

OPTIMAL CONTROL MODEL FOR OIL PALM BIOMASS INCORPORATING
FELLING RATE AND CARBON ABSORPTION

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ABSTRACT

The problem of optimal harvesting in controlling the outcome has often been considered, yet the optimal felling in the control of oil palm biomass has rarely been treated. The main objective of this thesis is to mathematically model the optimal control of oil palm biomass, incorporating felling rate and carbon absorption. Optimal control of oil palm biomass is mathematically a controlled system to optimize the amount of oil palm yield while absorbing carbon, controlled by state control (felling rate) and can be influenced by state variables. The first objective considered is the felling problem. A time-invariant linear quadratic optimal control model is developed for controlling the felling and harvesting rates of the oil palm biomass using the linear quadratic regulator approach. The optimal control solution is solved using Pontryagin's Minimum Principle. The second objective of this thesis is to develop a non-linear optimal control model of oil palm biomass incorporating carbon absorption, again assuming that the rate of felling can be controlled. The results indicate that felling rate affects the amount of biomass but not biomass growth rate. As the biomass growth rate is not affected by felling, separating the whole biomass into young and mature biomass and setting the biomass growth rate to be constant is the third objective accomplished in this thesis. Separating is considered vital since at the early age, no yield is produced and the absorption of carbon is quite low for young oil palm trees. A nonlinear system of ordinary differential equations is implemented for formulating the nonlinear optimal control models. To support the proposed models, a theoretical analysis of the positivity and boundedness of the solutions as well as their stability is carried out for each model. The equilibrium points are identified, and it is shown that a small perturbation of the felling rate near the equilibrium point decays exponentially, indicating that the model is stable with the existence of a felling rate. Model solutions are obtained using the control parametrization method and the Nelder–Mead method for both nonlinear models. The computational results show that the model is able to increase the amount of fruit yield and carbon absorption. This study has succeeded in developing models to control the oil palm biomass optimally in producing fruit whilst absorbing carbon from the atmosphere.

ABSTRAK

Masalah penuaian optimum dalam mengawal hasil tuaian sering dipertimbangkan, namun penebangan optimum dalam mengawal biojisim kelapa sawit jarang dikaji. Objektif utama tesis ini adalah untuk memodelkan secara matematik kawalan optimum bagi biojisim kelapa sawit, menggabungkan kadar penebangan dan penyerapan karbon. Kawalan optimum biojisim kelapa sawit adalah sistem yang dikawal secara matematik untuk mengoptimumkan jumlah hasil kelapa sawit sambil menyerap karbon, dikendalikan oleh kawalan negeri (kadar penebangan) dan boleh dipengaruhi oleh pemboleh ubah keadaan. Objektif pertama yang dipertimbangkan adalah masalah penebangan. Model pengendalian optimum kuadratik linear tak berubah masa dibangunkan untuk mengawal kadar penebangan dan penuaian biojisim kelapa sawit menggunakan pendekatan pengatur kuadratik linear. Penyelesaian kawalan optimum diselesaikan dengan menggunakan *Pontryagin's Minimum Principle*. Objektif kedua tesis ini adalah untuk membangunkan model kawalan optimum bukan linear biojisim kelapa sawit yang menggabungkan penyerapan karbon, sekali lagi mengandaikan bahawa kadar penebangan dapat dikawal. Keputusan menunjukkan bahawa kadar penebangan mempengaruhi jumlah biojisim tetapi bukan pertumbuhan biojisim. Oleh kerana pertumbuhan biojisim tidak terjejas oleh penebangan, memisahkan keseluruhan biojisim kepada biojisim muda dan matang dan menetapkan pertumbuhan biojisim menjadi malar adalah matlamat ketiga yang dicapai dalam tesis ini. Pemisahan dianggap penting kerana pada usia muda pokok, tiada hasil yang dihasilkan dan penyerapan karbon agak rendah. Satu sistem tidak linear bagi persamaan pembezaan biasa telah dilaksanakan untuk merumuskan model kawalan optimum bukan linear. Untuk menyokong model-model yang dicadangkan, analisis teori mengenai positifiti dan sempadan serta kestabilan telah dijalankan untuk setiap model. Titik keseimbangan dikenal pasti, dan ditunjukkan bahawa gangguan kecil kadar penebangan berhampiran titik keseimbangan merosot secara eksponen, menunjukkan bahawa modelnya stabil dengan adanya kadar penebangan. Penyelesaian model diperoleh menggunakan kaedah kawalan pemparameteran dan kaedah *Nelder-Mead* untuk kedua-dua model tidak linear. Keputusan pengiraan menunjukkan bahawa model dicadangkan dapat meningkatkan jumlah hasil buah dan penyerapan karbon. Kajian ini berjaya membangunkan model untuk mengawal biojisim kelapa sawit secara optimum dalam menghasilkan buah-buahan sambil menyerap karbon dari atmosfera.

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LIST OF ABBREVIATIONS

CDM	-	Clean Development Mechanism
CO ₂	-	Carbon Dioxide
CH ₄	-	Methane
CPO	-	Crude Palm Oil
FFB	-	Fresh Fruit Bunch
DDE	-	Delay Differential Equation
GHG	-	Greenhouse Gases
HFC	-	Hydrofluorocarbons
LQR	-	Linear Quadratic Regulator
MAS	-	Malaysian Airlines System
MPOB	-	Malaysian Palm Oil Board
MPOC	-	Malaysian Palm Oil Council
MyR	-	Malaysian Ringgit
N ₂ O	-	Nitrous Oxide
NPP	-	Net Primary Productivity
ODE	-	Ordinary Differential Equation
PFC	-	Perfluorocarbons
PDE	-	Partial Differential Equation
REDD	-	Reducing Emissions from Deforestation and Degradation
SF ₆	-	Sulphur Hexafluoride
UTM	-	Universiti Teknologi Malaysia
VCOS	-	Voluntary Carbon Offset Scheme

