THE DEVELOPMENT OF COMPUTATIONAL THINKING AND MATHEMATICS PROBLEM SOLVING SKILL THROUGH MATHEMATICS MODELLING ACTIVITIES

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DEDICATION

This project report is dedicated to the universe, for all the life lessons that you had taught me all this long.

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It was never an easy decision to commit into this journey of education given that I must manage well between my job and study.

To Associate Professor Dr. Halim Bin Abdullah, my supervisor, for being critical and practical in supervising this research project.

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Lastly, to this universe, for all the life lessons that you had taught me all this long.

ABSTRACT

Based on the latest TIMSS report, it could be observed that Year 8 students from Malaysia has not been performing well ever since participating in this global assessment on mathematics achievement. There were some studies stated that by equipping the students with computational thinking, it could improve the students' mathematics achievement. Likewise, some studies also claimed that computational thinking skill could be developed in a stimulated modelling environment. However, there were not many studies investigated about the relationship of mathematics modelling activities and the development of computational thinking skill and mathematics problem-solving competency. As a result, this study which adopted the qualitative design with the seven respondents ought to examine the development of computational thinking skill and mathematics problem-solving competency via mathematics modelling activities among secondary students. The modelling activities were conducted for the seven weeks including introduction of modelling, individual modelling task and group modelling task. The computational thinking test and mathematics test were conducted for more data collection. As for data analysis, the observation on works and interviews were executed, and the scores obtained from both tests were compared and analysed as well. Based on the findings of the study, it could be concluded that the students were able to develop their computational thinking skill and mathematics problem-solving competency via mathematics modelling activities with different progress. These results indicated the mathematics modelling could be conducted at the school for the benefits of educators and students. Few recommendations were listed for the usage of future research.

ABSTRAK

Berdasarkan laporan TIMSS terkini, ia itu dapat dilihat bahawa pelajar Tingkatan 2 dari Malaysia tidak menunjukkan prestasi yang baik sejak mengikuti penilaian global mengenai pencapaian matematik ini. Terdapat beberapa kajian yang menyatakan bahawa dengan melengkapkan pelajar dengan pemikiran komputasional, ia dapat meningkatkan pencapaian matematik pelajar. Begitu juga, beberapa kajian juga mendakwa bahawa kemahiran pemikiran komputasional dapat dikembangkan dalam persekitaran pemodelan yang terangsang. Walau bagaimanapun, tidak banyak kajian yang diteliti mengenai hubungan aktiviti pemodelan matematik dan pengembangan kemahiran pemikiran komputasionaldan kecekapan penyelesaian masalah matematik. Hasilnya, kajian ini yang menggunakan reka bentuk kualitatif dengan ketujuh-tujuh responden serta mengkaji perkembangan pemikiran komputasional dan perkembangan tentang kecekapan penyelesaian masalah matematik melalui aktiviti pemodelan matematik di kalangan pelajar menengah. Aktiviti pemodelan dilakukan selama tujuh minggu termasuk pengenalan pemodelan, tugas pemodelan individu dan tugas pemodelan kumpulan. Ujian pemikiran komputasi dan ujian matematik dijalankan untuk pengumpulan data yang lebih banyak. Untuk analisis data, pemerhatian terhadap karya dan wawancara dilakukan, dan skor yang diperoleh dari kedua-dua ujian itu dibandingkan dan dianalisis juga. Berdasarkan hasil kajian, dapat disimpulkan bahawa para pelajar dapat mengembangkan kemahiran berfikir komputasi dan kecekapan penyelesaian masalah matematik melalui aktiviti pemodelan matematik dengan kemajuan yang berbeza. Hasil ini menunjukkan bahawa pemodelan matematik dapat dilakukan di sekolah untuk faedah pendidik dan pelajar. Beberapa cadangan disenaraikan untuk penggunaan penyelidikan masa depan.

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LIST OF ABBREVIATIONS

AI	-	Artificial Intelligence
CAS	-	Computing at School
CCSSM	-	Common Core State Standards for Mathematics
CT	-	Computational Thinking
HOTS	-	Higher Order Thinking Skill
ICT	-	Information Communications Technology
KBSM	-	Kurikulum Bersepadu Sekolah Menengah
LPM	-	Malaysia Examination Board
MOE	-	Ministry of Education
MPS	-	Mathematics Problem Solving
NBA	-	National Basketball Association
NCTM	-	National Council of Teachers of Mathematics
PARCC	-	Partnership for Assessment of Readiness for College and Careers
PCK	-	Pedagogical Content Knowledge
PISA	-	Programme for International Student Assessment
PMR	-	Lower Secondary Assessment
PT3	-	Form 3 Assessment
SBAC	-	Smarter Balanced Assessment Consortium
SPM	-	Sijil Pengajian Malaysia
STEAM	-	Science, Technology, Engineering, Arts and Mathematics
UK	-	United Kingdom

