# NUMERICAL SOLUTION OF FOUR DIFFERENT STENOSIS LOCATIONS IN BIFURCATED ARTERY WITH HEAT TRANSFER

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# DEDICATION

Special dedication to my beloved parents

*Ahmad Jamali Bin Malin*

*Zaleha binti Amat*

To my respectful supervisor and co-supervisor

*Dr. Zuhaila Ismail*

*Prof. Dr. Norsarahaida S. Amin*

To my loving siblings and my supportive colleagues

To my friends and seniors

*Thank you for everything. Your presence made my journey truly memorable*

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## ABSTRACT

The development and progression of stenosis with a high probability of rupture can be characterised by changing the temperature distribution in the bifurcated artery. The purpose of this study is to investigate the behaviour of blood flow through four different locations of stenosis under the influence of heat transfer. In this study, the basic step of construction geometries (TYPE I, TYPE II, TYPE III, and TYPE IV that implies four possible morphologies formation of plaque from healthy to disease artery) are shown by using COMSOL Multiphysics 5.2. The blood flow is modelled as laminar, two-dimensional, steady, incompressible, and characterised as a Newtonian fluid. The classical Galerkin Weighted Residual (GWR) method is utilised to discretise the governing equations and boundary conditions. In addition, GWR is a convenient method to compute the solution since this method is compatible with circumventing the Babuska-Brezzi stability conditions. Firstly, a MATLAB source code is developed to solve the problem. Later, the results are compared with COMSOL Multiphysics 5.2 that based on the finite element method. The numerical validations are performed for the lid-driven cavity flow benchmark and the results of the axial velocity profile achieve a good agreement. This investigation focuses on the blood flow characteristics such as the velocity, temperature, pressure, streamline pattern, wall shear stress, and local Nusselt number, which have been discussed graphically and fundamentally. The parameters involved, such as Reynolds and Prandtl numbers, the maximum height of stenosis are very much affect the blood flow characteristics. Besides, TYPE IV shows the highest value of maximum velocity as compare to TYPE II and TYPE III. The backflow is formed in the post-stenotic region near the outer wall surface. Higher Reynolds number has enhanced the magnitudes of the wall shear stress predominantly in the downstream region of stenosis and reduced the pressure at the artery walls to some negative values resulted from the flow reversal. It shows that by increasing the maximum height of stenosis and Reynolds number, the shut of the peak for Nusselt number and also blood flow accelerations will increase rapidly.

## ABSTRAK

Pembentukan dan perkembangan stenosis dengan kebarangkalian memecah yang tinggi boleh disifatkan melalui pertukaran taburan suhu di dalam arteri bercabang. Tujuan kajian ini adalah untuk mengkaji sifat aliran darah melalui empat lokasi stenosis yang berbeza di bawah pengaruh pemindahan haba. Dalam kajian ini, langkah-langkah asas pembinaan geometri (JENIS I, JENIS II, JENIS III, dan JENIS IV yang mengimplikasikan empat pembentukan morfologi plak dari arteri yang sihat kepada yang berpenyakit) telah ditunjukkan melalui COMSOL Multiphysics 5.2. Aliran darah dimodelkan sebagai laminar, dua dimensi, mantap, bendalir tak termampat, dan disifatkan sebagai bendalir Newtonian. Kaedah Sisa Berpemberat Galerkin (GWR) digunakan untuk mencerakinkan persamaan menakluk dan syarat sempadan. Tambahan pula, GWR merupakan kaedah yang sesuai untuk mengira penyelesaian kerana kaedah ini dapat mengelakkan syarat kestabilan Babuska-Brezzi. Pertama, kod perisian MATLAB dibangunkan untuk menyelesaikan masalah yang terlibat. Kemudian, keputusan yang dihasilkan dibandingkan dengan COMSOL Multiphysics 5.2, yang berasaskan kaedah unsur terhingga. Pengesahan berangka dilaksanakan mengikut piawaian aliran rongga berpenutup bergerak dan keputusan halaju paksi mencapai persetujuan yang baik. Kajian ini menumpukan kepada sifat aliran darah seperti halaju, suhu, tekanan, corak garis lurus, tegasan ricih dinding, dan nombor Nusselt setempat yang telah dibincangkan secara grafik dan asas. Parameter yang terlibat seperti nombor Reynolds dan Prandtl, ketinggian maksimum stenosis sangat memberi kesan kepada sifat aliran darah. Selain itu, JENIS IV telah menunjukkan nilai halaju maksimum yang tertinggi berbanding dengan JENIS II dan JENIS III. Pembalikan aliran telah terbentuk pada akhir kawasan stenosis berhampiran permukaan dinding luar. Nombor Reynolds yang tinggi telah merangsang magnitud tegasan ricih dinding secara dominasi di kawasan hujung stenosis dan telah mengurangkan tekanan pada dinding arteri kepada beberapa nilai negatif yang terhasil daripada aliran berbalik. Ini menunjukkan bahawa dengan meningkatkan ketinggian maksimum stenosis dan nombor Reynolds, penutupan puncak untuk nombor Nusselt dan juga pecutan aliran darah akan meningkat dengan cepat.

