CFD Letters 14, Issue 9 (2022) 52-59

SEMARAK ILMU PUBLISHING

CFD Letters

Journal homepage:
https://semarakilmu.com.my/journals/index.php/CFD_Letters/index
ISSN: 2180-1363

Magnetohydrodynamics Mixed Convection of Viscoelastic Nanofluid Past a Circular Cylinder with Constant Heat Flux

Rahimah Mahat[1,*], Muhammad Saqib[2], Imran Ulah[3], Sharidan Shafie[2], Sharena Mohamad Isa[4]

[1] Advanced Facilities Engineering Technology Research Cluster (AFET), Universiti Kuala Lumpur Malaysian Institute of Industrial Technology, Persiaran Sinaran Ilmu, Bandar Seri Alam, 81750 Masai, Johor, Malaysia
[2] Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 81310 Johor Bahru, Malaysia
[3] College of Civil Engineering, National University of Sciences and Technology, 44000, Islamabad, Pakistan
[4] Manufacturing Engineering Technology Section, Universiti Kuala Lumpur Malaysia Italy Design Institute (UniKL MIDI), Kuala Lumpur, 56100, Malaysia

ARTICLE INFO

Article history:
Received 26 May 2022
Received in revised form 26 July 2022
Accepted 17 August 2022
Available online 30 September 2022

Keywords:
Magnetohydrodynamics (MHD); Viscoelastic; Nanofluid; Constant heat flux

ABSTRACT

A mathematical model for viscoelastic nanofluid flow approaching a linear horizontal circular cylinder in magnetohydrodynamics (MHD) has been developed. In the analysis, the cylinder with a constant heat flux is shown with a magnetic field. In this research, we employed the Tiwari and Das Nanofluid model to learn more about the impacts of nanofluids, and sodium carboxymethyl cellulose containing copper (Cu) nanoparticles was used as the base fluid. Dimensional linear equations are converted into dimensionless expressions using the appropriate transformations. The Keller box technique approach is used to handle the governing dimensionless concerns. Investigations are conducted on how a select few parameters affect flow and heat transfer. It includes and analyses the skin friction and heat transfer coefficients. When the obtained results are compared to the available data in the limiting situation, there is a great deal of congruence. It was discovered that the viscoelastic nanofluid's velocity, temperature, skin friction, and heat transfer coefficients heavily depend on the viscosity and thermal conductivity combined with the magnetic field and nanoparticles volume fraction.

1. Introduction

Heat transfer enhancement, augmentation, or intensification is a term used by researchers to describe a variety of strategies for improving heat transfer performance. Therefore, the addition of nanofluid in fluid flow is to enhance the heat transfer. The study on nanofluids have been studied widely by Waini *et al.*, Mahat *et al.,* Patil *et al.,* Murad *et al.*, and Shorbagy *et al.,* [1-7].

The study of the flow of an electrically conducting fluid in a magnetic field is known as magnetohydrodynamics (MHD). Fundamentally, magnetic fields produce currents in a flowing conductive fluid, which tend to form Lorentz force drag forces on the fluid flow. The fluid flow will be affected by the Lorentz force. MHD is being used in astronomy and geophysics, nuclear fission and

* *Corresponding author.*
E-mail address: rahimahm@unikl.edu.my (Rahimah Mahat)

https://doi.org/10.37934/cfdl.14.9.5259

fusion, metallurgy, and direct energy conversion, among other fields. Many researchers, such as Jamel *et al*., Hussain *et al*., Tlili *et al.,* Wakif *et al*., Hatami *et al.,* Cao *et al*., Liaqat *et al.,* El-Shorbagy *et al.,* and Jamel *et al.,* [8-14], have undertaken extensive research on MHD of nanofluids. The benefits of MHD's influence on mixed convection flow via a horizontal circular cylinder are evident in the technology's growth in industrial and technical applications.

The geometry for this study has been a horizontal circular cylinder since it is often utilized by researchers and has a wide range of technical and industrial applications. Therefore, the cylinder must be longer to get a steadier, consistent, and effective result [15]. This is because it could lessen the flow's potential impacts of drag or turbulence [16]. The flow and heat transfer via a horizontal circular cylinder have been the subject of several theoretical and experimental research. A regular occurrence in many industrial processes, fluid flow and heat transfer past circular cylinders have drawn a lot of attention in the literature.