LIST OF SYMBOLS

α	-	The weighted carbon absorption
$\mathbf{x}(t)$	-	Vector of state variables
$\mathbf{u}(t)$	-	Vector of control variables
A	-	A time-invariant state matrix of an LQR model
B	-	A time-invariant control matrix of an LQR model
$\dot{\lambda}$	-	Costate equation
\dot{x}	-	Differentiation of state variables with respect to t , $(\frac{dx}{dt})$
\dot{u}	-	Differentiation of control variables with respect to t , $(\frac{du}{dt})$
d	-	Natural depletion rate
η	-	The transition rate from young to mature palm oil trees
γ	-	The rate of fruit production
λ	-	Scalar multiplier
ω_1	-	The weighted parameter of carbon absorption
ω_2	-	The weighted parameter of fruit production
\mathbb{R}	-	The set of real numbers
\mathbb{R}^n	-	Real coordinate space of n dimensions
ρ	-	Parameter of natural mortality rate
τ	-	The maturation delay
τ^*	-	Parameter of felling on the growth rate
$f(t)$	-	The felling of mature palm oil trees
H	-	The Hamiltonian function
J	-	The performance index of the LQR model
k	-	Carrying capacity
n	-	Number of samples
P	-	Scalar multiplier $\lambda(t) = Px(t)$, $P(t_f) = S$
Q	-	Symmetric state weighting matrix for the LQR model
R	-	Symmetric control weighting matrix for the LQR model

r	-	The intrinsic growth rate of the young / immature trees
$r_0(t)$	-	Maximum growth rate
S	-	Symmetric weighting matrices
t_0	-	The initial value of time
t_f	-	The final time
m	-	The natural depletion rate of mature palm oil trees
h	-	The harvesting rate
X	-	Indicator function
ξ_1	-	Parameter to be estimated
ξ_2	-	Parameter to be estimated
ξ_3	-	Parameter to be estimated
ξ_4	-	Parameter to be estimated

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CHAPTER 1

INTRODUCTION

The demand for the forest products is reported to exceed the existing supply, which will make the carbon stock deficient for future generations. Re-planting a forest for future use is being actively carried out, but takes a very long time to generate a new forest products. Apart from forest products, the forest ecosystem is an important mechanism of the terrestrial carbon cycle [1], it acts as a carbon reservoir that stores carbon in numerous components, such as biomass carbon and soil carbon [2]. Carbon sequestration is a process of capturing carbon dioxide (CO₂) or removing carbon from the atmosphere and depositing it in a long-term forest reservoir. However, forests worldwide have been reported to be decreasing due to deforestation at a net rate of about 9.4 million hectares per year. This unabated deforestation mechanically increases the volume of greenhouse gasses (GHG), which results in the release of 1.6–1.7 Petagram (Pg) of carbon per year into the atmosphere [2], where one Petagram is equivalent to one billion tonnes per year.

Deforestation and the global warming phenomenon have given rise to great concern and numerous research on the importance of carbon sequestration and its values for the ecosystem [3]. There are few studies in forest management that consider carbon sequestration in their model system. Instead, studies have focused on developing and improving forest models, such as finding optimal revenues, reducing the cost function, maintaining forest productivity, improving ecological sustainability, and other related forest issues [1]. To date, researchers have reported that uncontrolled and unmanaged forest strategies by irresponsible parties have reduced the production of forest values, which also affects the carbon sequestration cycle [4, 5, 6, 7].

As deforestation issues become global, the attention of researchers has shifted to the search for the initial cause of this problem. It is claimed that trees should stay noticeably longer on the ground to absorb more CO₂ by reducing harvesting activities

[8] or by postponing harvesting to the age of biological maturity, which may result in the formation of a large carbon sink [9]. Thornley and Cannell [10] have reported that maximizing timber harvest and carbon sequestration are inversely related, and not complementary to each other. However, Gray and Whittier [1] argue that regular harvesting has little impact on carbon sequestration. At the same time, the economic and business point of view do not support the idea of expanding the timber age, and suggests regular forest harvesting [11].

1.1 Motivation

Various alternatives are being investigated to find a potential forest replacement, such as using an agricultural plantation, that still allows the carbon to be sequestered in biomass and sustain the ecosystems. However, agricultural plantation should not replace natural forests and impact the livelihood of the forest [12], but should balance the natural ecosystem and allow for the protection of a well-conserved forest by producing new sources with a short rotation. The advantages and disadvantages of forest is summarized in Figure 1.1.

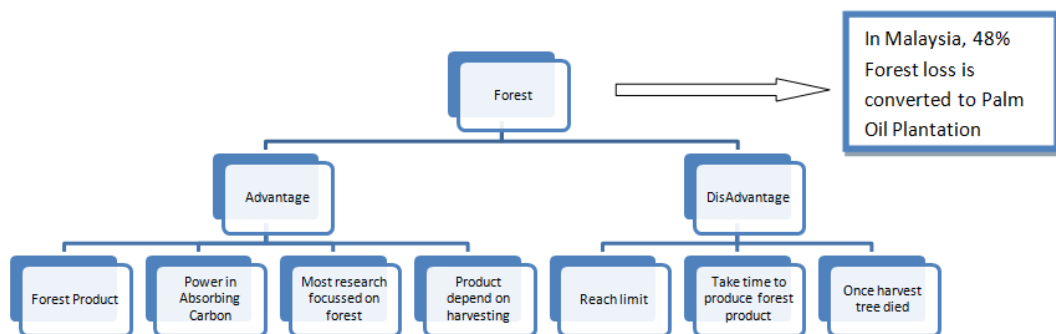


Figure 1.1 Advantages and disadvantages of forest

Oil palm is one of the world’s edible oil sources. This kind of plantation could be one of the alternatives to forests in sequestering carbon. Generally, such a plantation needs proper management to maintain its fruit production and sustain the environment. In order to produce more oil, the larger land is needed for the expansion of the production of oil palm. One of the issues faced by such a plantation is clearing the forest to establish an area for the oil palm plantation. The more oil palm trees are

planted, the more edible oil can be produced. However, the clearing and felling of an area will release more carbon than will be sequestered by the growing palm trees [13]. Yet, as stated by Wahab *et al.* [14] and Khamis *et al.* [15], the oil palm tree is efficient at sequestering carbon. Although not as good as a virgin forest, it has a very high productivity. At the same time, the global potential of the oil palm is well known as a major source of raw material for the world's food, oleochemical, and biofuel industries.