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## LIST OF ABBREVIATIONS

| | | |
|---|---|---|
| CVD | - | Cardiovascular Disease |
| FEM | - | Finite Element Method |
| MHD | - | Magnetohydrodynamics |
| PIV | - | Particle Image Velocimetry |
| CCA | - | Common Carotid Artery |
| ICA | - | Internal Carotid Artery |
| UTM | - | Universiti Teknologi Malaysia |
| ECA | - | External Carotid Artery |
| CFD | - | Computational Fluid Dynamics |
| DOF | - | Degree of Freedom |
| GWRM | - | Galerkin Weighted Residual Method |
| CV | - | Control Volume |
| CS | - | Control Surface |
| RTT | - | Reynold Transport Theorem |

## LIST OF SYMBOLS

| | | |
|---|---|---|
| $x$ | - | Axial coordinate |
| $y$ | - | Radial coordinate |
| $R_1(x)$ | - | Radii of the outer wall |
| $R_2(x)$ | - | Radii of the inner wall |
| $a$ | - | Radii of the mother artery |
| $r_1$ | - | Radii of daughter artery |
| $r_0$ | - | Radii of curvature for the lateral junction |
| $r_0^{'}$ | - | Radii of curvature for the flow divider |
| $d$ | - | Onset of the stenosis |
| $l_0$ | - | Length of the stenosis at distance $d$ from the origin |
| $x_1$ | - | Location of the onset and offset of the lateral junction |
| $x_2$ | - | Location of the offset of the lateral junction |
| $x_3$ | - | Apex |
| $\tau_m$ | - | Maximum height of stenosis |
| $\beta$ | - | Half of the bifurcation angle |
| $\rho$ | - | Density of blood |
| $u$ | - | Axial velocity component |
| $v$ | - | Radial velocity component |
| $w$ | - | Tangential velocity component |
| $t$ | - | Time |
| $\mu$ | - | The respective molecular viscosity coefficient |
| $m$ | - | Fluid consistency coefficient |
| $\Omega$ | - | Domain |
| $p$ | - | Pressure |
| $h$ | - | Length of the inlet |
| $\mathrm{Re}$ | - | Reynold number |
| $\mathrm{Pr}$ | - | Prandtl number |
| $T$ | - | Temperature |