In the current research, the effects of MHD mixed convection of viscoelastic nanofluid past a horizontal circular cylinder is theoretically investigated. The effects of magnetic field, nanoparticle volume fraction and how these affect the thermal characteristics of the system are of particular interest.

2. Methodology

The Cartesian coordinate (x, y) is used and the dimensional gravitational acceleration is $g_x = g\sin(\bar{x}/a)$, where $\bar{x}$ is the distance from the lower stagnation point. The dimensional velocity outside the boundary layer is $\bar{u}_e(\bar{x}) = U_\infty \sin(\bar{x}/a)$ and the constant free stream velocity is $(1/2)U_\infty$ as mentioned by Merkin [17]. Tiwari and Das model [18] have been chosen in this study and the model is defined as a single-phase model that use Brickman viscosity model. Under the above assumptions and by considering the nanofluid model, the dimensional governing equations of momentum equation and energy equation can be expressed as:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{1}$$

$$\begin{aligned}\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \bar{u}_e\frac{\partial \bar{u}_e}{\partial \bar{x}} + \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{k_o}{\rho_{nf}}\left[\frac{\partial}{\partial \bar{x}}\left(\bar{u}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) + \bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3} - \frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}}\right] \\ + g\beta_{nf}(T - T_\infty)\sin\left(\frac{\bar{x}}{a}\right) - \sigma B_0^2 \bar{u},\end{aligned} \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial \bar{y}^2}, \tag{3}$$

with the boundary conditions:

$$\begin{aligned}&\bar{u} = 0,\ \bar{v} = 0,\ T = -\frac{q_w}{k_{nf}} &&\text{at} \quad \bar{y} = 0,\ \bar{x} \geq 0, \\ &\bar{u} = \bar{u}_e(\bar{x}),\quad \frac{\partial \bar{u}}{\partial \bar{y}} = 0,\quad T = T_\infty &&\text{at} \quad \bar{y} \to \infty,\ \bar{x} \geq 0,\end{aligned} \tag{4}$$

where k_{nf} is the thermal conductivity of nanofluid, *T* is the fluid temperature and q_w is the constant heat flux. The numerical values of the thermophysical properties of base fluid and nanoparticles are given in Table 1.

Table 1
Thermophysical properties of nanoparticles and base fluid

Physical Properties	$\rho\left(\text{kg } m^{-3}\right)$	$C_p\left(\text{J kg}^{-1}\text{K}^{-1}\right)$	$k\left(\text{Wm}^{-1}\text{K}^{-1}\right)$	$\beta\times10^5\left(\text{K}^{-1}\right)$
Base Fluid (CMC)	997.1	4179	0.613	21
Nanoparticle (Cu)	8933	385	401	1.67

The dimensionless variables are introduced to simplify the complexity of the governing equations. Based on Anwar *et al.,* [19], the dimensionless variables are defined as:

$$x=\bar{x}/a,\ y=\text{Re}^{1/2}\left(\bar{y}/a\right),\ u=\bar{u}/U_\infty,\ v=\text{Re}^{1/2}\left(\bar{v}/U_\infty\right),$$
$$u_e\left(x\right)=\bar{u}_e\left(\bar{x}\right)/U_\infty,\ \theta=\left(T-T_\infty\right)/\left(T_f-T_\infty\right), \tag{5}$$

where *Re* is Reynolds number. By substituting Eq. (5) into Eqs. (1) - (3), the dimensionless system below is yielded:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{6}$$

$$\left[\left(1-\phi\right)+\phi\frac{\rho_s}{\rho_f}\right]\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=\left[\left(1-\phi\right)+\phi\frac{\rho_s}{\rho_f}\right]\sin x\cos x+\frac{1}{\left(1+\phi\right)^{2.5}}\frac{\partial^2 u}{\partial y^2}$$
$$-K\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right)+v\frac{\partial^3 u}{\partial y^3}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right]+\left[\left(1-\phi\right)+\phi\frac{\left(\rho\beta\right)_s}{\left(\rho\beta\right)_f}\right]\lambda\theta\sin\left(x\right)-\frac{\sigma B_0^2 a}{\rho_f U_\infty}u, \tag{7}$$

