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Flat EEG Image Segmentation by Fuzzy Entropy-Based Multi-Level Thresholding

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Abstract Thresholding is a type of image segmentation that deals with the conversion of an image with many gray levels into another image with fewer gray levels. It classifies grayscale pixels into two categories which creates a binary image. However, the output image is not always satisfying due to several factors such as inherent image vagueness as uncertainty arises within the gray values of an image. In this paper, a multi-level image thresholding based on fuzzy entropy is applied on Flat Electroencephalography (Flat EEG) image. The outcomes are compared visually with global thresholding.

Keywords Flat EEG; thresholding; uncertainty; fuzzy set; entropy.

Mathematics Subject Classification 94D05

1 Introduction

Image segmentation in this age is very important and a good challenge to start the study for image analysis as well as interpreting images with higher levels such as in the medical field and identification of an object's shape. The main purpose for image segmentation is to divide the image into sections by comparing the intensity, color, tone and image textures [1-3]. There are many ways to segment images such as thresholding, clustering, edge detection and region extraction. Thresholding is a process of partitioning image into a foreground and background. It is a common preprocessing step in medical visual system and may be classified into global and local thresholding. Global thresholding involves a single threshold value for the entire image. Meanwhile, for local thresholding there are multiple threshold values in an image. Generally, thresholding may be done in two ways which are classical and fuzzy approaches. Classical thresholding is applied to image pixels that are considered precise and the regions of the image are well defined. On the other hand, fuzzy thresholding considered imprecise regions with unclear boundaries [4]. Fuzzy set is an extension of the classical set that was introduced by Zadeh [5] in 1965. Therefore, fuzzy approach is crucial in handling the uncertainties and imprecise information whereby membership degree is introduced to each pixel of the image.

2 Basic Concepts

In this section, the concepts of Fuzzy Topographic Topological Mapping (FTTM), Flat EEG and digital Flat EEG will be introduced.

2.1 Flat EEG

A non-invasive technique known as FTTM is used to solve neuromagnetic inverse problem. FTTM (see Figure 1) aims to accommodate static and simulated, experimental magnetoencephalography (MEG), and recorded electroencephalography (EEG) signals. It consists of four components namely Magnetic Contour Plane (MC), Base Magnetic Plane (BM), Fuzzy Magnetic Field (FM), and Topographic Magnetic Field (TM). EEG is a system that measures and records electrical activity of the brain such that it reads voltage differences on the head relative to a given point [6]. Meanwhile, Flat EEG is a method for mapping high dimensional signal, namely EEG into a low dimensional space [7]. The image of Flat EEG which is in grayscale form is obtained from digital Flat EEG by using fuzzy approach [8]. Zakaria [7] has formulated fundamental idea to describe an epileptic seizure as a system that represented by its motion or a dynamic physical process. Flat EEG is a method to flatten EEG signals (see Figure 2) from a high dimensional signal into a low dimensional signal. Thus, the EEG signals can be viewed on the Cartesian plane as depicted in Figure 3.

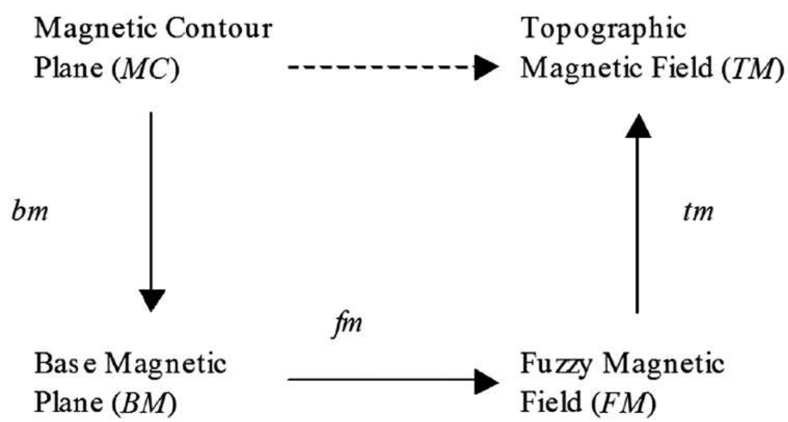


Figure 1: FTTM

The EEG coordinate system is defined as [7]

$$C_{EEG} = \left\{ \left((x, y, z), e_p \right) : x, y, z, e_p \in \Re \quad \text{and} \quad x^2 + y^2 + z^2 = r^2 \right\}, \tag{1}$$

where r is the radius of a patient head. The mapping of C_{EEG} to a plane is defined as follows:
$S_t : C_{EEG} \rightarrow MC$ such that

$$S_t \left((x, y, z), e_p \right) = \left(\frac{rx + iry}{r + z}, e_p \right) = \left(\frac{rx}{r + z}, \frac{ry}{r + z} \right)_{e_p(x,y,z)}, \tag{2}$$

where $MC = \left\{ \left((x, y)_0, e_p \right) : x, y, e_p \in \Re \right\}$ is the first component of FTTM. Both C_{EEG} and MC were designed and proven as 2-manifolds. Meanwhile S_t is designed to be a one-to-one function as well as being conformal.

