NUMERICAL SIMULATION OF THE NONLINEAR SCHRODINGER EQUATION USING STABLE IMPLICIT FINITE DIFFERENCE METHODS

ABEER AYED KHALAF ALANAZI

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DEDICATION

This thesis is dedicated to my mother, may the God have mercy on him and placed him into his havens, who instilled in me a love of knowledge and the search for it. It is also dedicated to my father who always motivates me to pursue my career with a lot of patience when I go through an adversity. Also, I dedicate this thesis to my husband who was the best collaborator on this achievement.

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ABSTRACT

Nonlinear Schrodinger equation (NLSE) in the context of drift wave packet is a difficult partial differential equation to solve without any approximations or transformations. Numerical computation must be taken into account to solve this complicated problem and the interplay between the first and second-order chromatic dispersions (CDs) and Kerr nonlinear effect needs to be considered. Although the NLSE in the absence of the first-order CD parameter term has been solved using various numerical and analytical methods, but the influential parameters, second-order CD, and self-phase modulation (SPM) have yet to be examined. Therefore, in this thesis the influence of these factors on new wave forms and on related conserved quantities was investigated. The NLSE was studied numerically by using finite difference methods. The Crank-Nicolson, which is second-order in time and space, was used. A high accuracy method that is fourth-order in space and second-order in time and known as the Douglas idea was also used to solve the NLSE. The accuracy and stability of the obtained schemes were analyzed. The conserved quantities mass, momentum, and energy were also computed. NLSE solutions were analyzed to illustrate the complex interfered model medium properties such as dispersion, dissipation, and nonlinearity. The impacts of the first and second-order CDs, nonlinearity on the structure of one soliton, interactions between two and three solitons, dark soliton, soliton-like periodic and dissipative, and shock waves were numerically inspected. It was found that these parameters affect not only the width and amplitude of the wave but also shock strength over the time evolution. On the other hand, new important waves existence can propagate by increasing time and become the effective wave propagation. These waves may be periodic, supersoliton, and oscillatory shock forms. Furthermore, a comparison of the obtained results of different techniques in this study confirmed that they are consistent with each other as well as with previous studies, indicating the accuracy of the numerical programming used. The findings of this study have the potential to improve communication performance through the development of physical parameters in the used model.

ABSTRAK

Persamaan Schrodinger tak linear (NLSE) dalam konteks bingkisan gelombang hanyut adalah persamaan terbitan separa yang sukar diselesaikan tanpa sebarang penghampiran atau transformasi. Pengiraan berangka perlu dipertimbangkan bagi menyelesaikan masalah rumit tersebut dan interaksi antara serak kromat (CD) tertib pertama dan kedua dan kesan tak linear Kerr perlu diambil kira. Walaupun NLSE tanpa kehadiran sebutan parameter CD tertib pertama telah diselesaikan dengan pelbagai kaedah berangka dan analitik, namun, semua parameter yang berpengaruh, CD tertib kedua, dan pemodulatan fasa diri (SPM) masih belum dikaji. Oleh itu dalam tesis ini, pengaruh semua faktor tersebut ke atas bentuk gelombang baharu dan ke atas kuantiti terabadi yang berkaitan telah diperiksa. NLSE telah dikaji secara berangka menggunakan kaedah beza terhingga. Skim Crank-Nicolson, yang merupakan tertib kedua bagi masa dan ruang telah digunakan. Kaedah ketepatan yang sangat tepat iaitu tertib keempat bagi ruang dan tertib kedua bagi masa yang dikenali sebagai idea Douglas juga digunakan untuk menyelesaikan NLSE. Ketepatan dan kestabilan skim yang diperoleh telah dianalisa. Kuantiti terabadi jisim, momentum, dan tenaga juga dikira. Penyelesaian NLSE telah dianalisis untuk menggambarkan sifat medium gangguan model yang kompleks seperti serakan, lesapan dan ketaklinearan. Kesan CD tertib pertama dan kedua, ketaklinearan terhadap struktur satu soliton, interaksi antara dua dan tiga soliton, soliton gelap, soliton jenis berkala dan lesapan, dan gelombang kejutan telah diperiksa secara berangka. Didapati bahawa parameter ini mempengaruhi bukan sahaja lebar dan amplitud gelombang tetapi juga kekuatan kejutan disepanjang evolusi masa. Sebaliknya, kewujudan gelombang baharu yang penting boleh dirambatkan dengan meningkatkan masa dan menjadi rambatan gelombang yang efektif. Gelombang ini mungkin berbentuk berkala, supersoliton, dan kejutan berayun. Seterusnya, perbandingan keputusan yang diperoleh dari teknik yang berlainan dalam kajian ini didapati konsisten antara satu sama lain serta konsisten dengan kajian sebelumnya, menunjukkan ketepatan program berangka yang digunakan. Keputusan yang diperoleh dari kajian ini berpotensi untuk meningkatkan prestasi komunikasi melalui pembangunan semua parameter fizikal dalam model yang digunakan.

