ON THE SOLUTION OF BLASIUS PROBLEM BY COMBINED ADOMIAN DECOMPOSITION METHOD-INTEGRAL TRANSFORMS

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ABSTRACT

A boundary layer problem is a study of motion of fluid at a very thin layer. It is a single perturbation problem derived from the Navier-Stokes equations which are known as equations of motions for fluid in which the solution need to be solved. The Blasius equation is one of the basic equations in fluid dynamics that describes the steady flow of incompressible fluids over a semi-infinite flat plate. The aim of this study is to solve Blasius problem for two different boundary conditions. The first approach is to transform the Blasius boundary value problem into an initial value problem that introduces $f''(0) = \sigma$ as a new initial condition. The method proposed to solve this problem for the two cases is by combining the Adomian Decomposition Method (ADM) with two integral transforms which are the Laplace and Elzaki transforms. Padè approximation is applied to determine the value of $f''(0) = \sigma$. The values obtained are substituted into the respective Blasius series solutions and the behaviour of the solutions are studied. It is found that the Blasius solution of $f(\eta)$ and $f'(\eta)$ for both cases agree well with solutions from previous studies.

ABSTRAK

Masalah lapisan sempadan adalah kajian pergerakan bendalir pada lapisan yang sangat nipis. Ia merupakan masalah usikan tunggal yang diperolehi daripada persamaan Navier-Stokes yang dikenali sebagai persamaan gerakan cairan yang perlu diselesaikan. Persamaan Blasius adalah salah satu persamaan asas dalam dinamik bendalir yang menggambarkan aliran mantap cairan tidak dapat dimampatkan di atas plat rata separa tak terbatas. Tujuan kajian ini adalah untuk menyelesaikan masalah Blasius untuk dua syarat sempadan yang berlainan. Pendekatan pertama adalah untuk mengubah masalah nilai sempadan Blasius ke masalah nilai awal yang memperkenalkan $f''(0) = \sigma$ sebagai keadaan awal baru. Kaedah yang dicadangkan untuk menyelesaikan masalah ini untuk kedua-dua kes adalah dengan menggabungkan ‘Adomian Decomposition Method’ (ADM) dan penjelmaan kamiran seperti Laplace dan Elzaki. Anggaran Padè digunakan untuk menentukan nilai $f''(0) = \sigma.$ Nilai-nilai yang diperoleh digantikan ke dalam penyelesaian siri Blasius dan tingkah laku penyelesaiannya dikaji. Didapati bahawa penyelesaian Blasius bagi $f(\eta)$ dan $f'(\eta)$ sangat tepat dengan penyelesaian yang diperoleh sebelum ini.

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LIST OF ABBREVIATIONS

ADM	-	Adomian Decomposition Method
BVP	-	Boundary Value Problem
DTM	-	Differential Transformation Method
HPM	-	Homotopy Pertubation Method
IVP	-	Initial Value Problem
VIM	-	Variational Iteration Method

LIST OF SYMBOLS

x	-	Horizontal coordinates of the plane
y	-	Vertical coordinates of the plane
u	-	Horizontal velocity component
v	-	Transverse velocity component
U_∞	-	Stream velocity parallel to x-axis
p	-	Fluid pressure
ρ	-	Density
$\frac{dp}{dx}$	-	Velocity of potential flow
t	-	Time from start of motion
δ	-	Thickness of boundary layer
ψ	-	Stream function
L	-	Length of boundary layer
Re	-	Reynold's number
η	-	Similarity variables related to vertical coordinates of the plane and boundary layer thickness
$f(\eta)$	-	Non-dimensional function related to stream function
σ	-	Skin friction around the plate

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CHAPTER 1

INTRODUCTION

1.1 Background of the Study

Fluid dynamics has a wide range of applications in engineering problems and the one most important advance in fluid dynamics is the deduction of the boundary layer equations. The fluid flow analysis in a boundary layer adjacent adjoining the wedge is an essential part in the area of fluid dynamics. Many problems in the field of mathematical physics and also in fluid mechanics can be modelled by different boundary value problems. Blasius equation arises in the study of laminar boundary layer exhibiting similarity properties [1]. The equation has been the subject of considerable research nowadays due to its importance in boundary layer theory.

A boundary layer problem is the study of motion of fluid at a very thin layer referred as boundary layer close to the surface body. The primary application of the boundary layer theory is in finding the skin-friction drag which acts on a body as it is travelled through a fluid; for instance the drag of a turbine blade, of an airplane wing or a complete ship [2]. The concept of the boundary layer, hence, infers that the flows at high Reynolds numbers can be distributed up into two unequally large regions. The two different flow forms can both happen within the boundary layer which is the flow can be either laminar or turbulent [3]. Boundary layer problem is a single perturbation problem derived from Navier-Stokes equations which are known as equations of motions for fluid in which the solution need to be solved [1,3].