| | | |
|---|---|---|
| $Nu$ | - | Nusselt number |
| $U_0$ | - | Averaged mean inflow velocity |
| $C_p$ | - | Specific heat |
| $k$ | - | Thermal conductivity |
| $T_{in}$ | - | Inlet temperature |
| $T_{wall}$ | - | Wall temperature |
| $N_i$ | - | Trial function |
| $n$ | - | The number of degrees of freedom |
| $R$ | - | Residual |
| $w_m$ | - | Weight function |
| $w_a$ | - | Weight function for the momentum equation |
| $w_b$ | - | Weight function for the continuity equation |
| $\tau$ | - | Tolerance value |
| $e_i$ | - | Element $i$ |
| $e$ | - | Internal energy |
| $\mathrm{Re}$ | - | Reynold number |
| $\mathrm{Pr}$ | - | Prandtl number |
| $E$ | - | Energy per unit mass |
| $F(x,t)$ | - | Body force |
| $P_h$ | - | Hydrostatic pressure |
| $F_{surface}$ | - | Surface force |
| $P_m$ | - | Mean pressure |
| $\overline{q}$ | - | Heat flux vector |
| $\sigma_{ij}$ | - | Stress tensor |
| $\varepsilon_{ij}$ | - | Strain rate tensor |
| $\delta_{ij}$ | - | Kronecker Delta |
| $V(t)$ | - | Material volume |

## LIST OF APPENDICES

# CHAPTER 1

# INTRODUCTION

## 1.1 Research Background

World Health Organization (WHO 2017) has confirmed that the cardiovascular diseases (CVDs) are the world biggest killer by accounting for more people to die annually compare with any other causes. An estimated number of death of 17 million deaths per year, representing 85% of death is caused by malfunction of organ, heart attack and stroke (World Health Organization, 2018). The understanding of blood flow behaviour is essential to investigate the connection between flow and disease development such as cardiovascular diseases. CVDs disorders of the heart and blood vessels are mainly due to the blockage that prevents blood from flowing to the heart or brain. To understand and discover the genesis of this vascular disease, special attention and interest has been given by many researchers all around the world. Generated knowledge in this area may lead to the design and advances of medical tools, which provide a greater prediction of disease progression and helps to increase the number of patients treated percutaneously.

### 1.1.1 Blood Composition

Blood is essential to the human body and other animals that delivers substances to the cells. Basically, the blood receives oxygen from the lungs and pushed through the body by the action of rhythmic pump of heart. Blood also transports heat to the skin to help regulate body temperature. Besides, blood is a complex mixture of cells, protein, lipoprotein, and ions. Blood can be classified as connective tissue and consisting the plasma, which is a clear extracellular fluid with the formed elements composed of Erythrocytes, known as red blood cells (RBCs), Leukocytes, known as white blood cells (WBCs), and platelets. These components can be centrifuged in a tube and separate the plasma to form elements according to their density. The total volume of 55% plasma is packed into the top of the tube in the form of pale-yellow colour. RBC makes up 45% of the total volume packed at the bottom of the tube, known as haematocrit. WBCs and platelets will be above RBC with the form of a narrow cream-coloured coat. The details of blood constituent can be referred in (Caro, C.G., Pedley, T.J., Schroter, R.C., 1978).