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LIST OF ABBREVIATIONS

BEC	-	Bose-Einstein condensation
CD	-	chromatic dispersion
CDs	-	chromatic dispersions
CNLSEs	-	coupled nonlinear Schrodinger equations
GVD	-	group velocity dispersion
GPE	-	Gross-Pitaevskii equation
KdV	-	Korteweg-de Vries
MI	-	modulational instability
NLSE	-	nonlinear Schrodinger equation
PMD	-	polarized mode dispersions
SPM	-	self-phase modulation
XPM	-	cross phase modulation

LIST OF SYMBOLS

Roman Letters

a	–	amplitude of the waves
$\dot{a}$	–	derivative of a with respect to time
a_{in}	–	coefficients of inter-model dispersion
a_s	–	s-wave scattering length
a_0	–	length of the harmonic oscillator ground state
a_1, a_2, a_3	–	amplitude of the waves
$\mathbb{A}$	–	amplitude function
A_a	–	diagonal z-dependent matrix
$\mathbb{B}$	–	complex-valued amplitude function
c, c_1, c_2, c_3	–	phase velocity
c_m	–	discrete velocity of absolute value of amplitude function
$D\,(\mathbf{\Phi})$	–	$N \times N$ block tridiagonal matrix
$\mathbb{D}$	–	dissipation factor
e	–	minus of unit matrix
$E(\boldsymbol{r}, t)$	–	electric field
f_j	–	nonlinear functions
$\boldsymbol{F}$	–	vector of nonlinear functions
$\mathbb{G}$	–	smooth function with continuous partial derivative
h	–	space step size
$\hbar$	–	Plank constant
h_a	–	very small constant

H	–	amplification matrix
$\mathbb{H}$	–	non-homogeneity factor
i	–	imaginary number
I	–	unit matrix
J	–	Jacobian matrix
k	–	time step size
$\mathbb{k}_0, \mathbb{k}_1$	–	matrix operators
L	–	Number of particles
L_{bi}	–	normalized strength of the linear birefringence
L_D	–	dispersion length
L_∞	–	maximum error
L_2	–	total of errors
m_a	–	real constant
m_s	–	atomic mass
N	–	number of points in the grid
N_a	–	number of atoms
$O(h)$	–	order of firtst-order of h
p	–	dispersion coefficient
p_1	–	higher order dispersion coefficient
p_2	–	Raman scattering coefficient
p_3	–	nonlinear dispersion parameter
$\mathbb{P}(x_m, t_n,)$	–	dependent variable
P	–	$N \times N$ block tridiagonal matrix
q	–	cubic nonlinear coefficient
q_1	–	quadratic nonlinearity parameter
q_2	–	quintic laws of nonlinearity

q_3 – self-steepening phenomena

Q – complex-valued function

$\boldsymbol{r}$ – spatial vector

R – real numbers group

$\mathbb{R}_a$ – largest eigenvalue

R^+ – positive real numbers group

Re – real part of a complex amount

s – wavenumber

S – $N \times N$ block tridiagonal matrix

S_C – scalar-valued state variable

S_p – spring constant

t – time variable

$\mathbb{T}$ – temperature

$\mathbb{T}_c$ – critical temperature

T_m^n – truncation error

tol – small specified value

$\boldsymbol{u}$ – 2×1 vector

$U, \mathbb{U}$ – complex-valued functions

v, V – real functions

$\mathbb{V}$ – smooth function with continuous partial derivative

V_{ex} – external trapping potential

v_g – group velocity

V_l – linear potential

w, W – real functions

x – spatial transverse variable

$x_1 = x_L,\ x_N = x_R$ – first and last points grid

X_{ph} – cross-phase modulation

z – longitudinal variable representing propagation distance

z_e – zero matrix

–

Greek Letters

α – linear gain(amplification) or loss(attenuation)