Blasius equation is one of the fundamental equations in fluid dynamics that defines the steady flow of incompressible fluids over a semi-infinite flat plate. Engineers, physicists and mathematicians have special interest in studying this equation as a result of the application of Blasius equation in fluid flow [4].

There are two types of Blasius equation that show up in the fluid mechanics theory where both forms are represented by the same differential equation but with different boundary conditions where each is subjected to special physical conditions [4-8]. Consider the Blasius equation of two-dimensional laminar viscous flow past a semi-infinite plate represented by a nonlinear ordinary differential equation

$$f'''(\eta) + \frac{1}{2}f(\eta)f''(\eta) = 0,$$

where the boundary conditions are given as

$$f(0) = 0, f'(0) = 0, f'(\infty) = 1$$

or

$$f(0) = 0, f'(0) = 1, f'(\infty) = 0.$$

This differential equation is well known as the mother of all boundary-layer equations in fluid mechanics. In 1908, Blasius provides a power series solution. It was noted that the solution is not given closed-form because the value of $\sigma = f''(0)$ is not known. Howarth in 1938 gained a more accurate value of σ by means of numerical technique [9]. In order to improve this value, many studies have been conducted throughout the years.

Many numerical, analytical and semi-analytical methods have been investigated for solving this equation. Some of these methods are finite difference, Adomian Decomposition, perturbation methods, differential transform and variational iteration methods. These methods are used to obtain the series solution that converges to the exact solution. Meanwhile, combination and modifications of approximate methods with integral transforms have also been introduced in order to accelerate the convergence of the solution and enlarge the convergence radius [4,10].

This study focuses on solving Blasius problem by using the combination of Adomian Decomposition Method (ADM) with two integral transforms such as Laplace Transform and Elzaki Transform for two different boundary conditions. Padè approximation is also studied in this research to handle the complex boundary conditions at infinity.

1.2 Problem Statement

In the past few decades, nonlinear problems have been derived from various problems found in science and engineering for which their solutions are important in determining the properties and behaviour of the physical system. However, the exact solution of a nonlinear problem is sometimes difficult to determine since the problem is usually complex as presented by Blasius problem. This study is conducted to investigate a two-dimensional viscous laminar flow over a flat plate which is described by the Blasius problem. The focus of this research is solving Blasius problem with one of the boundary condition is at infinity for two different types of boundary conditions. There are several combined methods that have been proposed to solve this equation. In this research, new method will be introduced to solve Blasius initial value problem for two different boundary conditions which is the Adomian Decomposition Method and its combination with two integral transforms which are Laplace transform and Elzaki transform.

1.3 Objectives of the Study

The objectives of the study are:

i. To transform the Blasius boundary value problem into initial value problem.
ii. To solve Blasius problem by using ADM-Laplace transform and ADM-Elzaki transform.
iii. To apply Padè approximant to the Blasius solution.
iv. To study the performance of the combined method.

1.4 Scope of the Study

This study focuses on solving Blasius problem by using the combination of Adomian Decomposition Method (ADM) with two integral transforms namely the Laplace transform and Elzaki transform. Two different boundary conditions are considered. Then, the performance of Blasius solutions with the application of Padè approximation will be analyzed.

1.5 Significance of the Study

The Blasius problems are among the most generally used problems in scientific studies which need to be solved as accurate as possible. By taking the advantage of combining the two powerful methods for finding exact solutions for nonlinear equations, it is very likely to improve the effectiveness of the Blasius solution. It is also to study and prove the applicability of this combined method on both types of boundary conditions. Hence, the understanding of this method is useful so that the combined methods can be applied to solve other nonlinear problems arising from another physical phenomena.

1.6 Operational Framework

The operational framework is illustrated in Figure 1.1 to get a clear picture of how this research is carried out.