$$\left[\left(1-\phi\right)+\phi\frac{\left(\rho C_p\right)_s}{\left(\rho C_p\right)_f}\right]\left[u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}\right]=\frac{\left(k_s+2k_f\right)-2\phi\left(k_f-k_s\right)}{\left(k_s+2k_f\right)+\phi\left(k_f-k_s\right)}\frac{1}{\text{Pr}}\frac{\partial^2\theta}{\partial y^2}, \tag{8}$$

with the new boundary conditions as:

$$u=0,\quad v=0,\quad \frac{\partial\theta}{\partial y}=-1,\quad \text{at}\quad y=0, x\geq 0,$$
$$u=u_e\left(x\right),\quad \frac{\partial u}{\partial y}=0,\quad \theta=0,\quad \text{as}\quad y\to\infty, x\geq 0, \tag{9}$$

where $\text{Pr}=\mu_f C_p/k_f$ is Prandtl number, $M=\sigma B_o^2 a/\rho_f U_\infty$ is magnetic field, $K=k_o U_\infty/\mu_f a$ is viscoelastic parameter, and λ is mixed convection parameter.

3. Mathematical Solution

In order to solve Eqs. (6) - (8), subject to the boundary conditions Eq. (9), the following variables have been considered:

$$\psi = xF\left(x,y\right), \qquad \theta = \theta\left(x,y\right), \tag{10}$$

are introduced where ψ is the stream function defined as:

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}. \tag{11}$$

By substituting Eq. (10) and Eq. (11) into Eqs. (6) - (8), obtained:

$$\begin{aligned}
&\left[(1-\phi)+\phi\frac{\rho_s}{\rho_f}\right]\left[\left(\frac{\partial F}{\partial y}\right)^2 + x\frac{\partial F}{\partial y}\left(\frac{\partial^2 F}{\partial x \partial y}\right) - x\frac{\partial F}{\partial x}\frac{\partial^2 F}{\partial y^2} - F\frac{\partial^2 F}{\partial y^2}\right] \\
&= \left[(1-\phi)+\phi\frac{\rho_s}{\rho_f}\right]\frac{\sin x \cos x}{x} + \frac{1}{(1+\phi)^{2.5}}\frac{\partial^3 F}{\partial y^3} + \left[(1-\phi)+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right]\lambda\theta\frac{\sin x}{x} - M\frac{\partial F}{\partial y} \\
&+K\left[2\frac{\partial F}{\partial y}\frac{\partial^3 F}{\partial y^3} - F\frac{\partial^4 F}{\partial y^4} - \left(\frac{\partial^2 F}{\partial y^2}\right)^2 + x\left(\frac{\partial^2 F}{\partial x \partial y}\frac{\partial^3 F}{\partial y^3} - \frac{\partial F}{\partial x}\frac{\partial^4 F}{\partial y^4} + \frac{\partial F}{\partial y}\frac{\partial^4 F}{\partial x \partial y^3} - \frac{\partial^2 F}{\partial y^2}\frac{\partial^3 F}{\partial x \partial y^2}\right)\right],
\end{aligned} \tag{12}$$

$$\left[(1-\phi)+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right]\left[x\frac{\partial F}{\partial y}\frac{\partial \theta}{\partial x} - x\frac{\partial F}{\partial x}\frac{\partial \theta}{\partial y} - F\frac{\partial \theta}{\partial y}\right] = \frac{k_{nf}}{k_f}\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2}, \tag{13}$$

which are subject to the following boundary conditions:

$$\begin{aligned}
&F = 0, \quad \frac{\partial F}{\partial y} = 0, \quad \frac{\partial \theta}{\partial y} = -1, \quad \text{at} \quad y = 0, x \geq 0, \\
&\frac{\partial F}{\partial y} = \frac{\sin x}{x}, \quad \frac{\partial^2 F}{\partial y^2} = 0, \quad \theta = 0, \quad \text{as} \quad y \to \infty, x \geq 0,
\end{aligned} \tag{14}$$