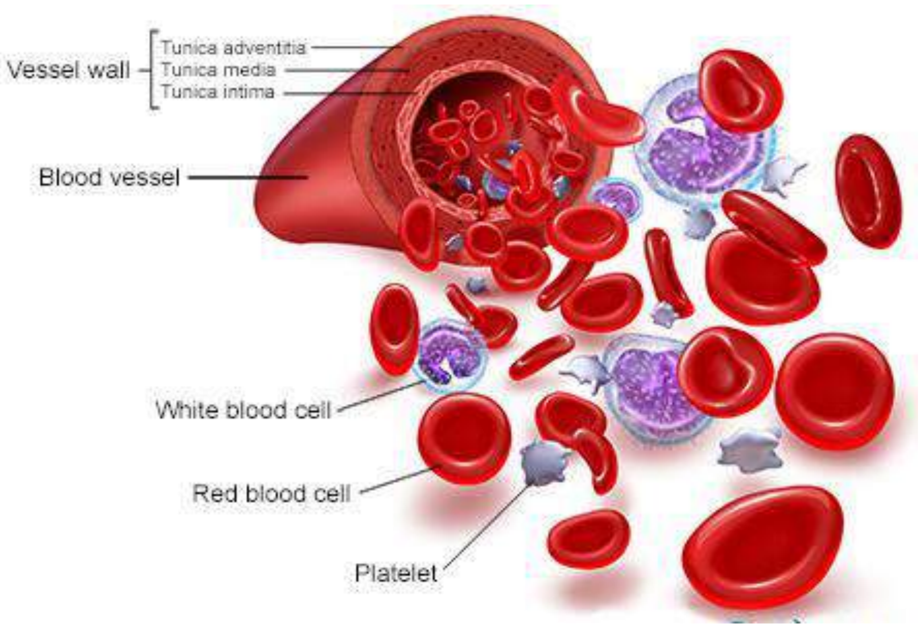


Figure 1.1 Composition of blood in a human vessel (Health Engine, 2019)

RBCs are small semisolid particles that increase blood viscosity and affect fluid behaviour. Blood is approximately four times more viscous than water. Despite, blood does not exhibit a constant viscosity at all flow rates especially non-Newtonian fluid type in the microcirculatory system. The non-Newtonian behaviour is most evident at meager shear rates when the red blood cells clump together into larger particles, (Ku, 1974). Blood is assumed as non-Newtonian behaviour in small capillaries, where the cell-free skimming layer reduces the effective viscosity through the tube. However, in most arteries, blood exhibits a Newtonian behaviour, and the viscosity is assume to be constant of, four centipoises. Zaman et al., (2015) had concluded that in a large radius of the artery, blood is assumed as a Newtonian fluid for values of a shear rate greater than 100 $s^{-1}$. Based on this, it clear shows that Newtonian fluid viscosity is always constant with shear rate and has high potential of cardiovascular diseases (Rabby et al., 2014).

### 1.1.2 Atherosclerotic Plaque

Cardiovascular diseases such as heart attack and stroke that are caused by atherosclerosis. The Atherosclerosis lesion is one of the most widespread diseases in human beings that lead to death in many countries. Typically, the walls of the healthy artery are smooth, which allowing blood to flow through the artery, for transportation of oxygen received from the lung and transmissions of nutrients from the intestine to tissues (Back, L., H., et al., 1977). There is an evidence that vascular fluid dynamics play an essential role in the development and progression of Atherosclerotic arterial, (Chakravarty and Sen, 2005). Atherosclerosis is a condition of the narrowing of the artery lumen as the result of plague deposition, known as stenosis (Zain and Ismail, 2017; Wu, J., et al., 2019). This leads to abnormal pattern of blood flow which severely reduces the flow of blood to the other organs and tissues. In some cases, plaque act differently and may lead to different blood rheology. Plaque might grows to finite size and eventually stops without narrowing the vessel in which the wall artery might resized their external diameter to provide sufficient space for the plaque, (Ikbal et al., 2009). Finally, the silent plaque can only be detected through an intravascular ultrasound device since it never caused any symptoms of the plaque formation that does not even block blood flow. Besides, there is an evidence shows that early atherosclerosis may lead to change in vessel wall size, and deposition of platelet thrombi may occur near the branching entrance. These conditions will disturb a flow where, from a fluid mechanical point of view, this can be related to the formation of eddies and separation of streamlines near the vessel wall (Genuardi et al., 2020).

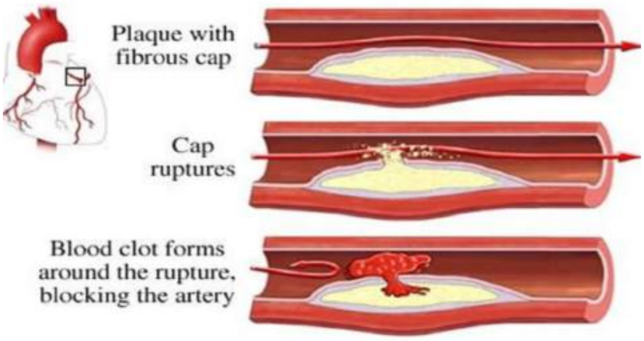


Figure 1.2 Atherosclerotic plaque build-up causes constriction of the vessel lumen, expose to arterial thrombosis (DPRC Hospital, 2017)