β – second-order chromatic dispersion

$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ – Laplacian operator

$\nabla_t^2 = \partial_x^2 + \partial_y^2 + \partial_t^2$ – operator

χ – linear potential

χ_r – function

δ – first-order chromatic dispersion

$\delta_x^2 \phi_m = \phi_{m-1} - 2\phi_m + \phi_{m+1}$ – operator

ϵ – interaction length

$\eta(\omega)$ – permeability

γ – self-phase modulation

λ_j – eigenvalues of the amplification matrix H

μ – real constant

Ω – shallowness

ω – frequency

ω_0 – carrier frequency

$\omega_x, \omega_x, \omega_z$ – trap frequencies in x, y, z directions

$\overline{\phi}$ – complex conjugate

Φ – approximation solution to complex slowly varying pulse amplitude envelope

ϕ	–	complex slowly varying pulse amplitude envelope
σ	–	Bond number
τ	–	time variable
Υ_D	–	zero-dispersion wavelength
ε	–	constant
$\varepsilon(\omega)$	–	dielectric constant
ϖ	–	bounded constant
ς	–	positive real number
	–	

Subscript/Superscript

m	–	point number in the grid
n	–	number of time step
	–	

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CHAPTER 1

INTRODUCTION

1.1 Problem Background

Many phenomena in applied fields are described by nonlinear partial differential equations. The exact solution of the partial differential equations is required since it gives a clear understanding of the physical situation of these phenomena in nature which has become an underlying topic in mathematical physics [1]. One of the most important partial differential equations is the nonlinear Schrodinger equation (NLSE) which is applicable to classical and quantum mechanics [2].

The linear Schrodinger equation was educed in 1925 and published in 1926 by Erwin Schrodinger, an Austrian physicist [3]. When a system includes L quantum particles, $3L + 1$ dimensions will be produced in the Schrodinger equation. Solving the Schrodinger equation for dynamics of L particles with $L > 10$ is problematic due to such high dimensions. This difficulty was overcome by using Hatree-Fork method which converted the linear Schrodinger equation with $3L + 1$ dimensions to a $3 + 1$ dimensions NLSE. Although the new obstacles in the NLSE due to the nonlinearity, the dimensions were shrunken dramatically in comparison with the main system. This provided an opportunity to investigate dynamics of L particles at large values of L [4].

The NLSE can be derived from Bose-Einstein condensation (BEC) and wave propagation [4]. In view of mathematics, dealing with NLSE, even with the linear one, may be precise and critical because such equation possesses a mixture of the properties of parabolic and hyperbolic equation [5].

In theoretical physics, the NLSE rises in describing nonlinear waves like transmission of the laser beam over a medium with refraction index influenced by the amplitude, plasma waves, water waves of an ideal fluid at the free surface and light propagation through nonlinear optical fibers [2, 6]. In general, the NLSE is regarded as a cosmopolitan equation that dominates the transmission of slowly varying packets of quasi-monochromatic waves in dispersive and weakly nonlinear media. The NLSE supposes nonlinearities to be weak, whereas a dispersion is to be finite at the scale of the carrying wave. While in other situations, a reductive perturbative expansion, when both dispersion and nonlinearities are equally weak, results long-wavelength equations like the Korteweg–de Vries (KdV), the Benjamin–Ono or the Kadomtsev–Petviashvili equations in multi dimensions. Near collapse, the validity of the NLSE breaks down where the fundamental assumptions of large-scale modulation and small amplitude, in comparison to the frequency and the wavelength of the carrier, are no longer satisfied [2, 7]. Between the stability and instability, there is a group of critical points called singular points. The attitude of solutions around these points changes significantly to form numerous solution forms in these regions, such as solitons, periodics, shocks, solitons of dark and bright envelopes and rogue waves. Around the singular points, the directions of these solutions are changing [8].