When $x \approx 0$, Eq. (12) and Eq. (13) reduce to the following ordinary differential equations:

$$\begin{aligned}
&\frac{1}{(1+\phi)^{2.5}} f''' - \left[(1-\phi)+\phi\frac{\rho_s}{\rho_f}\right]\left[f'^2 - ff''\right] + K\left(2f'f''' - ff^{iv} - f''^2\right) \\
&\qquad + \left[(1-\phi)+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right]\lambda\theta - Mf' = 0,
\end{aligned} \tag{15}$$

$$\frac{\left(k_s+2k_f\right)-2\phi\left(k_f-k_s\right)}{\left(k_s+2k_f\right)+\phi\left(k_f-k_s\right)}\frac{1}{\Pr}\theta''+\left[\left(1-\phi\right)+\phi\frac{\left(\rho C_p\right)_s}{\left(\rho C_p\right)_f}\right]f\theta'=0, \tag{16}$$

with the boundary conditions:

$$\begin{aligned} &f(0)=0, \quad f'(0)=0, \quad \theta'(0)=-1, \\ &f'(\infty)=1, \quad f''(\infty)=0, \quad \theta(\infty)=0, \end{aligned} \tag{17}$$

4. Results

The behavior of fluid flow for viscoelastic nanofluid affected by magnetic field *M* is analyzed. The validation for numerical solutions is done by comparing the skin friction and heat transfer coefficients with the results from Nazar *et al.,* [20]. Good agreements have been obtained from the results as shown in Table 2.

Table 2
Comparison values of heat transfer coefficient when $K=0$, $\Pr=1$, $\phi=0$, $M=0$, and different values of λ

λ	Nazar *et al.,* [20]	Present $f''(0)$	Nazar *et al.,* [20]	Present $\theta(0)$
-0.2	1.0340	1.033028	1.8157	1.816890
0.4	1.5747	1.573759	1.7018	1.702823
3.0	2.4913	2.489892	1.4015	1.402232
10.0	5.7730	5.777805	1.1770	1.178456

Figures 1 and Figure2 demonstrate how skin friction and heat transfer coefficients vary with M. Skin friction rises as M increases, as shown in Figure 1. Figure 2 shows, however, that heat transport reduces as M increases. Figure 3 and Figure 4 depict the effects of M on velocity and temperature curves. As the value of M increases, the fluid velocity decreases (see Figure 3). Lorentz force is created by the existence of a transverse magnetic field that acts parallel to the flow. The flow is then slowed due to the resistance caused by this force. It's worth noting that this force has an unusual effect. It's worth noting that this force causes the fluid to suffer resistance by increasing friction between its layers, which causes the flow's temperature to rise, as seen in Figure 4. The temperature profile climbs significantly as the M value rises.