According to Rabby et al., (2014), Alimohamadi et al,. (2014), and Jahangiri et al., (2015), as plaque deposition such as cholesterol, fatty substance and smooth muscle cells continues to accumulate. The plaque will be formed on the inner wall of the artery, which is known as the endothelium. This may cause a bleeding when the plaque breaks away into emboli (particles) inside an artery. As a result, blood clots (thrombus) may form around the plaque and blood flow will be blocked and disrupted (Ikbal et al., 2009). As a consequence, patients of this case experience usual symptoms like pain on exertion. In severe cases, the patients will suffer from the neurological system (stroke) and unstable angina or myocardial infarction (heart attack) due to blockage of blood flow in the carotid artery and coronary artery, respectively. The malfunction of the cardiovascular system reveals that people is exposed to these cardiovascular diseases due to lack of awareness is emphasized on controlling their dietary and food intake. The risks such as smoking, high blood pressure and blood cholesterol is eight times more likely to develop atherosclerosis infection than a person without these risk factors. Hence, to elucidate the possible connection between blood flow behaviour and development of thrombosis and atherosclerosis, there is a growing need of work, both experimental and theoretical, to understand basic knowledge of artery disease treatment so that society awareness toward cardiovascular disease will be improved.

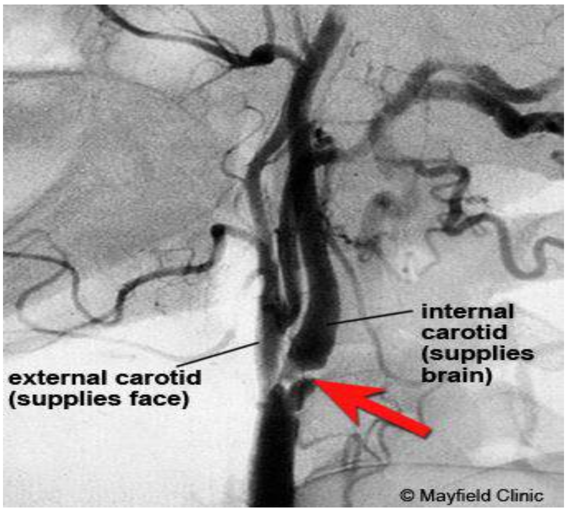


Figure 1.3 The carotid artery bifurcation site prone to atherosclerotic plaque deposition in the internal carotid arteries (Mayfield Clinic, 2018)

### 1.1.3 Arterial Geometry

Apart from that, an essential dynamics of blood flow factor that needs to be considered in current analyses is the arterial geometry. Studying the effect of these factors on the blood flow characteristics with heat transfer will help in a better understanding of the roles of blood dynamical in the development and progression of arterial diseases. Arteries and arterioles that consist of curvatures, junctions, and bifurcations were reported by Rabby et al., (2014), Srinivasacharya and Rao (2017), Zain and Ismail (2017), and Jamali and Ismail (2019). They claimed that the arterial geometry in bifurcation and bend shape exceptionally, would be the initiation of plaque accumulation. This suggests why the flow in certain regions has a high potential to be disturbed by flow disturbances like flow reversal and stagnation. As observed in the previous study, the flow reversal and recirculation zones are formed downstream of stenosis and along the edge of the daughter artery (Zain and Ismail, 2019). It is believed that the existence of flow recirculation in the cardiovascular system can cause danger to the health of a person, especially an atherosclerosis patient since the blood is moving slowly in this zone. The fruitful study has been classified as the geometry of the bifurcated artery according to the angulation between mother and daughter artery and also the location of plaque. T- shaped bifurcated artery is classified when the angulation is greater than $70^o$ and plagued shifting to the bifurcation branched is more difficult. However, Y-shaped bifurcated artery is more pronounced because the angulation of mother to daughter artery is less than $70^o$, (Lefèvre et al., 2000).