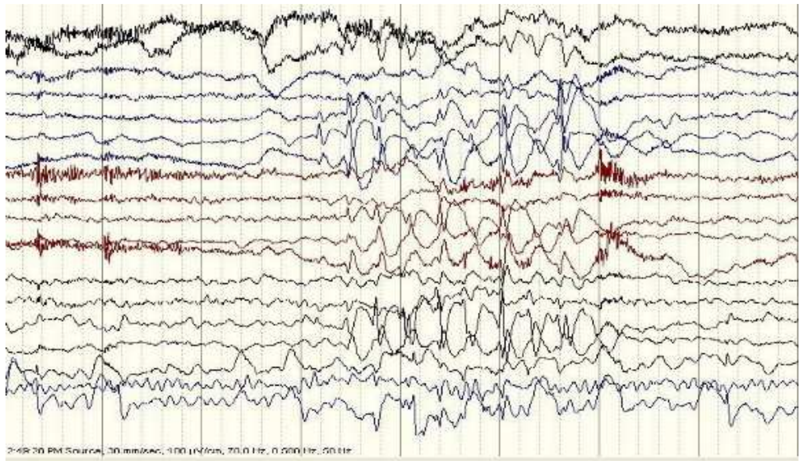


Figure 2: EEG Signal

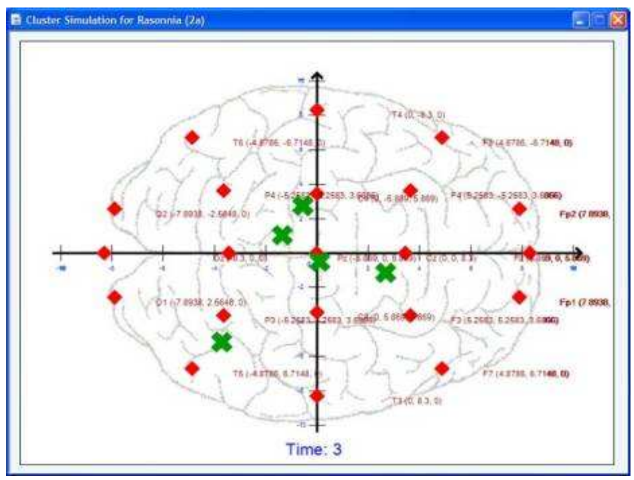


Figure 3: Flat EEG

2.2 Digital Flat EEG

Flat EEG that is obtained via the flattening method has to be digitized in order to convert it to a form that can be stored in a computer. It has been successfully digitized into grid by using Voronoi digitization. The digital Flat EEG is transformed into image by using fuzzy approach [8-10]. There are three main steps that are involved in the transformation of Flat EEG into image as follows:

(a) Flat EEG is divided into pixels (see Figure 4)

(b) The membership value for each pixel is determined in a cluster centre and the maximum operator of fuzzy set is implemented (see Figure 5)

(c) The membership value of pixel is transformed into image data.

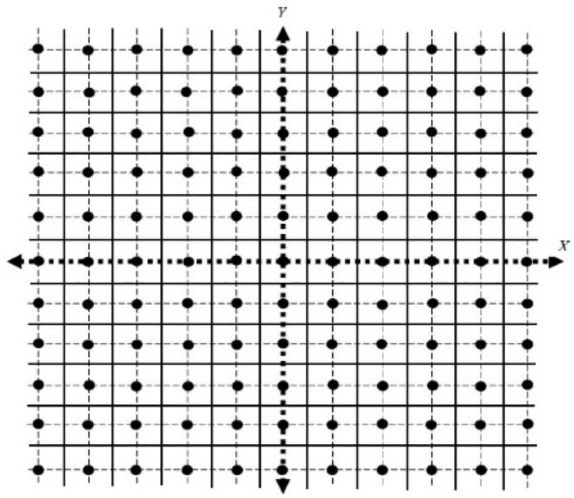


Figure 4: Flat EEG Pixels

3 Methodology

Fuzzy entropy is a measure of quantity of fuzzy information whereby vagueness is defined by a membership function [2-3].