Managing oil palm plantation to reach the maximum carbon absorption along with maximum fruit production is challenging. However, it has a long lifespan and able to absorb lots of carbon from the atmosphere even without producing yield. The ability of oil palm biomass to absorb carbon from the atmosphere is considered significant yet rarely thought of. The biomass is the mass of living biological organisms in a given area or ecosystem at a given time such as plants, animal or microorganisms [16]. The biomass or volume is referred to as the biomass or volume of living trees with a stem diameter at breast height [17]. Biomass is any organic matter that can be used as an energy source. Wood, crops yard and animal waste are examples of biomass [18]. Palm oil is the oil that produced by oil palm trees [19]. In the context of this thesis, oil palm biomass is defined as the mass of standing oil palm trees in a given area at a given time. Since this plantation takes a few years to be fully grown before producing fresh fruit, the fundamental challenges to be faced are the felling and replacement of the oil palm trees, affecting the amount of oil palm biomass in producing fresh fruit and absorbing carbon.

In Malaysia, an estimated 651,757 hectares of forest loss between 1988 and 2012 had been converted to plantations agricultural by 2014 [20] and nearly half (48%) of the natural forest loss area represents conversion to tree plantations primarily oil palm plantation [21]. On top of that, due to the fact that the expansion of area in Malaysia has reached its limits, it is impossible to increase the biomass productivity together with sequestering the carbon. The advantages and disadvantages of oil palm is summarized in Figure 1.2.

The management of an oil palm involves land preparation, planting new trees, harvesting, felling, and regular operation and maintenance [19]. The felling and regular

harvesting are considered as the main aspects to improve and sustain palm oil production [22, 23]. The beauty of this oil palm biomass is having a lifespan of around 25 to 30 years, with continuous felling management [14, 15]. The effect of felling and harvesting in maintaining the plantation, has a cost that has increased tremendously year by year as illustrated in Figure 1.3 [24].

Less mathematical modelling has been developed for oil palm plantations than for forest plantations. It can be observed that in order to achieve optimal production in an oil palm plantation, re-planting, harvesting, and felling activities are needed. All these activities are important to smooth the plantation system [25].

The present thesis attempts to show that oil palm biomass is capable of being one of the potential plantations in absorbing carbon from the atmosphere while continuously maintaining the biomass in producing fruit yield. Thus, by assuming that the felling rate can be controlled, the intention of this thesis is to develop optimal control models for oil palm biomass incorporating carbon absorption. As agreed by several researchers, sequestration of carbon is important not only for environmental benefits but has a big potential for commercial benefit [26]. Even though clearing the forest to establish the plantation releases more carbon than will be sequestered by the growing oil palms, the dynamic relationship will definitely overcome this and have positive implications for economic sustainability. Considering felling as a control state, oil palm biomass may increase the carbon sequestration process and optimize oil palm production.

1.2 Background of the problem

There are many comprehensive models of forest that have been developed to handle logging or felling problems. Examples of such models were developed by Sohngen *et al.* [27], Hritonenko *et al.* [28], Gaoue *et al.* [29], and Chaudhary *et al.* [30], in which these studies used ordinary differential equation. These models however, focused on maximizing the yield of harvesting from the forest and assume that the biomass of forest tree is available as it is replaced by natural plantation. The availability of agricultural plantation depends on the activities of felling old trees and re-planting new trees [31, 25]. Gaoue *et al.* [29] adopted a logistic growth model by adding non-lethal, lethal harvesting (felling) and a variable τ to their model, which

represents the average lifespan. Gaoue *et al.* [29] suggested that in order to maintain a population, the optimal non-lethal and lethal felling should not exceed 40% of the total population density, and suggested the life history as an indicator, not the lifespan, for harvesting and felling. However, Gaoue *et al.* [29] disregard how lethal harvesting (felling) may affect plant productivity and did not consider the potential of a plant to absorb carbon from the atmosphere for sustaining the environment. These models of [27, 28, 29, 30], can be applied to oil palm plantation due to their similar characteristics such as the natural growth of the population, harvest to obtain yield, natural depletion and lifespan. However, oil palm plantation needs unique model due to its own lifespan, requiring regular harvesting and felling, as well as different carrying capacity of each area.

Previously, a mathematical model to predict the specific growth rate and biomass concentration of an oil palm plantation has been formulated by Khamis *et al.* [15]. He reported that based on statistical testing and goodness of fit, the best model is the logistic model, followed by the Gompertz model, Morgan–Mercer–Flodin, Chapman–Richard (with initial stage), and log-logistic growth models [15]. Studies on quantifying the carbon stock of a tree biomass have often been urged in oil palm plantation [32, 33, 34, 35, 36]. These researchers believed that the amount of carbon in the biomass increased linearly with time. In agreement with this, Kongsager *et al.* [35] quantified the carbon sequestration potential of tree crop plantations for cocoa, oil palm, rubber and orange. While Sanquetta *et al.* [32] calculated the values of carbon in the oil palm biomass, which lies within the empirical range from 25 Metagramme (Mg) C ha⁻¹ to over 50 Metagramme (Mg) C ha⁻¹ towards the end of the plantation [32]. A Metagramme (Mg) Carbon (C) ha⁻¹ is equal to a tonne Carbon (C) ha⁻¹. The estimated carbon content of the oil palm trees is calculated based on the equation of Khalid *et al.* [37]. Thus Khalid *et al.* [37] and Kongsager *et al.* [35] have proven that the amount of carbon is proportional to the amount of biomass in the plantation. However, these studies only concentrated on quantifying the carbon stock of standing biomass and ignored the effect of uncontrolled felling towards the plantation and carbon sequestration. Other methods include artificial neural network (ANN), fuzzy logic, genetic algorithm (GA), linear programming and regression are also being applied to solve the carbon sequestration problem [38, 39, 40]. However, these studies are applied to virgin forest and less focused on mathematical model. Later, a comprehensive model of the optimal control of tree

biomass was considered by Chaudary *et al.* [30], who proposed an age-structured forestry biomass to distinguish between premature and mature trees. However, this model works well for the forestry biomass problem only and the variables are not suitable for oil palm biomass.

Thus, there is a gap in the literature, which pay less focus on felling trees that may affect productivity. Most mathematical models developed are for the forest but not crop plantation such as oil palm plantation. On top of that, previous literature discussed the goodness of oil palm biomass in absorbing carbon but less mathematical models have been developed to show that this plant is able to effectively absorb carbon. This has motivates the researcher to mathematically model the oil palm plantation problem incorporating carbon absorption via controlling the felling activity namely on how to increase the fruit production fruit while optimally absorbing carbon from the atmosphere.