In quantum mechanics, the one-dimensional NLSE is a particular case of the classical nonlinear Schrodinger field, which in turn is a classical limit of a quantum Schrodinger field [2]. The quantum NLSE respects the bose field's commutation relations, including the resulting uncertainty relations between observables. In this sense, it is an exact equation within the range of validity of the Born approximation. Generally, it governs the full quantum field theory of bosons in one dimension, beyond any mean-field description [9]. In addition, the quantized vortex stretching in superfluid HeII turbulence is studied by using the NLSE [10].

Moreover, at present, the study of nonlinear propagations in dispersion-dissipation medium plays a very motivating role to investigate physical problems of fluids, superfluids, ocean, geophysics, hydrodynamics, quantum fluids and fiber-optical new communications technology applications [11–20]. Dissipative and dispersive progressing waves are unstable trains that rise automatically in nonlinear weak dispersion media. A soliton will be formed when the dispersion dominates over the dissipation due to the balance between nonlinearity and dispersion. Furthermore, vice versa, when the dissipation is superior, a shock wave is constructed because of the combination between the nonlinearity and dissipation effects. The front steepening induced by nonlinear effects develops a gradient catastrophe. When a gradient catastrophe is developed, dispersion will encourage the beginning of oscillatory shock profiles [14, 21–23]. Experimentally, such dispersive breaking was first discovered at the beginning of the year 1970. The oscillatory nature of the shock appearing in the extremely rarefied (collisionless) plasma was firstly predicted by Sagdeev and coworkers [24]. On another point of view, nonlinear unlocalized waves structure such as dissipative shock applications can be noticed in space fluid, laboratory experiments and scientific observations [25–34].

It is worth mentioning that the NLSE displays two essential types of breaking according to the formula of the initial solution used. For a bright wave, the dispersive effects display two propagating envelopes that are the type of breaking occurs in fibers. On the contrary, when a dark waveform represents the initial solution, the breaking arises at time equals to zero [21, 22, 24]. The resulting oscillatory waves exhibit one narrower central zero velocity soliton with symmetric pairs [21, 22].

Previously, the exact solution of the NLSE was obtained analytically by writing this equation in real forms through some transformations and then using some methods like Jacobi elliptic expansion, tanh-function method, Cole-Hopf transformation, Hirota bilinear method, inverse scattering method and others. Lately, direct methods are utilized to find the exact wave solutions of the NLSE such as, complex tanh-function method, complex hyperbolic-function method, sub-ordinary differential equation method, complex ansatz method, complex Jacobi elliptic method and so on [1].

Furthermore, there are various numerical methods for solving the NLSE, including the well-known spectral and pseudospectral methods which depend on the use of fast Fourier transforms, the finite difference methods which are flexible and easily implemented, space-time finite element methods, and the improved quadrature discretization method [35].

The NLSE has two kinds from the viewpoint of the features of their solutions; one is called self-focusing which has opposite signs of dispersion and nonlinear term and possesses bright soliton solutions whose modulus grows from a certain value at infinity and tend to the same value. On the contrary, in the other kind the de-focusing NLSE, the dispersion and nonlinear terms have the same signs and its solution is called a dark soliton whose modulus is less than the one of the uniform solution where the dark soliton propagates [36, 37]. Solitons are exact solutions of the NLSE that represent a wave of permanent form even when interacting with other solitons as if the principle of the superposition was valid [38]. In addition, the NLSE has a plane-wave solution provided that the dispersive relation between wavenumber and amplitude of the complex wave function is satisfied [4].

Besides the importance of the NLSE, the study of the coupled nonlinear Schrodinger equations (CNLSEs) has received a great deal of attention recently because of their appearance as governing equations in many physical areas like nonlinear optics, including optical communications, bio-physics, multi-component BEC at zero temperature, etc. Specifically, soliton pulse type transmission in fiber arrays and multimode fibers is ruled by a system of N-NLSEs which is often not integrable [39].