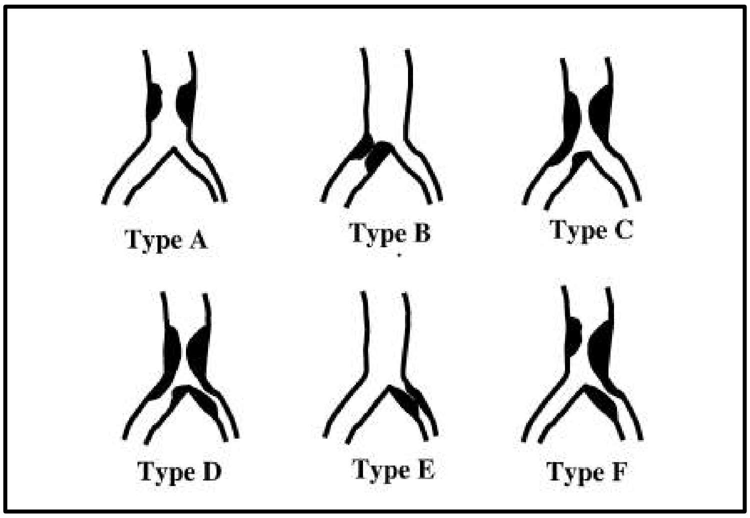


Figure 1.4 Classifications of bifurcations artery according to plaque burden (Lefèvre et al., 2000)

### 1.1.4 Medical Application

The study of blood flow in the artery has its importance in medical and engineering areas (Sharma et al., 2011). Heat transfer is also an important domain of thermal engineering in blood flow problem. Several literatures have to practice its principles to explore a variety of information on how the body possibly transfers the heat. Interesting to be noted that the heat source produced by the human body supplies to various part of the organs of the body. This energy is utilized for the nutrients metabolism process that ceaselessly takes place. It has been reported by a medical practice that, in a human body at rest, about two-thirds of the total heat is generated by the metabolic activities of the internal organs in the thorax, the abdomen, and the brain. The share of the heat generated in the brain is around 16 % of the total heat generated in the whole body (Sinha et al., 2016).

The mathematical models of heat transfer have been developed and are very useful in the application of modern treatment, such as laser surgery as well as cryosurgery. This physiotherapy technique involves a certain amount of temperature and directly affects the portion of the human body. The normal temperature of human blood is about 37°C. Thus, irreversible ill effects will occur in the proteins of blood, which can cause death after such high fever (Chato, 1980).

Figure 1.5 Hyperthermia surgery and Cryosurgery Machine

Moreover, hypothermia or hyperthermia is widely used for many purposes such as open-heart surgeries and cancer treatment. The role of the temperature is substantially essential, refer to Figure 1.5. For a rising of 1°C, the time of cure is reduced to half for a particular biological result like the decrease of cancer cells of a tumor (Lin, Yen, Chen, Jin, and Shieh, 1999). The objective of hyperthermia in cancer therapy is to raise the temperature of cancerous tissue above a therapeutic value 42°C, while maintaining the surrounding normal tissue at a suitable temperature value. The heat exchange between the living tissues and the blood network that passes through it depends on the geometry of the blood vessels, the blood flow through it, and the properties of the blood and surrounding tissue, (Chato, 1980). Furthermore, the observed effects of heat transfer in blood flow influence the stenosis deformation and may disturb the flow behaviour. From medical point of view, the presence of stenosis in the artery may lead to blood flow acceleration (velocity increase), and the heat transfer coefficient is increases at the stenotic region and drop as the flow pass through the offset of stenosis (Audaa and Ph, 2018). Therefore, the presence of stenosis in artery enhances the rate of heat transfer. Formulation and analysis of mathematical models on heat transfer during blood flow have been found to be very useful.