1.3 Statement of the problem

Most existing oil palm models discussed the importance of harvesting in producing fruit yield but not the importance of felling in maintaining the biomass. Since the activity of felling could reduce or increase the lifespan of unproductive palm trees, there have been concerns about the issues of uncontrolled felling activity. The most relevant models to be referred to are forest models due to forest model considering felling in its model equation. The forest model are Chaudary *et al.* [30] and Gaoue *et al.* [29] models. However, the state variables in a forest model are less suitable to be applied for an oil palm plantation. The unsuitability of the forest model is due to the trees are totally removed from the ground for harvesting. While for oil palm, the harvesting only collects fruit and oil palm trees remains standing on the ground. These two models is considered in formulating mathematical model for oil palm biomass and added a new variable of carbon absorption is added and felling rate is used as the control state to the model equation.

On top of this, there are few available optimal formulation relevant to the oil palm industry problem, yet most of them require mathematical considerations to achieve an optimal solution [3]. There is a gap in the literature, to the filling of which this

thesis contributes, namely, how to increase oil palm biomass in producing fruit while optimally absorbing carbon from the atmosphere. The present thesis has identified a weakness of uncontrolled felling rate, which will result in a bleak future with a long term deficit in oil palm yield. To the best of researcher's knowledge, in the case of optimal control model, past studies have only concentrated on harvesting in improving the oil palm's productivity, ignored felling activities that may reduce the amount of biomass and did not touch on the potential of oil palm biomass for environmental benefit. Thus, this thesis intends to mathematically model the oil palm biomass, incorporating carbon absorption, via the optimal control of the felling rate. To verify these models, positivity, boundedness and stability of each models are established.

1.4 Objectives of the research

This thesis attempts to achieve a few objectives regarding the incorporation of carbon absorption in an oil palm plantation model. As per the above issues, the objectives of this thesis are as follows:

1. To propose a mathematical model of control problem for felling and harvesting rate using time-invariant linear quadratic optimal control approaches.
2. To formulate an optimal control model for oil palm biomass in maximizing the carbon absorption and fresh fruit bunch production via optimal control of the felling rate.
3. To examine the positivity of the solution and demonstrate the stability of the mathematical model.
4. To optimize the state function of the optimal control model.

1.5 Scope of the research

The optimal control theory for differential equations provides a path, that chooses a time-dependent management control to achieve a desirable solution. Generally, for such research, a model involving a system of differential equations is created based on logistic growth model to investigate the effect on the palm oil yield and carbon absorption rate from controlled felling rate.

This thesis is divided into three major parts. Firstly, the formulation of the model for the control of the felling and harvesting rate using linear Quadratic Regulator (LQR) approaches as a starting point to create a basic model. Then an optimal control model for oil palm biomass will be formulated and lastly the state function of the optimal control model of oil palm biomass incorporating carbon absorption will be developed.

A set of secondary data is used to estimate the parameters of the model. The source of data is taken from MPOB which covers the whole Peninsular Malaysia which falls under its jurisdiction. The data are the amount of oil palm biomass in tonnes per hectares, the amount of fruit yield in tonnes per hectares and the amount of carbon absorb in kilogram per hectares. This data was established from experimental study, conducted by the research department of MPOB. The determination of the control parameter depends on parameter estimation and simulation process. By controlling the parameters in the state and control variables, dynamics of the solution can be determined. As the edible oil is produced from the yield of fresh fruit bunch, the variable of fruit production in producing oil is used throughout this thesis.

To improve the performance of the formulated model, analysis of its positivity, boundedness and stability are carried out. The primary aim of this analysis is to validate the model. Even though this research incorporates carbon absorption in the oil palm model, the details of the carbon components released into the air, which consists of both that above the ground and that from the root system below the ground, is beyond the scope of this thesis.

1.6 Significance of this thesis

The significance of this thesis is primarily for the nation and the body of knowledge. Its contributions are as follows.

1.6.1 Contribution of the mathematical formulation

The existing literature seldom has an optimal control formulation for the oil palm industry problem. Therefore, the product of this thesis would be new formulation of models that can optimize the benefits of an oil palm plantation by controlling the

felling activities over the years. The contribution of this thesis is mainly in formulating the state equations of the biomass, considering the growth, felling, re-planting, and maturity of the biomass. The state equation is controlled by the control variable of the felling rate, that results in improving the productivity of the oil palm plantation. The optimal control model will be helpful in the formulation of new studies and is key to future developments that will lead to and contribute to a new body of knowledge.

1.6.2 Contribution of the theoretical analysis

The study of the model includes positivity solution, boundedness and stability analyses to ensure that the newly formulated models are efficient in representing the operation of an oil palm plantation.

1.6.3 Contribution in sustaining the environment

This research highlights the potential of the oil palm industry to absorb carbon and contribute to the society. Indeed, it is uncommon to find an optimal formulation for carbon absorption in tree crop plantations. This mathematical model acts as an oil palm biomass green potential model and helps in terms of increasing the value of the biomass plantation in the absorption of carbon in the future. The result of the model have proved that oil palm biomass is able to achieve high rates of carbon absorption even without adding or expanding any new area of the plantation. Furthermore, the model can give insight ideas not just for any agricultural plantation but also for other biological living.

1.7 Thesis organization

This thesis is organized as follows. The first chapter extensively describes the issue of felling activities that may affect the productivity of a plantation and detract from environmental aspects, for example, by increasing the deforestation rate. This issue has become the main concern of this research. This chapter starts with the motivation from the mathematical modelling to address these issues and it continues with the background for the thesis, the problem statement, the objectives of the research, its significance as well as its scope.

Chapter 2 reviews the previous literature related to the mathematical modelling of agricultural plantations. It reviews the biological processes related to the logistic growth model and the methodology used in the literature to obtain the relevant objectives. The discussion of the literature is vital to determine the optimal solution for the research problem.

In Chapter 3, the proposed methodology is discussed in detail. This chapter covers the development of the model, identifies the best parameter estimation, and the method used for solving the optimal control problem. At the end of this chapter, there is an explanation of the methodology used to measure the stability of the model for model validation.

Chapter 4 presents the issue of felling problems in oil palm plantations and proposes a linear quadratic regulator to solve the model, which is a system of ODEs. The model has two state variables: the biomass and the yield of fruit. Then the solution procedure of the resulting control problem is discussed. Finally, the chapter is summarized with some concluding remarks.