Nonlinear fiber optics describes the phenomena of nonlinear optical occurring inside optical fibers. To manufacture optics, two different substances are used to guide the light pulses to travel inside the cables that carry information sent from one side to another. Polymeric fibers are usually utilized through short-distance transfer and in installations rough surroundings, while glass fibers are used for high quality and long-distance data transfer [40]. The information holding pulses travels for hundreds, or even thousands, of kilometers and experiences various degeneration procedures [41]. Although the beginning of the optical fiber communications field dates back to 1960, the commercial usage of optical fibers became workable only when fiber losses were reduced significantly after 1970. The single-mode fibers are mainly used to study nonlinear effects, where single-mode fibers mean that the electromagnetic wave in the yz (transverse) directions has a stationary shape called a mode [5].

1.2 Motivation

In the telecommunications area, it has been needing to transfer more data in a faster time with appropriate bandwidth and frequency via high transmission [42, 43]. New communications technology has become essential according to fiber-optical applications in allowing longer transmission distances with higher bandwidths [5].

Likewise, in the infrared ray region, fiber optics is the most critical preferred transmission method. These fibers have been used in networks for the purpose of transoceanic and terrestrial long-haul to offer low losing, great bandwidths, and immune electromagnetic properties. Currently, optical communications have depleted all the degrees of freedom in single-mode fiber except the space dimension [44, 45].

Today, traveling pulses in optical fiber transfer most of the telecommunications of all data. Transmission in fibers are preferable in comparison to electrical cables because of two compelling features: waves in fibers suffer low power loss and fibers supply an extremely wide bandwidth [46].

Another aspect of comparison between optical and electrical transmission indicates that they share a defect wherein each kind of ducts, group velocity depends on frequency, thus transmitted signals suffer from distortion due to group velocity dispersion (GVD) because each signal needs a specific bandwidth. This distortion is linear which implies that the possibility of its compensation completely by using an opposite dispersion element [46].

Rahman studied the rarefactive (compressive) time damping soliton properties depending on the superthermal factor of electrons and positrons. Furthermore, the behavior of existing waves at critical double layers is taken into account [47]. In addition, the increased attention to dissipative and dispersive waves has provided opportunities to discover new systems to find new higher resolution data, predict new phenomena and counteract unphysical singularities in dispersive conservative media [24, 48].

It was thought that the incidence of shock waves is often referred to and investigated concerning higher-order cubic terms that affect ultrashort pulses. However, it turned out that the Kerr effect in partnership with standard nonlinear term in the NLSE represents the major-order influence responsible for wave-breaking under typical experimental conditions that involved pulses with a period in the range from a few psec to nsec [48].

The temporal-optical solitonic waves of the NLSE have been investigated theoretically, experimentally and by the simulation to examine their properties in communications using fiber optics [8, 41, 49–58]. Also, these optical profiles are introduced as a serious alternative to successive generations in ultrafast telecommunications systems [8,49,53,55]. The nonlinear form of NLSE became one of the major delegated ways for depicting the waves behavior in a large number of nonlinear physical applications i.e., plasmas, optics, fluids, deeper water, semiconductors, BEC and dynamical models [8,41,49–60].

Moreover, the NLSE has various applications in many fields, as it has been mentioned in the background. In the field of optical signal transmission, the universal known model NLSE enabled the studies and numerical calculations as well as it performs a definitely main role in nonlinear applications and studies because it has soliton solutions. Another significant property of the standard NLSE is that it has an infinite number of polynomial conservation laws [37]. The NLSE in the context of the drift wave packet and that describes optical pulses propagation has the following form

$$i\frac{\partial\phi}{\partial t}+i\delta\frac{\partial\phi}{\partial x}-\frac{\beta}{2}\frac{\partial^2\phi}{\partial x^2}+\gamma|\phi|^2\phi=0,\quad -\infty<x<\infty,\ \ t\geq 0, \tag{1.1}$$

where $\phi\,(x,t)$ is the complex slowly varying pulse envelope where $|\phi|^2$ is measured in W, $\delta\equiv\frac{1}{v_g}$ is the drift wave packet parameter, which is measured in units of (ps), of the pulse envelope moves at the group velocity v_g which is measured in in units of (m/s), $\beta\in\{R-\{0\}\}$ is the GVD parameter that measured in units of (ps^2/m), γ is the self-phase modulation (SPM) parameter that measured in units of $(Wm)^{-1}$ and $i=\sqrt{-1}$ [5,22,24,41,46,61,62].