Trapezoidal membership function is used to estimate the membership of m segmented regions. The following membership functions for m level thresholding are derived as in Equation (3) to Equation (5) [3] and [11]:

$$\mu_1(k) = \begin{cases} 1, & k \leq a_1 \\ \dfrac{k - c_1}{a_1 - c_1}, & a_1 \leq k \leq c_1 \\ 0, & k > c_1, \end{cases} \tag{3}$$

$$\mu_{m-1}(k) = \begin{cases} 0, & k \leq a_{m-2}, \\ \dfrac{k - a_{m-2}}{c_{m-2} - a_{m-2}}, & a_{m-2} < k \leq c_{m-2} \\ 1, & c_{m-2} < k \leq a_{m-2} \\ \dfrac{k - c_{m-1}}{a_{m-1} - c_{m-1}}, & a_{m-1} < k \leq c_{m-1} \\ 0, & k > c_{m-1}, \end{cases} \tag{4}$$

$$\mu_m(k) = \begin{cases} 1, & k \leq a_{m-1} \\ \dfrac{k - a_m}{c_m - a_n}, & a_{m-1} < k \leq c_{m-1} \\ 0, & k > c_{m-1}. \end{cases} \tag{5}$$

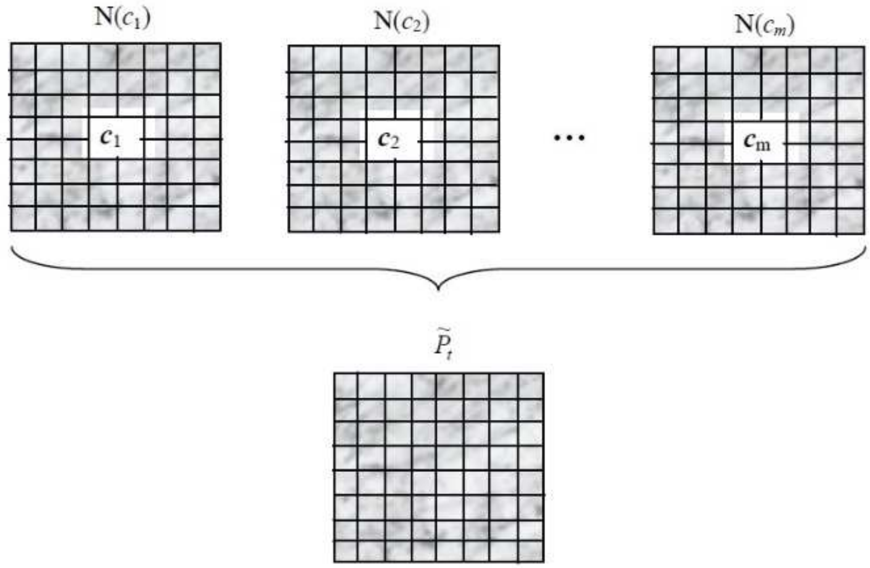


Figure 5: Fuzzy Neighborhood of Each Cluster Center c_j of a Flat EEG

The maximum fuzzy entropy for each segment of m-level segments are as follows

$$\begin{aligned} F_1 &= -\sum_{i=0}^{L-1} \frac{p_i * \mu_1(i)}{P_1} * \ln\left(\frac{p_i * \mu_1(i)}{P_1}\right), \\ F_2 &= -\sum_{i=0}^{L-1} \frac{p_i * \mu_2(i)}{P_2} * \ln\left(\frac{p_i * \mu_2(i)}{P_2}\right), \\ &\vdots \\ F_m &= -\sum_{i=0}^{L-1} \frac{p_i * \mu_m(i)}{P_m} * \ln\left(\frac{p_i * \mu_m(i)}{P_m}\right), \end{aligned} \tag{6}$$

whereby

$$P_1 = \sum_{i=0}^{L-1} p_i * \mu_1(i), \quad P_2 = \sum_{i=0}^{L-1} p_i * \mu_2(i), \ldots, P_m = \sum_{i=0}^{L-1} p_i * \mu_m(i).$$

By maximizing the total entropy, it gives optimum value of parameters as follows:

$$\varphi(a_1, c_1, \ldots, a_{n-1}, c_{n-1}) = \max(F_1(t) + F_2(t) + \ldots + F_n(t)). \tag{7}$$