Chapter 5 presents the carbon absorption control model for an oil palm biomass. The objective of this chapter is to maximize oil palm fruit production while absorbing carbon. As this research assumed that the rate of felling can be controlled, a model is constructed based on the ODE model. The model is known as Model 1, employing two state variables: biomass and growth of oil palm plantation. As fruit yield and carbon absorption are proportionate with the amount of oil palm biomass, thus increase of biomass will indirectly increase the amount of fruit yield and carbon absorption. Here, positivity, boundedness and the stability of the model are established to ensure the fitness of the solution. Then finally, a result discussion is given.

Chapter 6 presents the optimal carbon absorption control model of an oil palm plantation, known as Model 2. This chapter aims to optimize the state function of the optimal control model by optimally controlling the felling rate. Before the result is simulated, positivity, boundedness, and stability are established to ensure that the model is suitable in presenting the oil palm problem. Finally, the result is discussed.

Chapter 7 summarizes the study with a conclusion, stressing the contributions of the models as well as some suggestions for future development. The flow diagram of the conceptual link among the chapters is given in Figure 1.4.

REFERENCES

1. Gray, A. N. and Whittier, T. R. Carbon stocks and changes on Pacific Northwest national forests and the role of disturbance, management, and growth. *Forest Ecology and management*, 2014. 328: 167–178.
2. Lal, R. Forest soils and carbon sequestration. *Forest ecology and management*, 2005. 220(1-3): 242–258.
3. Gan, P. Y. and Li, Z. D. Econometric study on Malaysia s palm oil position in the world market to 2035. *Renewable and Sustainable Energy Reviews*, 2014. 39: 740–747.
4. Li, Y., Härdtle, W., Bruelheide, H., Nadrowski, K., Scholten, T., von Wehrden, H. and von Oheimb, G. Site and neighborhood effects on growth of tree saplings in subtropical plantations (China). *Forest Ecology and Management*, 2014. 327: 118–127.
5. Sanders, J. R. and Grabosky, J. C. 20 years later: Does reduced soil area change overall tree growth? *Urban Forestry & Urban Greening*, 2014. 13(2): 295–303.
6. Castagneri, D., Nola, P., Motta, R. and Carrer, M. Summer climate variability over the last 250 years differently affected tree species radial growth in a mesic Fagus–Abies–Picea old-growth forest. *Forest Ecology and Management*, 2014. 320: 21–29.
7. Başkent, E. Z., Keleş, S., Kadioğulları, A. İ. and Bingöl, Ö. Quantifying the effects of forest management strategies on the production of forest values: timber, carbon, oxygen, water, and soil. *Environmental Modeling & Assessment*, 2011. 16(2): 145–152.
8. Backéus, S., Wikström, P. and Lämås, T. Modeling carbon sequestration and timber production in a regional case study. 2006.
9. Alexandrov, G. A. Carbon stock growth in a forest stand: the power of age. *Carbon Balance and Management*, 2007. 2(1): 4.

10. Thornley, J. and Cannell, M. Managing forests for wood yield and carbon storage: a theoretical study. *Tree physiology*, 2000. 20(7): 477–484.
11. Köthke, M. and Dieter, M. Effects of carbon sequestration rewards on forest management—an empirical application of adjusted Faustmann formulae. *Forest policy and Economics*, 2010. 12(8): 589–597.
12. Wu, A. The role forest plantations play in carbon cycle and climate change. 2014.
13. Germer, J. and Sauerborn, J. Estimation of the impact of oil palm plantation establishment on greenhouse gas balance. *Environment, Development and Sustainability*, 2008. 10(6): 697–716.
14. Wahab, R., Samsi, H., Mohamed, A. and Sulaiman, O. Utilization potential of 30 year-old oil palm trunks laminated veneer lumbers for non-structural purposes. *Journal of Sustainable Development*, 2008. 1(3): 109–113.
15. Azme, K., Ismail, Z., Haron, K. and Ahmad, T. Nonlinear growth models for modeling oil palm yield growth. *Journal of Mathematical Statistics*, 2005. 1: 225–233.
16. Nagel, B., Dellweg, H. and Gierasch, L. Glossary for chemists of terms used in biotechnology (IUPAC Recommendations 1992). *Pure and Applied Chemistry*, 1992. 64(1): 143–168.
17. Petersson, H., Holm, S., Ståhl, G., Alger, D., Fridman, J., Lehtonen, A., Lundström, A. and Mäkipää, R. Individual tree biomass equations or biomass expansion factors for assessment of carbon stock changes in living biomass—A comparative study. *Forest Ecology and Management*, 2012. 270: 78–84.
18. Is biomass renewable. What is Biomass?, 2017. URL <http://www.aesenergy.net/biomass-energy.html>.
19. Overview of the Malaysian Oil Palm Industry 2014, 2018. URL http://palmoilis.mpob.gov.my/V4/wp-content/uploads/2020/03/Overview_of_Industry_2018.pdf.
20. Shevade, V. S. and Loboda, T. V. Oil palm plantations in Peninsular Malaysia: Determinants and constraints on expansion. *PloS one*, 2019. 14(2): e0210628.

21. Shevade, V. S., Potapov, P. V., Harris, N. L. and Loboda, T. V. Expansion of industrial plantations continues to threaten Malayan tiger habitat. *Remote Sensing*, 2017. 9(7): 747.
22. Idayu, I., Supriyanto, E. *et al.* Oil palm plantations management effects on productivity Fresh Fruit Bunch (FFB). *APCBEE procedia*, 2014. 8: 282–286.
23. Azhar, B., Saadun, N., Puan, C. L., Kamarudin, N., Aziz, N., Nurhidayu, S. and Fischer, J. Promoting landscape heterogeneity to improve the biodiversity benefits of certified palm oil production: Evidence from Peninsular Malaysia. *Global Ecology and Conservation*, 2015. 3: 553–561.
24. Cramb, R. A. and Ferraro, D. Custom and capital: A financial appraisal of alternative arrangements for large-scale oil palm development on customary land in Sarawak, Malaysia. *Malaysian Journal of Economic Studies*, 2017. 49(1): 49–69.
25. Alam, A., Er, A. and Begum, H. Malaysian oil palm industry: prospect and problem. *Journal of Food, Agriculture & Environment*, 2015. 13(2): 143–148.
26. Paul, K., Jacobsen, K., Koul, V., Leppert, P. and Smith, J. Predicting growth and sequestration of carbon by plantations growing in regions of low-rainfall in southern Australia. *Forest ecology and management*, 2008. 254(2): 205–216.
27. Sohngen, B. and Mendelsohn, R. J. An optimal control model of forest carbon sequestration. *American Journal of Agricultural Economics*, 2003. 85(2): 448–457.
28. Hritonenko, N., Yatsenko, R., Yand Goetz and Xabadia, A. Maximum principle for a size-structured model of forest and carbon sequestration management. *Applied Mathematics Letters*, 2008. 21(10): 1090–1094.
29. Gaoue, O., Jiang, J., Ding, W., Agosto, F. and Lenhart, S. Optimal harvesting strategies for timber and non-timber forest products in tropical ecosystems. *Theoretical Ecology*, 2016. 9(3): 287–297.
30. Chaudhary, M., Dhar, J. and Misra, O. A mathematical model for the conservation of forestry biomass with an alternative resource for