The chromatic dispersion (CD) of the first and second-order δ and β with the nonlinearity SPM considered in this NLSE (1.1) occupy a pivotal role in the propagation of short optical pulses in the nonlinear regime. The fiber dispersion arises due to the frequency dependence of the refractive index and accordingly different spectral components associated with the pulse travel at different speeds which gives the dispersion a major influence in the transmission of pulse in fibers. Also, the CD of the second-order β is responsible for pulse broadening. The nonlinear parameter SPM γ describes the self-induced phase shift occurring inside an optical field during the pulse propagation. The major effect of SPM is to modulate spectral broadening of ultrashort optical signals propagating through the fiber due to the intensity dependence of the refractive index [22].

In the nonlinear regime, which is considered in this work via the NLSE (1.1), the interplay between dispersion and nonlinearity can lead to qualitatively different behavior depending on the sign of the GVD. In the anomalous dispersion regime of fibers where $\beta < 0$, the SPM reduces spectral broadening of signals and contributes to developing of optical solitons. On the contrary, in the normal dispersion regime of fibers where $\beta > 0$, the SPM enhances the broadening rate [22].

Therefore, these features of the NLSE and its factors have motivated the study and discussion of the influence of the parameters considered in the NLSE (1.1) on the solitonic optical pulses and interaction between them through transmission in this work. Furthermore, the relation between NLSE and the emergence of shock waves has been a strong motivator for discussing the effects of the parameters on shock waves and oscillations behavior. The conserved quantities affected by these parameters are of interest in this work.

1.3 Problem Statement

Pulse propagation in a nonlinear optical fiber can be studied by solving the NLSE (1.1). It is clear that this NLSE in the context of the drift wave packet is a very complicated partial differential equation and cannot be solved without any approximations or transformations. So, numerical calculations and schemes must be taken into account to solve the complicated problem and to study the interplay among the first and second-order chromatic dispersions (CDs) and Kerr nonlinear effects in optical fibers [63–67]. Moreover, some researchers [5, 35, 46, 68, 69] turn to solve and study the NLSE with δ by using some transforms to convert it to the normalized standard form of the NLSE as follows

$$i\frac{\partial U}{\partial t} - \frac{\partial^2 U}{\partial x^2} + q|U|^2 U = 0, \tag{1.2}$$

where $U = U(t, x)$ is a complex-valued function and $q \in R$, or by ignoring the coefficient of the drift wave packet. These simplifications may produce less accurate solutions. Therefore, the main focus in this work is to solve the NLSE (1.1) numerically when ($\delta \neq 0$) by deriving a new scheme using an implicit finite difference method with high accuracy and stability in order to study soliton solutions and shock waves during transmission in single-mode optical fibers. In addition, this work has concentrated on examining the stability and accuracy of the obtained scheme. The resulting scheme has been applied on many numerical experiments to investigate and discuss the influence of the first order CD, δ on the shapes, amplitudes and the velocities of the solitons, new soliton-like periodic and new shock waves and on the conserved quantities like mass, momentum and energy of a system governed by the NLSE.

In addition, in this work, Eq.(1.1) when $\delta = 0.0$ has been investigated. In this case, many researchers [8, 35, 41, 52, 53, 56, 58, 68] have treated the resulting NLSE by using different numerical and analytical methods with no focus on the effects of the parameters considered in the resulting NLSE. But in this work, the NLSE in the field of single-mode of light progress in nonlinear fibers has been solved numerically by using two implicit finite difference methods to inspect the distinctive reliance of the waves forms and conserved quantities on GVD and SPM. Also, the movement and shape properties of one, two and three solitons interaction, new soliton-like periodic and new shock waves have been examined. Likewise, their conservation laws in terms of γ and β are taken into account.

In addition, the full discretization of the NLSE using the proposed implicit numerical methods produces nonlinear block tridiagonal systems [6, 38, 70] which need to be solved.