The $(n-1)$ number of threshold values is obtained by using fuzzy parameters as follows:

$$t_1 = \frac{(a_1 + c_1)}{2}, t_1 = \frac{(a_1 + c_1)}{2}, \ldots, t_{n-1} = \frac{(a_{n-1} + c_{n-1})}{2}. \tag{8}$$

4 Results and Discussion

The aforementioned method is implemented on Flat EEG images during epileptic seizure at time $t = 3$ and $t = 4$. Table 1 shows the data of the cluster centers that are transformed into image form at every time as given in Figure 6 and Figure 7, respectively. For testing and analysis, two grayscale input images of Flat EEG are used. The input image in Figure 6 shows that there are two cluster centres whereas there are three cluster centers that can be observed in Figure 7. The brightness of the vague boundaries represents the strength of the electrical potential for Flat EEG image.

Table 2 shows the comparison of output images by using global thresholding (classical method) and fuzzy entropy (fuzzy method). There are two threshold values that are tested for global thresholding which are Th=100 and Th=150. The output images show that classical approach creates black and white images out of grayscale images. Those pixels whose value is above a given threshold will be set to white and other pixels to black.

For fuzzy entropy, 2^{nd} and 3^{rd} level thresholding is used to obtain the output images. It partitioned the images into different segments that shows different regions of the electrical potential strength. It shows that 3^{rd} level thresholding gives better and more segmented region compared to 2^{nd} level thresholding.

Figure 6: Input Image at $t = 3$

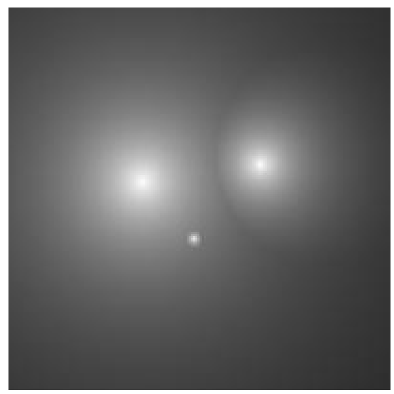
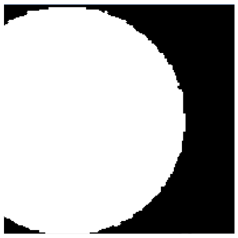
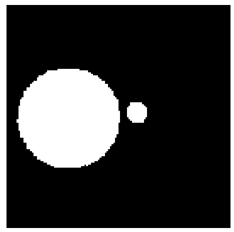
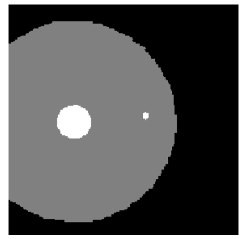
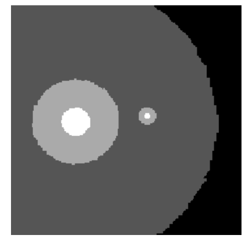
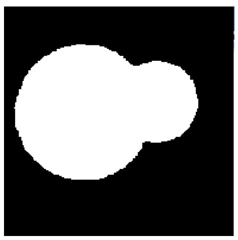
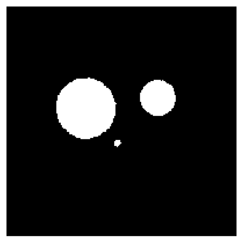
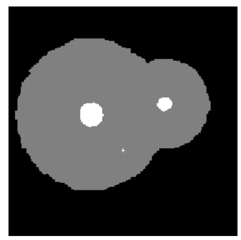
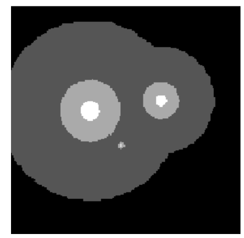


Figure 7: Input Image at $t = 4$

Table 1: Position and Electrical Potential of the Cluster Center at Two Different Time

Time (second)	**Position** x	**Position** y	**Electrical potential** (μV)
3	1.6918	−0.2614	40.0578
	−4.3997	0.2275	195.1227
4	−2.9509	−0.8576	112.0698
	3.2332	−1.7782	66.8746
	−0.2821	2.1374	12.4553

Table 2: Comparison of Segmented Images by Global Thresholding and Fuzzy Entropy

Time	**Global Thresholding**		**Fuzzy Entropy**	
	Th=100	Th=150	2-level	3-level
$t = 3$				
$t = 4$				

5 Conclusion

From the results, it shows that fuzzy entropy-based thresholding method for multi-level segmentation gives better results in case of visual comparison compared to global thresholding. For future work, other types of membership functions could be tested for better separation of the segmented regions.

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