- industrialization: A modified Leslie Gower interaction. *Modeling Earth Systems and Environment*, 2015. 1(4): 43.
31. Basiron, Y. Palm oil production through sustainable plantations. *European Journal of Lipid Science and Technology*, 2007. 109(4): 289–295.
 32. Sanquetta, C., Ellico Netto, S., Dalla Corte, A., Rodrigues, A., Behlin, A. and Sanquetta, M. Quantifying biomass and carbon stocks in oil palm (*Elaeis guineensis* Jacq.) in Northeastern Brazil. *African Journal of Agricultural Research*, 2015. 10: 4067–4075.
 33. Corley, R. and Tinker, P. *The Oil Palm*. Oxford: Blackwell. 2013.
 34. Aholoukpé, H., Dubos, B., Flori, D. P., A., Amadji, G., Chotte, J.-L. and Blavet, D. Estimating aboveground biomass of oil palm: Allometric equations for estimating frond biomass. *Forest Ecology and Management*, 2013. 292: 122–129.
 35. Kongsager, R., Napier, J. and Mertz, O. The carbon sequestration potential of tree crop plantations. *Mitigation and Adaptation Strategies for Global Change*, 2013. 18(8): 1197–1213.
 36. Kho, L. and Jepsen, M. Carbon stock of oil palm plantations and tropical forests in Malaysia: A review. *Singapore Journal of Tropical Geography*, 2015. 36(2): 249–266.
 37. Khalid, H., Zin, Z. and Anderson, J. Quantification of oil palm biomass and nutrient value in a mature plantation. I. Above-ground biomass. *Journal of Oil Palm Research*, 1999. 11(1): 23–32.
 38. Ahmad, A., Azid, I., Yusof, A. and Seetharamu, K. Emission control in palm oil mills using artificial neural network and genetic algorithm. *Computers & Chemical Engineering*, 2004. 28(12): 2709–2715.
 39. Sipöcz, N., Tobiesen, F. and Assadi, M. The use of artificial neural network models for CO₂ capture plants. *Applied energy*, 2011. 88(7): 2368–2376.
 40. Ubando, A., Culaba, A., Aviso, K., Ng, D. and Tan, R. Fuzzy mixed-integer linear programming model for optimizing a multi-functional bioenergy system with biochar production for negative carbon emissions. *Clean Technologies and Environmental Policy*, 2014. 16(8): 1537–1549.

41. Varkkey, H., Tyson, A. and Choiruzzad, S. Palm oil intensification and expansion in Indonesia and Malaysia: Environmental and socio-political factors influencing policy. *Forest Policy and Economics*, 2018. 92: 148–159.
42. Butler, R., Koh, L. and Ghazoul, J. REDD in the red: Palm oil could undermine carbon payment schemes. *Conservation letters*, 2009. 2(2): 67–73.
43. Ovando, P., Beguería, S. and Campos, P. Carbon sequestration or water yield? The effect of payments for ecosystem services on forest management decisions in Mediterranean forests. *Water Resources and Economics*, 2019. 28: 100–119.
44. Craven, C. The Honduran palm oil industry: Employing lessons from Malaysia in the search for economically and environmentally sustainable energy solutions. *Energy Policy*, 2011. 39(11): 6943–6950.
45. Henson, I., Ruiz, R. and Romero, H. The greenhouse gas balance of the oil palm industry in Colombia: A preliminary analysis. I. Carbon sequestration and carbon offsets. *Agronomía Colombiana, Univ. Nacional de Colombia*, 2012. 30(3): 359–369.
46. Ismail, A. and Mamat, M. The optimal age of oil palm replanting. *Oil Palm Industry Economic Journal*, 2002. 2(1): 11–18.
47. Dumrongsiri, A. Mathematical model for production, logistics and plant capacity planning of oil palm bunches. *Ist Mae Fah Luang Univ. Intern. Conf.*, 2012.
48. Wilcove, D. and Koh, L. Addressing the threats to biodiversity from oil-palm agriculture. *Biodiversity and conservation*, 2010. 19(4): 999–1007.
49. Banitalebi, A., Aziz, M., Aziz, Z. and Nasir, N. Modelling and optimization for palm oil plantation management. *AIP Conference Proceedings*, 2016. 1750(1): 030046.
50. Patthanaisaranukool, W., Polprasert, C. and Englande Jr, A. Potential reduction of carbon emissions from crude palm oil production based on energy and carbon balances. *Applied Energy*, 2013. 102: 710–717.

