

MASS RATIO EFFECT ON THE VORTEX INDUCED VIBRATION OF
CYLINDERS IN TANDEM ARRANGEMENT

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CYLINDERS IN TANDEM ARRANGMENT

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Dedicated to my mother who is no more in this world

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ABSTRACT

Vortex-Induced Vibration (VIV) is often regarded as the most complex fluid-structure interaction problem that is yet to be fully understood. This research investigated the impact of mass ratio on the mechanism of VIV of closely spaced cylinders. The mass ratio is a vital parameter which affects the VIV of a circular cylinder. There are few studies that discussed mass ratio effect on VIV of single cylinder and no significant research has been conducted to study the effect of mass ratio on VIV of closely spaced cylinders. In the study, numerical simulations were carried out to understand the nature of VIV of two cylinders with equal-diameter for different mass ratios in tandem configuration. VIV characteristics of two mass ratios were compared. Cylinder with mass ratio 2 represents lighter cylinder whereas mass ratio 8 represents a heavier cylinder. Only the upper or super upper response branch normally found between reduced velocities 5 to 8 was studied. The cylinders were exposed to uniform flows in subcritical flow regime and shear stress transport detached eddy turbulence model was employed for simulating the turbulent flow around these cylinders. The center to center spacing between cylinders was four times of cylinder diameter. A series of tests were conducted to validate the present numerical study. Vital VIV parameters with detailed discussions of flow patterns to scrutinize the influence of upstream cylinder's mass ratio on the VIV of the rear cylinder at resonance zone were presented. It was found that oscillation frequency of the upstream cylinder plays a significant role in the nature of VIV of downstream cylinder. For a relatively heavier upstream cylinder, VIV amplitude of downstream cylinder escalates at the lower limit of resonance zone. Noticeable VIV increment of rear cylinder can be found when natural frequency of the upstream cylinder is at least 14% lower than that of the downstream cylinder. The study of the mass ratio effect on VIV of closely spaced cylinders is significant in terms of designing aquatic clean energy converter widely known as VIVACE converter and assessing the collision risk and fatigue of cylindrical-shaped risers located close to each other.

ABSTRAK

Vortex-Induced Vibration (VIV) sering dianggap sebagai masalah interaksi struktur cecair yang paling kompleks yang masih belum dapat difahami sepenuhnya. Kajian ini mengkaji tentang kesan nisbah jisim kepada mekanisme VIV yang bertindak kepada silinder rapat. Nisbah jisim merupakan parameter penting yang memberi kesan kepada VIV silinder bulat. Terdapat beberapa kajian yang membincangkan kesan nisbah jisim pada VIV silinder tunggal dan tidak ada kajian penting telah dijalankan untuk mengkaji kesan nisbah jisim pada VIV silinder bulat. Dalam kajian itu, simulasi berangka telah dijalankan untuk memahami sifat VIV daripada dua silinder dengan sama-diameter dengan nisbah jisim yang berbeza dalam susunan seiring. Ciri-ciri VIV daripada dua nisbah jisim telah dibandingkan. Silinder dengan nisbah jisim 2 mewakili silinder yang lebih ringan manakala nisbah jisim 8 mewakili silinder yang lebih berat. Hanya *upper* atau *super upper response branch* yang biasanya ditemui antara pengurangan halaju 5-8 adalah dikaji. Silinder yang bertindak dengan aliran seragam dalam regim aliran subgenting dan model gelora *shear stress transport detached eddy* telah digunakan untuk simulasi aliran bergelora sekitar silinder ini. Jarak antara pusat dua silinder adalah empat kali daripada diameter silinder. Satu siri ujian telah dijalankan untuk mengesahkan kajian berangka ini. Parameter penting dalam penyelidikan VIV telah dibentangkan dan perbincangan terperinci dari segi corak aliran untuk meneliti pengaruh nisbah jisim silinder di hulu ke atas VIV silinder di hilir pada zon resonans. Penyelidikan ini mendapati bahawa kekerapan ayunan untuk silinder hulu memainkan peranan penting dalam kegiatan-VIV pada silinder hiliran. Bagi silinder hulu yang lebih berat, amplitud VIV silinder di hilir bertambah pada had bawah zon resonans. Untuk kenaikan VIV yang ketara di silinder belakang, frekuensi semulajadi silinder hulu adalah sekurang-kurangnya 14% lebih rendah daripada silinder di hilir. Kajian mengenai kesan nisbah jisim pada VIV silinder rapat adalah penting dari segi mereka bentuk akuatik bersih penukar tenaga yang dikenali sebagai VIVACE dan penilai risiko pelanggaran dan lesu penaik berbentuk silinder yang terletak berhampiran antara satu sama lain.