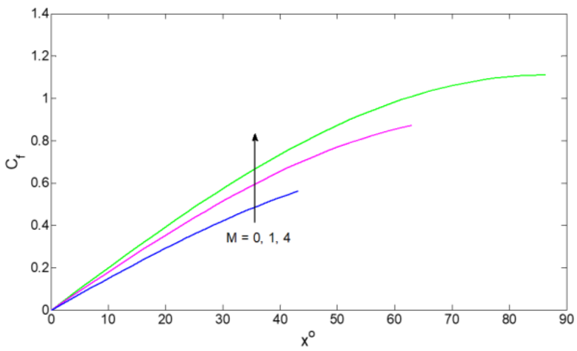

Fig. 1. Variation values of *M* for skin friction coefficient

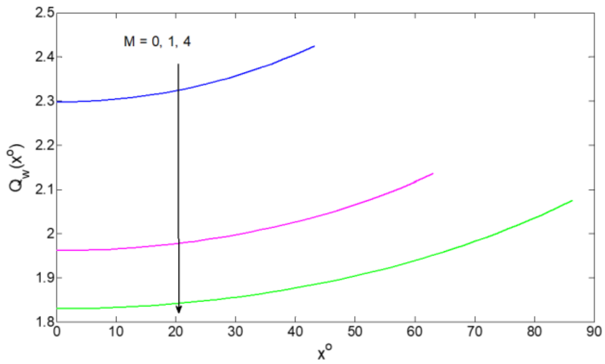

Fig. 2. Variation values of *M* for heat transfer coefficient

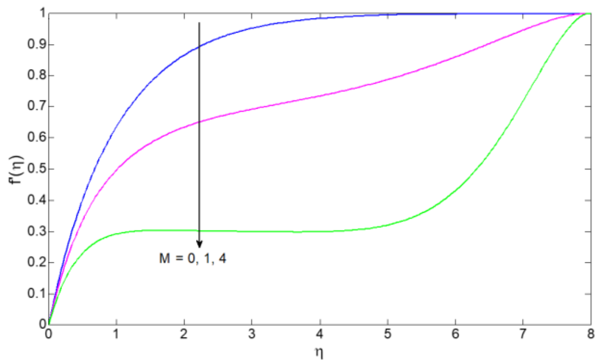

Fig. 3. Variation values of *M* for velocity profile

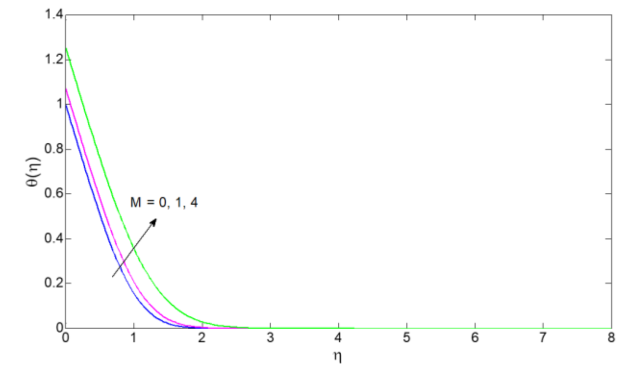

Fig. 4. Variation values of *M* for temperature profile

Figure 5 and Figure 6 illustrates the effects of the nanoparticles volume fraction, ϕ on the temperature and velocity curves. The graphic shows that the velocity profile starts to decline as the temperature profile rises with time. Physically, the basic fluid becomes more viscous when the nanoparticles ϕ are added, slowing the fluid flow. The thermal boundary layer steadily grows, as seen in Figure 6. This behaviour is consistent with the physical prediction that an increase in causes the fluid's temperature to rise by improving the fluid's thermal conductivity.

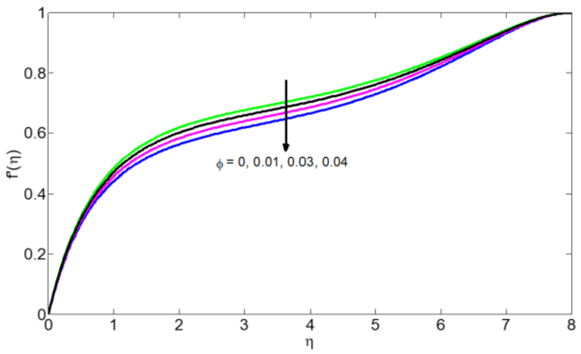

Fig. 5. Variation values of ϕ for velocity profile

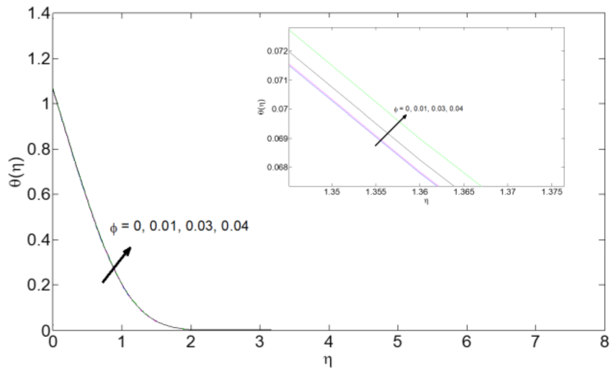

Fig. 6. Variation values of ϕ for temperature profile

Figure 7and Figure 8 plots the impacts of the nanoparticles volume fraction acting on the fluid as well as the coefficients of skin friction and heat transmission. According to observations, the values of skin friction and heat transfer coefficient, improve when the nanoparticles volume fraction increases. As the nanoparticles volume fraction rises, the fluid's thermal conductivity improves, increasing the coefficients of skin friction and heat transfer.