## 1.2 Problem Statements of Research

Atherosclerosis is one of the important and common causes of death and disability throughout the world (WHO). Atherosclerosis refer to physiological problem due to the malfunction of cardiovascular system and have close relationship with flow characteristics of blood deformability of the vessel wall.

Heat transfer is one of the crucial parameters that needs to be considered in blood flow investigation. Lack of a proper analysis of this factor will cause an irreversible effect in the proteins of blood and may lead to fatality. The abnormal growth that happens in the bifurcated artery, known as stenosis, increase blood flow and blood temperatures in the artery. Therefore, the energy equation needs to be considered in order to determine the behaviour of heat transfer by calculating the blood temperature in the bifurcated artery.

The existence of stenosis at various possible location in the artery, especially in the bifurcated lesion, will change the artery size at the specific location. So far, the existing literature investigated the flow structure and characteristic of heat transfer on Newtonian and non-Newtonian fluid models. The fluid behaves as a strong non-Newtonian characteristic in the small artery at a low shear rate (Zaman et al., 2015). However, it has been confirmed that the Newtonian model is considered as a good approximation as the shear rates are more than 100 $s^{-1}$ (reciprocal seconds), which have tendency to occur in a large artery.

Another important factor in haemodynamic analysis is the geometry of the artery. The arterial geometry in bifurcation and the shape of the bend will exceptionally, be the initiation of plaque accumulation since it has a sudden change on area and curvature. Various locations of stenosis in the bifurcation artery give a huge considerable effect to the blood characteristic and flow structure. It is believed that the existence of flow recirculation in the cardiovascular system can cause danger to the health of a person, especially atherosclerosis patients since the blood is moving slowly in this zone. Studying the effect of different location stenosis with heat transfer will help in better understanding of the development and arterial disease progression.

A numerical method is needed since the bifurcated artery geometry itself isin irregular shape and small scale. The finite element method (FEM) is proposed in this study to obtain the numerical results of the flow. This method has been demonstrated as one of the suitable numerical methods since it is easily applied to domains with complex geometry, film properties, and boundary conditions. The governing equation is discretized by using the Galerkin weighted residual (GWR) method, which is sufficient for computation the result in a large scale of convergence.

## 1.3 Objectives of Research

The main objective of this research is to develop the mathematical model of blood flow under the influence of heat transfer with the effect of different location of stenosis. This includes the construction of suitable mathematical models of the flow by considering appropriate governing equations and boundary conditions. The resulting governing equations are then solved using a Galerkin weighted residual (GWR). Specifically, the objectives are:

1. To develop appropriate governing equations that describe the blood flow through the bifurcated artery with the influence of heat transfer.
2. To generate MATLAB code based on the numerical discretization that has been performed using the Galerkin Weighted Residual method (GWR).
3. To investigate the effect of heat transfer at different locations of stenosis on the blood flow characteristics, such as velocity, streamlines, wall shear stress, and pressure drop.
4. To investigate the effect parameters such as Reynold number, Prandtl number, the maximum height of stenosis, and local Nusselt number inside the streaming blood.

## 1.4 Scope of Research

The fluid flow with heat transfer in the stenosed bifurcated artery is analyzed. Only four different types of stenosis location are considered. The fluid is considered as an incompressible Newtonian fluid. The Navier-Stokes equation is modelled with few assumptions imposed, which are the flow is steady, laminar, and two-dimension by considering a rigid wall with the no-slip condition. The geometry of arteries forming bifurcations is not symmetrical about the trunk axis, and bifurcated circular cylinders of finite length contain the stenosis at the outer wall and inner wall. As a numerical technique, the finite element method (FEM) is developed by using the Cartesian coordinate system to tackle the convergence rate for even large-scale transient simulation. The GWR method is utilized to discretize the governing equations and boundary conditions. In addition, the classical Galerkin weighted residual is a convenient method to compute the resulting governing equations since this method is compatible with circumventing the Babuski-Brezzi stability conditions. A MATLAB source code is developed to solve the problem. Then, the results are compared with COMSOL Multiphysics 5.2, a software based on the FEM. The comparison is found in satisfactory agreement with previous literature work.