51. Jones, D. and O'hara, K. Carbon density in managed coast redwood stands: Implications for forest carbon estimation. *Forestry*, 2011. 85(1): 99–110.
52. Golub, A., Hertel, T., Lee, H.-L., Rose, S. and Sohngen, B. The opportunity cost of land use and the global potential for greenhouse gas mitigation in agriculture and forestry. *Resource and Energy Economics*, 2009. 31(4): 299–319.
53. Huang, L., Liu, J., Shao, Q. and Xu, X. Carbon sequestration by forestation across China: Past, present, and future. *Renewable and Sustainable Energy Reviews*, 2012. 16(2): 1291–1299.
54. Jiao, Y., Ren, H.-E. and Dong, B.-Z. *Advances in Computer Science, Intelligent System and Environment*. 2011.
55. Maser, O., Garza-Caligaris, J., Kanninen, M., Karjalainen, T., Liski, J., Nabuurs, G., Pussinen, A., De Jong, B. and Mohren, G. Modeling carbon sequestration in afforestation, agroforestry and forest management projects: The CO2FIX V.2 approach. *Ecological Modelling*, 2003. 164(2): 177–199.
56. Kula, E. and Gunalay, Y. Carbon sequestration, optimum forest rotation and their environmental impact. *Environmental Impact Assessment Review*, 2012. 37: 18–22.
57. Lorenz, K., and Lal, R. *Carbon Sequestration in Forest Ecosystems*. 2009.
58. Yee, L., Devi, S., and Khin, E. Kyoto protocol and social accounting implication on global-warming in Malaysia: An action research approach. *African Journal of Agricultural Research*, 2011. 6(6): 1489–1499.
59. Chik, N., Rahim, K., Radam, A. and Shamsudin, M. Impact of Malaysian industrial energy use on carbon dioxide emissions. *Pertanika Journal of Social Science and Humanities*, 2013. 21: 13–28.
60. Saner, P., Loh, Y., Ong, R. and Hector, A. Carbon stocks and fluxes in tropical lowland dipterocarp rainforests in Sabah, Malaysian Borneo. *Public Library of Science, PloS one*, 2012. 7(1): e29642.
61. Wicke, B., Sikkema, R., Dornburg, V. and Faaij, A. Exploring land use changes and the role of palm oil production in Indonesia and Malaysia. *Land Use Policy*, 2011. 28(1): 193–206.

62. Hansen, U. and Nygaard, I. Sustainable energy transitions in emerging economies: The formation of a palm oil biomass waste-to-energy niche in Malaysia 1990–2011. *Energy Policy*, 2014. 66: 666–676.
63. Mekhilef, S., Siga, S. and Saidur, R. A review on palm oil biodiesel as a source of renewable fuel. *Renewable and Sustainable Energy Reviews*, 2011. 15(4): 1937–1949.
64. Wan, I., Ramli, A. and Sulaiman, M. Prediction model for estimating optimum harvesting time of oil palm fresh fruit bunches. *Journal of Food, Agriculture & Environment*, 2011. 9(3/4 part 1): 570–575.
65. Harun, N. H., Sidek, N., Aris, R. M., Ahmad, I., Wakiwaka, H. and Tashiro, K. Investigations on a novel inductive concept frequency for the grading of oil palm fresh fruit bunches. *Sensor*, 2013. 13: 2254–2266.
66. Ng, W., Lam, H., Ng, F., Kamal, M. and Lim, J. Waste-to-wealth: Green potential from palm biomass in Malaysia. *Journal of Cleaner Production*, 2012. 34: 57–65.
67. Shuit, S., Tan, K., Lee, K. and Kamaruddin, A. Oil palm biomass as a sustainable energy source: A Malaysian case study. *Energy*, 2009. 34(9): 1225–1235.
68. Clark, C. *Mathematical Bioeconomics*. New York: John Wiley & Sons. 1990.
69. Kang, M.-Z., Cournède, P.-H., De Reffye, P., Auclair, D. and Hu, B.-G. Analytical study of a stochastic plant growth model: Application to the GreenLab model. *Mathematics and Computers in Simulation*, 2008. 78(1): 57–755.
70. Pallas, B., Soulié, J.-C., Aguilar, G., Rouan, L. and Luquet, D. X-Palm, a functional structural plant model for analysing temporal, genotypic and inter-tree variability of oil palm growth and yield. *The Seventh International Conference on Functional Structural Plant Model*, 2013.
71. Syahrudin. The Potential of Oil Palm and Forest Plantations for Carbon Sequestration on Degraded Land in Indonesia. *Cuvillier Verlag*, 2005.
72. Pulhin, F., Lasco, R. and Urquiola, J. Carbon sequestration potential of oil palm in Bohol, Philippines. *Ecosystems and Development Journal*, 2018.

73. Sukuwan, S. Carbon Assessment Tool for New Oil Palm Plantings. *Kuala Lumpur, Malaysia: Round Table on Sustainable Palm Oil*, 2012.
74. Lamade, E. and Bouillet, J.-P. Carbon storage and global change: The role of oil palm. *OCL. Oléagineux Corps gras Lipides*, 2005. 12(2): 154–160.
75. Malthus, T. *An Essay on the Principle of Population*. Reeves & Turner. 1888.
76. Lewis, F., Vrabie, D. and Syrmos, V. *Optimal Control*. New York: John Wiley & Sons. 2012.
77. Xabadia, A. and Goetz, R. The optimal selective logging regime and the Faustmann formula. *Journal of Forest Economics*, 2016. 16(1): 63–82.
78. Kirk, D. *Optimal Control Theory: An Introduction*. Englewood Cliffs, NJ: Prentice-Hall. 1970.
79. Teo, C., K amd Goh and Wong, K. *A Unified Computational Approach to Optimal Control Problems*. Walter de Gruyter & Co. 1991.
80. Bryson, A. and Ho, Y.-C. *Applied optimal control: Optimization, estimation, and control (revised edition)*. Levittown, PA, USA: Taylor & Francis. 1975.
81. Rohanin, A. *Theory and Implementation of Gradient-Based Modifications of a Model–Reality Differences Algorithm for Solving Nonlinear Optimal Control Problems*. Doctor of philosophy. Universiti Teknologi Malaysia, Malaysia. 2005.
82. Bellman, R. *The Theory of Dynamic Programming*. Santa Monica, CA, USA: RAND Corp. 1954.
83. Pontryagin, L. *Mathematical Theory of Optimal Processes*. CRC Press. 1987.
84. Hocking, L. *Optimal Control: An Introduction to the Theory with Applications*. Oxford Univ. Press. 1991.
85. Dorato, P. and Levis, A. Optimal linear regulators: The discrete-time case. *IEEE Transactions on Automatic Control*, 1971. 16(6): 613620.
86. Lin, Q., Loxton, R. and Teo, K. The control parametrization method for nonlinear optimal control: A survey. *Journal of Industrial and Management Optimization*, 2014. 10(1): 275–309.