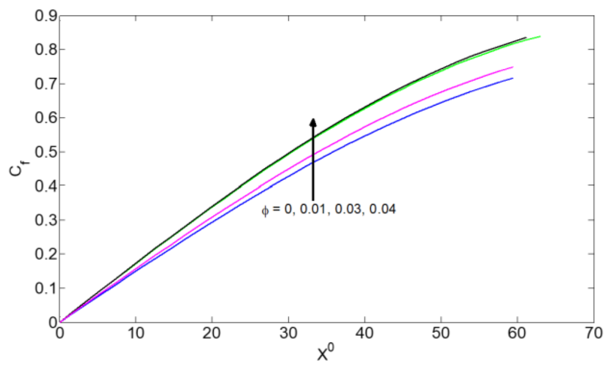

Fig. 7. Variation values of ϕ for skin friction coefficient

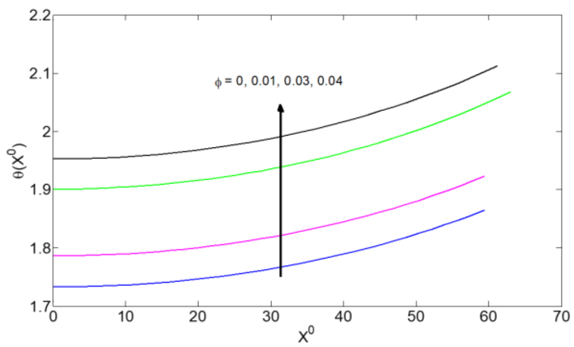

Fig. 8. Variation values of ϕ for heat transfer coefficient

5. Conclusions

The conclusions drawn from this modern research are as follows: Velocity slows down and increase with augmentation in magnetic parameters. In the meantime, skin friction increases while heat transfer coefficient decreases for the increasing values of *M*. Besides that, velocity decrease as the nanoparticles volume fraction increase. Other than that, the distributions are increase as the nanoparticles volume fraction increase.

Acknowledgement
The authors would like to acknowledge the Ministry of Higher Education (MOHE) for the financial support through vote numbers, FRGS/1/2019/STG06/UNIKL/03/1.

References

[1] Iskandar, Waini, Anuar Ishak, Grosan, T. and Pop, I. “Mixed convection of a hybrid nanofluid flow along a vertical surface embedded in a porous medium.” *International Communications in Heat and Mass Transfer* 114, no. 104565 (2020): 1-5. https://doi.org/10.1016/j.icheatmasstransfer.2020.104565
[2] Mahat, Rahimah, Sharidan Shafie, and Fatihhi Januddi. "Numerical analysis of mixed convection flow past a symmetric cylinder with viscous dissipation in viscoelastic nanofluid." *CFD letters* 13, no. 2 (2021): 12-28. https://doi.org/10.37934/cfdl.13.2.1228
[3] Mahat, Rahimah, Noraihan Afiqah Rawi, Abdul Rahman Mohd Kasim and Sharidan Shafie. "Mixed Convection Flow of Viscoelastic Nanofluid Past a Horizontal Circular Cylinder with Viscous Dissipation." *Sains Malaysiana* 47, no. 7 (2018): 1617–1623. http://dx.doi.org/10.17576/jsm-2018-4707-33
[4] Patil, P.M., Shashikant, A., and Hiremath, P.S. "Diffusion of liquid hydrogen and oxygen in nonlinear mixed convection nanofluid flow over vertical cone." *International Journal of Hydrogen Energy* 44, no. 31 (2019): 17061-17071. https://doi.org/10.1016/j.ijhydene.2019.04.193
[5] Nurul Musfirah Murad, Noraihan Afiqah Rawi, Sharidan Shafie, Yeou Jiann Lim and Rahimah Mahat. "Unsteady Falkner-Skan flow of Hybrid Nanofluid over a Nonlinear Moving Wedge." *Malaysian Journal of Fundamental and Applied Sciences* 18, no. 1 (2022): 116-123. https://doi.org/10.11113/mjfas.v18n1.2421
[6] Patil, P.M., Benawadi, S., and Shanker, B. " Influence of mixed convection nanofluid flow over a rotating sphere in the presence of diffusion of liquid hydrogen and ammonia." *Mathematics and Computers in Simulation* 194, April (2022): 764-781. https://doi.org/10.1016/j.matcom.2021.12.022
[7] El-Shorbagy, M.A., Farshad Eslami, Muhammad Ibrahim, Pouya Barnoon, Wei-Feng Xia and Davood Toghrai. " Numerical investigation of mixed convection of nanofluid flow in a trapezoidal channel with different aspect ratios in the presence of porous medium." *Case Studies in Thermal Engineering* 25, no. 100977 (2021): 1-14. https://doi.org/10.1016/j.csite.2021.100977
[8] Jamel Bouslimi, Abdelhafez, M.A., Abd-Alla, A.M., Abo-Dahab, S.M. and Mahmoud, K.H. “ MHD Mixed Convection Nanofluid Flow over Convectively Heated Nonlinear due to an Extending Surface with Soret Effect.” *Dynamic Analysis, Learning, and Robust Control of Complex Systems* 2021, Article ID 5592024 (2021): 1-20. https://doi.org/10.1155/2021/5592024