## 1.5 Significances of Research

Cardiovascular diseases are the number one killer disease in the world. More than 786,641 deaths of all 2515458 deaths are approximately caused by stroke, heart attack and other cardiovascular diseases (Mozaffarian, Benjamin, Go, Arnett, Blaha, and Howard, 2015). Therefore, the application of computational fluid dynamics is essential and economical in biomedical engineering to predict the roles of fluid dynamical factors in the development and progression of arterial diseases in various types of physiological artery geometries.

This research may help in better understanding the behaviour of blood flow in arterial bifurcation with stenosis under the effect of heat transfer. Hypothermia or hyperthermia is widely used for many purposes, such as open-heart surgeries and cancer treatment. The temperature is also substantially critical. Here, the numerical source code for the solution of Newtonian blood flow in arterial bifurcation will be established. The establishment of a numerical library can be used for validation purposes against future experimental and numerical results. This study can be beneficial for analyzing the behavior of temperature distribution inside the artery during hyperthermia and cryosurgery.

## 1.6 Outline of the Thesis

This thesis contains seven chapters. Chapter 1 contains general introduction of the research background to understand the connection between human blood flow and cardiovascular diseases. Explanation to human blood flow allows us to understand cardiovascular diseases such as heart attacks and strokes. This chapter also highlight problem, statement research objectives, scope, and the significance of the study. The literature review of the interesting problems is elaborated and provided in Chapter 2. It consists of discussions of some basic rheology of blood, heat transfer, and types of the stenosed artery, particularly with the location of stenoses. The achievements of researchers of all existing problems in blood flow, as well as the method used to accomplish their study are presented. The finite element method is implemented, as a solving method for the proposed problem. The fundamental steps of using the finite element method are provided in Chapter 3. Triangular Taylor-Hood element is proposed in this chapter where the domain with the shape of triangular or rectangular is discretized to solve the nonlinear equation by applying the iteration manner. Subsequently, the computer implementation of the finite element method has been explored in this chapter as well. Chapter 4 will provide basic knowledge of how to create geometry in COMSOL Multiphysics 5.2. Then, the procedure is continued by collecting the nodes from the constructed geometry from COMSOL and transferred into MATLAB.

The investigation on the effect of different morphologies of the stenosed bifurcated artery with heat transfer is presented in Chapter 5. The finite element method is validated with the integrated software COMSOL Multiphysics 5.2 and previous works for the axial velocity over the specific location of the artery. The alteration of the fluid flow inside different artery geometry with involving parameters, such as Reynold number, Prandtl number and occlusion of stenosis as well as Nusselt number, are explored and discussed. The blood flow characteristics, such as velocity, streamlines, wall shear stress, temperature and pressure drop are scrutinized in this chapter. Chapter 6, the last chapter, consists of a summary of the study and several suggestions for future work.

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# LIST OF PUBLICATIONS AND PAPERS PRESENTED

The findings from this research have been published or presented in conferences as listed below:

Published Papers and Proceedings in Conference

1. Jamali, M. S. A., and Ismail, Z. (2019) 'Simulation of Heat Transfer on Blood Flow through a Stenosed Bifurcated Artery', *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 60, 2(2019), 310-323.

2. Jamali, M. S. A. and Ismail, Z. (2019) 'Generalized Power Law Model of Blood Flow in a Stenosed Bifurcated Artery', *Annals of Mathematical Modeling*, *1*(2).
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3. Muhammad Sabaruddin Ahmad Jamali, Zuhaila Ismail and Norsarahaida Saidina Amin. 'Heat Transfer of Newtonian Blood Flow in a stenosed Bifurcated Artery', *Proceedings of 6th International Graduate Conference on Engineering, Science and Humanities 2018 (IGCESH 2018)*. August 13-15, 2018. Johor Bahru, 598-600.