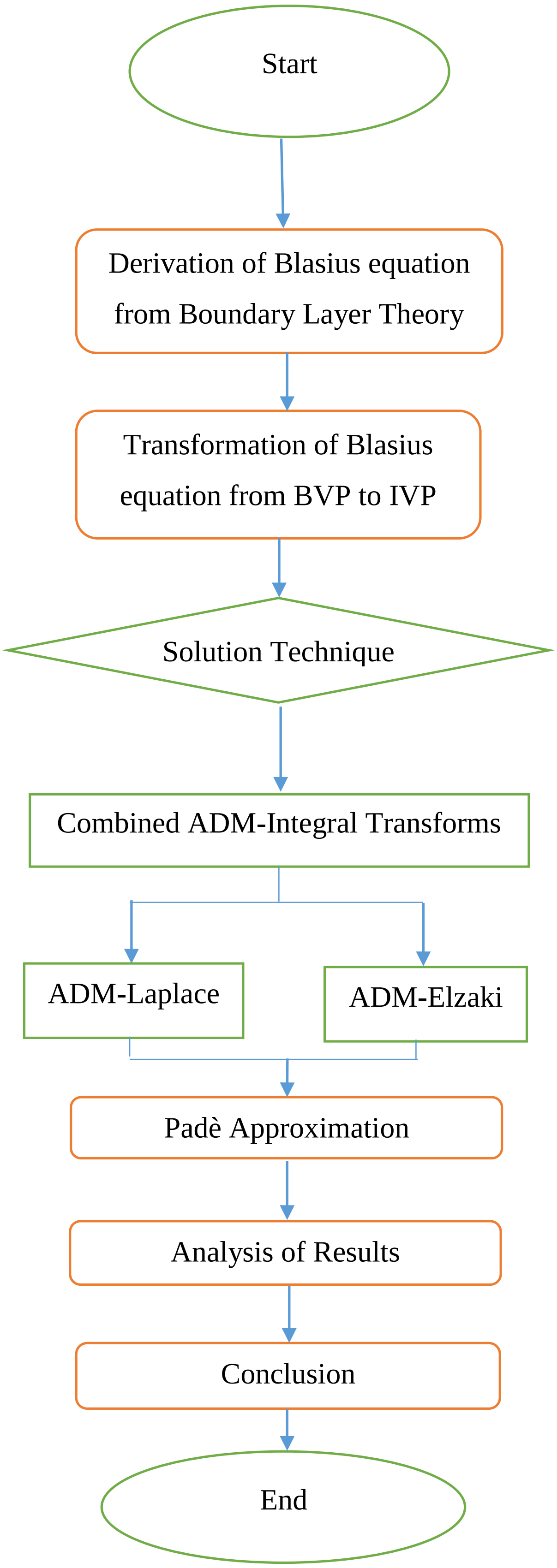

Figure 1.1 Summary of the study of ADM-Integral transforms for Blasius problem

1.7 Summary of the Chapter

This chapter begins by studying the background of this research where many problems in the field of fluid mechanics and also in mathematical physics can be modelled by different boundary value problems. A boundary value problem is the study of motion of fluid at a very thin layer close to the surface body. The focus of this research is solving Blasius problem with one of the boundary condition is at infinity for two different types of boundary conditions by using the combination of Adomian Decomposition Method and integral transforms. This objectives of this research are to transform the Blasius boundary value problem into initial value problem, to transform Blasius problem by using combination of ADM and integral transforms, applying Padè approximant to the Blasius solution and to study the performance of the combined method. The operational framework also is shown in this chapter to know how this research is carried out.

Stokes equations for steady, two dimensional, incompressible flows where the density is kept to be constant. Navier formulated the equations in Cartesian coordinate system (x, y), which are horizontal and vertical coordinates together with velocity components (u, v) where u and v are respectively the horizontal and vertical fluid velocities.

So, for a two-dimensional steady state Navier-Stokes equations, the basic equations governing incompressible fluid are the following:

Continuity:
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.1}$$

Momentum:
$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \tag{2.2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right). \tag{2.3}$$

where ρ is the density, p is the pressure and ν is the kinematic viscosity at a point of the fluid. In this study, we will consider the flat plate and assume that the plate is oriented along the x-axis, the pressure gradient may be neglected.

It is classified by Xavier et. al that the Navier-Stokes equation can be reduced to the boundary layer approximations [12]. He showed that the continuity and momentum equations for laminar flow that are derived from the boundary layer approximation becomes an ordinary differential equation in u and v as the number of independent variables is reduced from two (x and y) to one (say η).

2.2.1 Boundary Layer over a Semi-Infinite Flat Plate

A steady and stationary flow is considered as shown in Figure 2.2 to impose tangentially on a vertical flat plate of semi-infinite length. In addition, if the fluid flows in the x-direction in the half-space $x < 0$ at the constant velocity U and the plate is positioned along the half-plane $y = 0, x > 0$ associated with the earlier flow. Based on the solution for an impulsive flow over an infinite plate, it can be assumed that the

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