[9] Hussain, S., Mehmood, K. and Sagheer, M. "MHD mixed convection and entropy generation of water–alumina nanofluid flow in a double lid driven cavity with discrete heating." *Journal of Magnetism and Magnetic Materials* 419, (2016): 140-155. https://doi.org/10.1016/j.jmmm.2016.06.006

[10] Tlili, I., W. A. Khan, and K. Ramadan. "MHD flow of nanofluid flow across horizontal circular cylinder: steady forced convection." *Journal of Nanofluids* 8, no. 1 (2019): 179-186. https://doi.org/10.1166/jon.2019.1574

[11] Abderrahim Wakif, Zoubair Boulahia, Farhad Ali, Mohamed R. Eid and Rachid Sehaqui. "Numerical Analysis of the Unsteady Natural Convection MHD Couette Nanofluid Flow in the Presence of Thermal Radiation Using Single and Two-Phase Nanofluid Models for Cu–Water Nanofluids." *International Journal of Applied and Computational Mathematics* 4, no. 81 (2018): 1–27. https://doi.org/10.1007/s40819-018-0513-y

[12] Hatami, M., Nouri, R. and Ganji, D.D. "Forced convection analysis for MHD Al2O3–water nanofluid flow over a horizontal plate." *Journal of Molecular Liquids* 187, (2013): 294-301. https://doi.org/10.1016/j.molliq.2013.08.008

[13] Yan Cao, Hamdi Ayed, Fahd Jarad, Hussein Togun, Hajar Alias, AlibekIssakhov, Mahidzal Dahari, Makatar Wae-hayee and Mohamed Hechmi El Ouni. "MHD natural convection nanofluid flow in a heat exchanger: Effects of Brownian motion and thermophoresis for nanoparticles distribution." *Case Studies in Thermal Engineering* 28, no. 101394 (2021): 1-12. https://doi.org/10.1016/j.csite.2021.101394

[14] Liaqat Ali, Xiaomin Liu, Bagh Ali, Saima Mujeed and Sohaib Abdal "Finite Element Simulation of Multi-Slip Effects on Unsteady MHD Bioconvective Micropolar Nanofluid Flow Over a Sheet with Solutal and Thermal Convective Boundary Conditions." *Coatings* 9, no.12 (2019): 1-22. https://doi.org/10.3390/coatings9120842

[15] Badr, H. M. "On the effect of flow direction on mixed convection from a horizontal cylinder." *International journal for numerical methods in fluids* 5, no. 1 (1985): 1-12. https://doi.org/10.1002/fld.1650050102

[16] Maïga, Sidi El Bécaye, Cong Tam Nguyen, Nicolas Galanis, and Gilles Roy. "Heat transfer behaviours of nanofluids in a uniformly heated tube." *Superlattices and Microstructures* 35, no. 3-6 (2004): 543-557. https://doi.org/10.1016/j.ijheatfluidflow.2005.02.004

[17] Merkin, J. H. "Mixed convection from a horizontal circular cylinder." *International Journal of Heat and Mass Transfer* 20, no. 1 (1977): 73-77. https://doi.org/10.1016/0017-9310(77)90086-2

[18] Tiwari, Raj Kamal, and Manab Kumar Das. "Heat transfer augmentation in a two-sided lid-driven differentiallyheated square cavity utilizing nanofluids." *International Journal of Heat and Mass Transfer* 50, no. 9-10 (2007):2002-2018. https://doi.org/10.1016/j.ijheatmasstransfer.2006.09.034

[19] Anwar, Ilyana, Norsarahaida Amin, and Ioan Pop. "Mixed convection boundary layer flow of a viscoelastic fluid over a horizontal circular cylinder." *International Journal of Non-Linear Mechanics* 43, no. 9 (2008): 814-821. https://doi.org/10.1016/j.ijnonlinmec.2008.04.008

[20] Roslinda Nazar, Amin, N. and Pop, I. " Mixed convection boundary-layer flow from a horizontal circular cylinder with a constant surface heat flux." *Heat and Mass Transfer* 40, (2004):219-227. https://doi.org/10.1007/s00231-002-0367-y