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LIST OF ABBREVIATIONS

1 DOF	-	One Degree Of Freedom
2 DOF	-	Two Degree Of Freedom
ALE	-	Arbitrary Lagrangian – Eulerian
CFD	-	Computational Fluid Dynamics
CEL	-	CFX Expression Language
DAAS	-	Data Acquisition System and Analysis System
FIM	-	Flow induced motion
FFT	-	Fast Fourier transform
FSI	-	Fluid Structure Interaction
DES	-	Detached Eddy Simulation
DNV	-	Det Norske Veritas
LES	-	Large Eddy Simulation
PSD	-	Power spectral density
RANS	-	Reynolds Averaged Navier-Stokes
RMS	-	Root mean square
WIV	-	Wake induced vibration
VIV	-	Vortex induced vibration
VIVACE	-	Vortex Induced Vibration for Aquatic Clean Energy
UTM	-	Universiti Teknologi Malaysia

LIST OF SYMBOLS

A_x	-	Inline amplitude in X direction
A_y or A	-	Cross line amplitude in Y direction
C_D	-	Drag Coefficient
C_L	-	Lift Coefficient
C_m	-	Added mass coefficient
D	-	Diameter of the cylinder
f or f_{osc}	-	Oscillation frequency
f_n	-	Natural frequency
f_s	-	Vortex shedding frequency
F_L	-	lift force acted on the cylinder
F_x	-	Inline force in X direction
F_y	-	cross line force in Y direction
L	-	Length of the cylinder
L/D	-	Inline amplitude in X direction
m	-	Cylinder mass
m_a	-	Added mass

m^+ or m^*	-	Mass ratio
Re	-	Reynolds number
U	-	Free stream mean velocity
U_r	-	Reduced velocity
K	-	Spring constant
E_{mech}	-	Converted energy by VIVACE
ω_z	-	Circular natural frequency of the cylinder
ρ	-	Water density
ω	-	Cylinder circular natural frequency
Φ	-	Phase difference
ζ	-	Damping factor
ν	-	Kinematic viscosity
μ	-	Dynamic viscosity
$\overline{\rho u_i u_j}$	-	Reynolds stresses.

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CHAPTER 1

INTRODCTION

1.1 Background

Vortex-Induced Vibration widely known as VIV is a complicated hydrodynamic phenomenon. VIV is often regarded as the most complex fluid-structure interaction problem that happens when fluid flows over a certain structure. During the interaction of flow and the structure, at the boundary layer region due to viscous drag, energy dissipates and the flow lacks adequate kinetic energy. The lack of energy causes flow separation at boundary layer and vortices are formed at the wake region of the body as shown in figure.1.1. These vortices disperse periodically

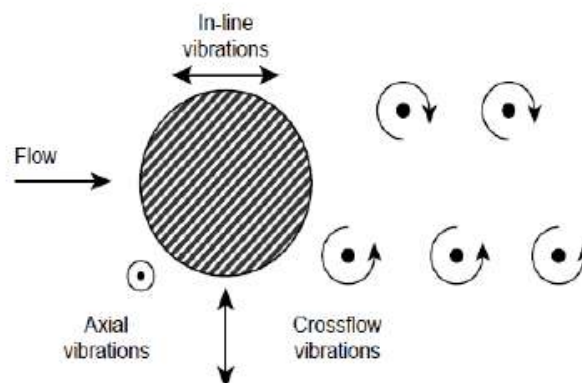


Figure 1.1: Vortex induced vibration of cylinder (Blevins,1990)

from each side of the body and produce the time dependent non-uniform pressures that are distributed around the object.

Lift forces that is dependent on time, are originated around the body by these non-uniform pressures. As a result, the structure oscillates in both inline and cross flow direction depending upon the degree of freedom. Vortices shed are unsteady flow by nature that depends on the mass and profile of the body, it also relies on the velocity of the free stream flow. Accurate calculation of the separation point is the key, surface roughness and Reynolds number play a significant role in terms of governing the nature of separation. When the shedding frequency is close to the natural frequency of the object, higher VIV amplitudes can be found. These higher VIV amplitudes can occur for a range of reduced velocities which is defined as lock-in zone. Vikestad (1998) showed this synchronization mainly occurs due to the variation of hydrodynamic mass. Vortex-induced vibration can be defined as hydro-elastic phenomenon if cylinders are subjected to water flows, where non-dimensional parameter mass ratio normally remains between 1 to 10. Mass ratio is nothing but the ratio between oscillating structure's mass and mass fluid that is displaced by the structure.

Most of the research works about VIV of a circular cylinder or other blunt objects discuss cases where the body was only freed to oscillate in cross flow direction. In other words, single degree of freedom cases dominate VIV related research works. However, there are very few recent researches that studied two degree of freedom cases and it was found that the nature of VIV for two degree of freedom cases where cylinder was designed to vibrate both cross flow and stream wise direction can be significantly different. Though there are numbers of research papers about VIV simulation of a circular cylinder, most of them were related to 2D flow and lower Reynolds number, numerical simulation of vortex induced vibration at high Reynolds number can be very challenging. Suitable mesh, appropriate turbulence model, strong computational resources are tricky issues to be dealt with before considering 3D simulation.