87. Iwasaki, T. and Skelton, R. All controllers for the general control problem: LMI existence conditions and state space formulas. *Automatica*, 1994. 30(8): 1307–1317.
88. Nelder, J. and Mead, R. A simplex method for function minimization. *Computer Journal*, 1965. 7(4): 308–313.
89. Wright, M. Nelder, Mead, and the other simplex method. *Documenta Mathematica*, 2010. 7: 271–276.
90. Singer, S. and Singer, S. Efficient implementation of the Nelder–Mead search algorithm. *Applied Numerical Analysis & Computational Mathematics*, 2004. 1(2): 524–534.
91. B Ashein, G. and Enns, M. Computation of optimal controls by a method combining quasi-linearization and quadratic programming. *International Journal of Control*, 1972. 16(1): 177–187.
92. Goetz, R.-U., Hritonenko, N., Mur, R., Xabadia, A. and Yatsenko, Y. Forest management and carbon sequestration in size-structured forests: The case of *Pinus sylvestris* in Spain. *Forest Science*, 2010. 56(3): 242–2565.
93. Amelia, L., Wahab, D. and Hassan, A. Modelling of palm oil production using fuzzy expert system. *Expert Systems with Applications*, 2009. 36(5): 8735–8749.
94. Thieme, H. *Mathematics in Population Biology*. Princeton Univ. Press. 2018.
95. Kumar, V. and Lal, S. Stability analysis of logistic growth model with immigration effect. *International Journal of Science and Research*, 2017. 16(1): 112–119.
96. LaSalle, R. *The Stability and Control of Dynamic Processes*. Berlin: Springer-Verlage. 1986.
97. May, R. M. *Stability and Complexity in Model Ecosystems*. Princeton Univ. Press. 2001.
98. Goetz, R., Hritonenko, N., Mur, R., Xabadia, A. and Yatsenko, Y. Forest management for timber and carbon sequestration in the presence of climate change: The case of *Pinus Sylvestris*. *Ecological Economics*, 2013. 88: 86–96.

99. Klipp, E. *Systems Biology in Practice: Concepts, Implementation and Application*. John Wiley & Sons. 2008.
100. Bellman, R. On the Poincaré–Lyapunov theorem. *Nonlinear Analysis: Theory, Methods & Applications*, 1980. 4(2): 297–300.
101. Smith, G. *Numerical Solution of Partial Differential Equations: Finite Difference Methods*. Oxford Univ. Press. 1985.
102. Edelstein-Keshet, L. *Mathematical Models in Biology*. Philadelphia: SIAM. 1988.
103. Sharma, M. Sustainability in the cultivation of oil palm—Issues & prospects for the industry. *Journal of Oil Palm, Environment and Health (JOPEH)*, 2013. 4.
104. Tsoularis, A. and Wallace, J. Analysis of logistic growth models. *Mathematical Biosciences*, 2002. 179(1): 21–55.
105. Subbaram Naidu, D. *Optimal Control System*. Idaho State Univ. 2002.
106. Anderson, B. and Moore, J. *Optimal Control: Linear Quadratic Methods*. Courier Corporation. 2007.
107. Bagheri, S., Jafarov, T., Freidovich, L. and Sepehri, N. Beneficially combining LQR and PID to control longitudinal dynamics of a SmartFly UAV. *Information Technology, Electronics and Mobile Communication Conference*, 2016: 1–6.
108. Beck, M. Identification, estimation and control of biological waste-water treatment processes. *Proceedings D—Control Theory and Applications*, 1986. 133(5): 254–264.
109. Baker, F. *Item Response Theory: Parameter Estimation Techniques 2nd. ed.* New York: Marcel Dekker. 2004.
110. Beerli, P. Comparison of Bayesian and maximum-likelihood inference of population genetic parameters. *Bioinformatics*, 2005. 22(3): 341–345.
111. Marquardt, D. An algorithm for least-squares estimation of nonlinear parameters. *Journal of the Society for Industrial and Applied Mathematics*, 1963. 11(2): 431–441.

112. Srinivasan, V. and Mason, C. Nonlinear least squares estimation of new product diffusion models. *Marketing Science*, 1986. 5(2): 169–178.
113. Chen, F. On a nonlinear nonautonomous predator–prey model with diffusion and distributed delay. *Journal of Computational and Applied Mathematics*, 2005. 180(1): 33–49.
114. Protter, M. H. and Charles Jr, B. *A first course in real analysis*. Springer Science & Business Media. 2012.
115. Silalertruksa, T., Bonnet, S. and Gheewala, S. Life cycle costing and externalities of palm oil biodiesel in Thailand. *Journal of Cleaner Production*, 2012. 28: 225–232.
116. Gaoue, O., Ngonghala, C., Jiang, J. and Lelu, M. Towards a mechanistic understanding of the synergistic effects of harvesting timber and non-timber forest products. *Methods in Ecology and Evolution*, 2016. 7(4): 398–406.
117. Nath, V. and Mitra, R. Robust pole placement using linear quadratic regulator weight selection algorithm. *International Journal of Scientific Research Engineering and Technology*, 2014. 3(3): 329–333.
118. Jordan, D. W. and Smith, P. *Nonlinear ordinary differential equations: an introduction to dynamical systems*. Oxford University Press, USA. 1999.
119. Kolmanovskii, V. and Shaikhet, L. Some conditions for boundedness of solutions of difference Volterra equations. *Applied Mathematics Letters*, 2003. 16(6): 857–862.

LIST OF PUBLICATIONS

Indexed Journal (SCOPUS)

1. **Nasir, N.**, Abd. Aziz, M. I., Banitalebi, A. (2019). Carbon absorption control model of oil palm plantation. *Journal of Sains Malaysiana UKM*,48(4),921–925.
2. **Nasir, N.**, Abd. Aziz, M. I., Hamzah, A. S. (2018) Stability Analysis of Palm Oil Control Model with Felling Effect. *International Journal of Advanced Computer Technology*, 7(11), 2916–2920.

Non-Indexed Journal

1. **Abd. Aziz, M. I.**, Nasir, N., Banitalebi, A. (2019). The Optimal Felling Rate in The Palm Oil Plantation System. *Journal of MATEMATIKA*, 35(1), 95–104.
2. **Nasir, N.**, Abd. Aziz, M. I., Banitalebi, A. (2017). Linear Optimal Control Model for Felling the Oil Palm Trees. *Jurnal Teknologi*, 79(4), 123–128.

Indexed conference proceedings

1. **Banitalebi, A.**, Abd. Aziz, M. I., Nasir, N. (2016, June). Modelling and optimization for palm oil plantation management. In *Advances in Industrial and Applied Mathematics: Proceedings of 23rd Malaysian National Symposium of Mathematical Sciences (SKSM23) Universiti Teknologi Malaysia*. (Vol. 1750, No. 1, p. 030046). AIP Publishing.