1.4 Scope of the Research

An optical fiber is a thin glass thread consisting of a cladding around a central core made of fused silica with an ultra-low loss. In a single-mode fiber, the internal core is microscopic with a radius of less than $5\mu m$. In this work, single-mode optical fibers are regarded to transmit signals from one end to the other rather than metal wires because signals pass through the fibers with lower loss and block electromagnetic interference. Another feature of single-mode optical fibers is that various applications, including optical communications, depend on the use of them almost exclusively [5,71].

In the slowly varying envelope approximation, optical pulses propagation along single-mode fibers has been studied in this work by solving the NLSE (1.1) numerically. This study is considered in the presence of the first and second-order CDs and SPM coefficients that play an important role in producing and propagating of different types of solitons and shock waves depending on the values and signs of these parameters. The choice of nonlinear parameter γ and CDs δ and β is prompted by their convenience to produce pulses soliton shape during propagating in fiber for optic telecommunication systems and by their usage in the previous studies and applications.

Moreover, in weakly dispersive nonlinear media, the dispersion factor and Kerr effect associated with the nonlinear term in the NLSE perform a significant role in the forming of an unlocal oscillatory structure with short-wavelength. The transition of the wave-like between two edges leading and trailing is generally called a dispersive shock. In such a frame, it should be clear that dispersive shock waves compose the dispersive counterpart of viscous regularization of classical shock waves which arise when the dissipation surpasses the dispersive effects [24, 72]. Thus, in this work, the properties of the dispersive and dissipative shock waves have been discussed via the schemes obtained from solving the NLSE (1.1) when $\delta \neq 0$ and when it is neglected by using implicit finite difference methods.

In this work, the trivial Neumann boundary condition has been used to solve the regarded models where they are boundary value problems. In addition, the two implicit finite difference methods, Crank-Nicolson and Douglas, have been used to solve the NLSE as they are beneficial and coinciding with the ultrashort optical signals propagation which have very short widths that is the signal includes a few optical cycles. Another feature of both of Crank-Nicolson and Douglas methods is that they are unconditionally stable which ensures that there is no limitation on the time step size [22, 38, 65, 73–75]. Many authors used the Crank-Nicolson technique for numerical estimation of nonlinear complicated partial differential models such as heat conductions, fluid mechanics and optics because of its accuracy and stability in obtaining a solution that agrees with real data and observations [76, 77] In addition, under the same grid and coefficients, the implicit approximations are dramatically more accurate than the explicit ones, although the high cost of solving a system of algebraic equations at each time step. Although the common use of the split-step Fourier method for investigating nonlinearity in optical fibers, its implementation consumes time significantly when solving the NLSE that simulates the evolution of wavelength-division-multiplexed light pulse systems. These reasons make the finite difference methods very attractive for solving the NLSE [22, 38, 65, 73–75]. The stability of the resulting schemes has been proven by using von-Neumann stability analysis. Furthermore, the accuracy of the resulting schemes has been shown by evaluating the truncation error with Taylor's series expansion. Also, the validation of the numerical solutions has been confirmed by studying the stability and accuracy of the resulting schemes analytically and by comparing some of the obtained findings with previous results. Also, the consistency and harmony in the representation of the solutions at very large times validate the resulting schemes and codes.

Likewise, Newton's method has been used to solve the resulting block nonlinear tridiagonal systems after applying the former numerical methods. All the computational processes have been implemented by using FORTRAN software.

1.5 Research Objectives

This work aims to achieve the following objectives

1. To develop two computer codes in order to solve the NLSE (1.1) in the absence of the first order CD ($\delta = 0$). The first code has been derived by using the Crank-Nicolson technique, where the resulting scheme is second order in time and space. The second code has been derived by using the Douglas technique, where the resulting scheme is fourth-order in space and second-order in time.
2. To develop a computer code in order to solve the NLSE (1.1) when $\delta \neq 0$ by using the Crank-Nicolson scheme.
3. To prove the accuracy and stability, by using von-Neumann stability analysis, of the resulting numerical schemes mentioned in the Objectives (1) and (2).
4. To investigate the effects of the CDs of the first and second-order and SPM on solitons and shock waves properties and on their conserved quantities.

1.6 Contribution of the Research

The tremendous development in the uses and applications of the nonlinear phenomena in the field of communication via optical fibers and what may affect waves during transmission motivate this study to address some of the effects by using a mathematical numerical computational treatment.

