution. Sensitivity analysis could be used to uncover possibilities to improve the management and operations systems of a business. If essential modifications are implemented, XYZ company can expand and improve its service sector nationally and internationally. The proper way of operation can help them be more capable of overcoming any obstacles that the business may face.

References

Ahmed, M. M., Khan, A. R., Uddin, Md. S., & Ahmed, F. (2016) A New Approach to Solve Transportation Problems. *Open Journal of Optimization*, 5, 22-30. http://dx.doi.org/10.4236/ojop.2016.51003

Aizemberg, L., Kramer, H. H., Pessoa, A., & Uchoa, E. (2014). Formulations for a Problem of Petroleum Transportation. *European Journal of Operational Research*, 82-90.

Rafi, F. S., & Islam, S. (2020). A Comparative Study of Solving Methods of Transportation Problem in Linear Programming Problem. *Journal of Advances in Mathematics and Computer Science,* 35(5), 45-67.

Fauziah, A. R., & Siti Nurhafiza, A. (2015). Transportation Model in Delivery Goods Using Railway Systems**.** *e-Proceeding of the Global Conference on Economics and Management Science 2015,* 978-967.

Ghazali, Z., Abd Majid, M., & Mohd Shazwani. (2012). Optimal Solution of Transportation Problem Using Linear Programming: A Case of a Malaysian Trading Company. *Journal of Applied Science*, 2430-2435.

Pang, H. E., & Chandrashekar, R.. (2019). Transportation Cost Optimisation through Distribution Network Model: A Case Study at XYZ Company. *Journal Communications in Computational and Applied Mathematics,* 8-11.

Jayaraman, V. (1998). Transportation, facility location and inventory issues in distribution network design: An investigation. *International Journal of Operations & Production Management*, 18(5), 471-494.

Juman, Z. A. M. S., & Nawarathne, N. G. S. A. (2019). An efficient alternative approach to solve a transportation problem. *Ceylon Journal of Science*, *48*(1), 19–29. DOI: http://doi.org/10.4038/cjs.v48i1.7584

Latunde, T., Richard, J. O., Esan, O., & Dare, D., (2020). Optimal Feasible Solutions to A Freight Transportation Problem. *Malaysian Journal of Computing*, 456-468.

Muhammad, K., (2012). Direct Methods for Finding Optimal Solution of a Transportation Problems are not Always Reliable. *International Refereed Journal of Engineering and Sciences*, 46-52.

Khan, M. A., (2014). Transportation Cost Optimization Using Linear Programming. *International Conference on Mechanical, Industrial and Energy Engineering*. 26-27 December 2014.

Nur Syafiqah A. S., (2016). Distribution of sugar based on transportation problem algorithm. *International Journal of Accounting & Business Management*, 2289-4519.

Reeb, J., & Leavengood, S. (2002). *Transportation Problem: A Special Case for Linear Programming Problems.* Operations Research, Oregon State University.

Shaiful Bakri, I., & Dayangku Farahwaheda, A.M. (2019). Routes Transportation Problem for Waste Collection System at Sitiawan, Perak, Malaysia. *International Journal of Innovative Technology and Exploring Engineering (IJITEE),* 2278-3075.