The study of vortex dynamics and characteristics of fluid flow around a circular cylinder is important to understand the nature of flow around relatively large and complex structures. A circular cylinder is considered one of the fundamental shapes of structures that are used in many engineering designs and often a group of circular cylinders are used for designing complex structures. When two bodies are placed near to each other, thought-provoking fluid phenomena can be observed, so the VIV study of cylinders near to each other became an interesting and important subject of basic research in fluid mechanics. Vortex induced vibration can be observed in many engineering objects, such as offshore structures, production risers, bridges, aircraft control surfaces, thermo wells, engines, heat exchangers etc. The goal of this research work is to examine the nature of VIV of a pair of circular cylinders positioned in tandem; we specially focused on the effect of upstream (front) cylinder's mass ratio on the behavior of VIV of downstream (rear) cylinder.

1.2 Problem statement

There are comparatively very little available experimental or numerical research works that study VIV of a pair of cylinders. The initial hypothesis about VIV of closely spaced cylinders may be behind this lack of references for an oscillating pair of cylinders. Initially, it was assumed that when two closely spaced cylinder's experience VIV, the VIV of each cylinder resembles that of isolated cylinder. However, recent researches indicate that assumption was far from the reality. Recent literature also discussed the role of mass ratio on single cylinder's VIV and its significance, but the influence of mass ratio on the VIV of cylinders positioned near to each other has not been studied to the best of our knowledge. The study of cylinders in tandem arrangement is also important for designing VIVACE device. So far, no conclusive studies were done to study mass ratio effect for VIVACE device.

1.3 Objectives of the study

The goal of this research work is to examine the nature of VIV of a pair of circular cylinders positioned in tandem and develop new knowledge in terms of finding factors that govern rear cylinder's VIV. So, this research work has following objectives:

- To assess the mass ratio effect on the VIV of a circular cylinder at high Reynolds number.
- To determine the mass ratio effect on the VIV of two cylinders in the tandem arrangement.
- To identify the fluid dynamic effects of upstream cylinder's mass ratio on downstream cylinder in details by analyzing flow patterns.

1.4 Scope of the study

This research work contains subjects related to hydrodynamic nature of Vortex-induced vibration (VIV) of single cylinder and cylinders positioned near to each other. Procedures / guidelines / formulas to analyze VIV of single cylinder are established through literature study. The vortex induced vibration of closely spaced cylinders in tandem configuration was studied based on that analysis. Intensive numerical simulations were conducted for this study. Numerical simulations were validated by comparing obtained results with available experimental results. The influence of the mass ratio of the upstream cylinder over rear (downstream) cylinder's VIV was studied in details for 4D spacing. Only upper or super upper response branch (resonance zone) which is generally found between reduced velocity 5 to reduced velocity 7 was studied for all cases.

1.5 Significance of the study

As mentioned earlier, VIV is one of the most complicated fluid-structure interaction problems. There are many parameters that affect the nature of VIV of circular cylinders; mass ratio is one of those factors. Hydrodynamic engineers faced enormous challenge while designing riser systems that are used for extracting oil and gas from sea bed due to its destructive nature in terms of causing severe fatigue damage within small period of time. Vortex Induced Vibration can also be used to generate clean energy. Aquatic Clean Energy Converter or better known as VIVACE uses VIV to generate clean renewable energy from ocean currents (Soo, 2013). In most cases numbers of closely spaced circular cylinders are used to design VIVACE. Therefore, determination of mass ratio effect on vortex dynamics of closely spaced circular cylinders is important especially for designing riser system and VIVACE device.

1.6 Organization of the thesis

This thesis is divided into eight chapters. The current chapter gives the outline of the thesis. The objectives along with the scope of the research work are presented in chapter one. Significances of this study are also mentioned in this current part of the thesis.

Chapter two reports a thorough review of the already conducted research works that are related to the present study. For more vibrant presentation, the literature review was divided into several groups' i.e. basic definition of VIV, VIV mechanism of single and pair of cylinders, Usage of CFD to study VIV, the effect of mass ratio and motivation of the study.

In chapter three research methodologies have been described. Experimental and numerical research methods are presented. The experimental methodology

describes the facilities, test set-up of experiment and procedure of test. It also describes the mathematical model used for numerical simulation.

Chapter four presents the validation of numerical simulation by comparing experimental results that are obtained through towing tank tests conducted by the author and from other researcher's work.

Chapter five explains the mass ratio effects on single cylinder for both 1DOF and 2DOF systems.

Natures of vortex induced vibration of cylinders in the tandem arrangement for different mass ratio are described in chapter six. Here both cylinders are allowed to vibrate for both 1DOF and 2DOF systems for different mass ratios. Both cylinders have same mass ratio.

In chapter seven, the vortex induced vibration of cylinders in tandem arrangement with varying mass ratio. Here both cylinders are allowed to vibrate for both 1DOF and 2DOF systems. Upstream cylinders mass ratio have been altered but downstream cylinders mass ratio remain same. This chapter basically describes the most important contribution of this research.

Finally, Chapter eight presents the major conclusions obtained from this research work. In addition, recommendations for future studies have been presented.

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