Some nonlinear effects control the transmission of waves in fiber optics; it is imperative to study them carefully. Examples of these influences are the intensity dependence of medium refractive index and the phenomenon of inelastic-scattering. Dispersion properties of the fiber medium play numerous roles in the investigation of communicating nonlinear wave existence in optical fibers as the relatively optical wave modes. These modes' propagations can either reinforce or frustrate the impacts of different mode conditions and coupling as they range from the balance of linear-nonlinear phase shifts in soliton formations to damped and super continuum generations.

In this work, the spatial equivalents of dispersions as diffractions and dissipations are used in defocusing nonlinearity to form different forms of dispersive shock propagating waves. Furthermore, the unliked dissipative wave shocks which absorb energy make dispersion and nonlinearity perform in the same directions creating a damped dissipative soliton train. Here, the considered dispersions, dissipations and nonlinearity interactions have been described numerically by the NLSE. NLSE has been established to describe waves displaying weak self-defocusing nonlinear waves, self-phase modulations, chromatic dispersions and cross-phase modulations. So, this equation has been attractive for the consideration of dispersive and dissipative wave shock phenomena.

Numerically, this study has the following contributions:

i A numerical code has been produced for solving the NLSE (1.1) when $\delta = 0$ using Crank-Nicolson method. The novelty is using this code to investigate the impacts of a wide range of values of β and γ factors on several input waves properties including solitons, interaction between them, new soliton-like periodic and dissipative and new shock waves as well as on the conserved quantities. It has been proven that the accuracy of the numerical results is second-order in space and time. It has been proven that the resulting scheme is unconditionally stable.

ii A numerical code has been produced for solving the NLSE (1.1) when $\delta = 0$ using Douglas method to validate the obtained findings using Crank-Nicolson method in the point (i) and it has been found that all the numerical results obtained using Douglas method are consistent with those obtained using Crank-Nicolson method. In addition, it has been confirmed that Douglas method offers results with higher order accuracy as it is fourth-order in space and second-order in time, thus Douglas method is better than Crank-Nicolson one. It has been proven that the resulting scheme is unconditionally stable.

iii A novel numerical code has been produced for solving the NLSE (1.1) when $\delta \neq 0$ using Crank-Nicolson method. Another novelty is using this new code to investigate the influence of a wide range of δ values and overlap impacts among first-second-order CDs and SPM factors on several input waves properties including solitons, interaction between them, novel soliton-like periodic and dissipative and novel shock waves as well as on the conserved quantities. It has been proven that the accuracy of the new numerical results is second-order in space and time. It has been proven that the new resulting scheme is unconditionally stable.

The produced codes have enabled to overcome the difficulties in solving the NLSE (1.1) and provided the opportunity to study a lot of applications, especially when $\delta \neq 0$ in this equation, which was not solved in advance except by using some transforms or by neglecting the first-order CD factor. The numerical results can be used to guide the specialists in fiber manufacture to avoid some values that lead to oscillations that cause the quality of communications to decrease.

1.7 Outline

The thesis is organized as follows; the first chapter has established the problem background, motivation of the research, problem statement, scope of the research, research objectives and research contribution. Chapter two has presented some basic information in the field of optical fibers, including solitons and dispersive-dissipative shock waves. Also, the second chapter has given a review of the literature related to the forms of NLSE in various fields and to solve this equation numerically and analytically. In addition, a review of the usage of the implicit Crank-Nicolson method and Douglas technique in the literature has been displayed in Chapter two. Chapter three has provided the methodology of the research and some important related concepts. In the fourth chapter, the NLSE has been solved by using the Crank-Nicolson method as well as several new initials have been presented. In Chapter five, the NLSE has been solved by using Douglas idea and some numerical experiments have been conducted. In Chapter six, the NLSE when $\delta \neq 0$ has been solved by using the Crank-Nicolson method in addition to displaying a comparison between the two models of the NLSE for the considered new initial conditions. Lastly, a summary of the research has been demonstrated.

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Non-Indexed conference proceedings

1. Oral presentation in the 7th International Conference and Workshop on Basic and Applied Sciences (ICOWOBAS 2019) held from 16th-17th July 2019, Johor Bahru, Malaysia.