LIST OF SYMBOLS

a	-	Acceleration
c	-	y-intercept
cms^{-1}	-	Centimetre per second
m	-	Gradient
ms^{-1}	-	Metre per second
v	-	Velocity
x	-	x value
y	-	y value

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CHAPTER 1

INTRODUCTION

1.1 Introduction

On a global context, Programme for International Student Assessment (PISA) (2006) stressed on the importance of producing the young graduates with high mathematics literacy to enable the development of the country in the aspects of technology and economy. Similarly, according to Curriculum Development Centre (CDC) (2004), in terms secondary mathematics syllabuses, they intended to enable the students to apply mathematics literacy as the knowledge base intensely to improve decisions thus solve problems. Mathematics literacy referred to the ability to apply mathematics skills to think creatively and critically to solve daily problem (Department of Education, 2003). It can be concluded that mathematics literacy is the measurement of competency of thinking mathematically. For instance, several components of mathematics literacy include making connections between variables, identifying patterns and modelling.

The main goal of mathematics curriculum in Malaysia is to discover and develop the learners who could think mathematically (Ministry of Education, 2003). The definition of thinking mathematically is not cleared. To illustrate, it was explained and defined as the process of consuming mathematics thinking extensively to distinguish the relationships between the variables to resolve the problem (Aydin & Ubuz, 2014). On the other hand, Stacey (2006) stated that the process of thinking mathematically involves solving problem with different approaches and strategies. Likewise, in terms of mathematics syllabus, it is essential to design the mathematics problem which is engaging to the learners and being related to the world (Boaler, 2002). Ultimately, Ministry of Education of Malaysia aimed to produce young graduates who can think independently and critically to solve real world problem.

Consequently, the mathematics process shall be realised to support the learners in handling the problem. Leveraging on various approaches of mathematics to solve real world problem is associated with the process of putting decision into effects which is regarded as application of mathematics (ICMI Study 14, 2002). It is vastly connected between the usage of mathematics as the knowledge base and the implementation of the plans to tackle world issues. It is dynamic to link real world issues to mathematics in the classroom instruction (Smith & Morgan, 2016). This is to enhance the learners' understanding to be proactive in mastering mathematics (Gainsburg, 2008). Likewise, when a mathematics problem is combined with the components of real-world problem whereby the students failed to solve the problem within the calculation of the mathematics syllabuses then it would be recognised as an applied problem (Bergman, 2009). The endeavours in establishing the linkage between mathematics and real-life issues has been highlighted for the development of mathematics thinking Carrejo & Marshall, 2007). Linking mathematics with real world setting is not just about producing more problem based on recent real issues but the efforts in strengthening the relationship between these two components shall be the main focus in the mathematics curriculum (Maaß, 2004).

Therefore, problem solving is being emphasised a lot in teaching and learning of mathematics in the school in Malaysia. For instance, in terms of teachers training, "Boston Model" was introduced in the teacher's trainees' training to highlight about thinking process and metacognition (Nagappan, 2001). Numerous approaches were provided by the Ministry of Education in the school curriculum namely trial and error, recognising patterns, using a table, working backwards and rational reasoning. Furthermore, i-THINK (thinking maps) was introduced to promote students' higher order thinking skills (Yusop & Mahamod, 2016). Thinking could be defined as the progression of intellectual interpretation via several mental actives such as assessment, visualisation and problem solving. Problem solving motivates the students discovering information and theories created by the problem hence mastering the technique of approaching the problem and acquiring the proper mindset towards problem solving (Schwartz, 2013). Problem-solving is described as a useful and significant learning methods with which students must examine the tactics to resolve the issues by themselves (Liu, 2011). Bellanca (2010) indicated that the core value of education is

to create capable problem solvers who can think critically and rationally to solve 21st century problem. Problem-solving is defined as a revolution from an unwanted original state to a required state (Beecher, 2017) by eliminating the difficulties. It required huge number mental process of logical thinking and reasoning (Spector & Park, 2012). There are seven stages often stated in the problem-solving process, namely:

i. the acknowledgement of an issue
ii. the intellectual description of the issue
iii. the enhancement of a tactic to resolve the issue
iv. the formation of knowledge related to the issue
v. the distribution of intellectual and physical resources to resolving the issue
vi. the growth observation concerning the objective
vii. the assessment of the resolution

In the recent years, the advancement of technology has changed the way how education and teaching shall be delivered from traditional pedagogy to the modern method in the classroom. For instance, online classroom instead of face-to-face classroom; smartboard instead of whiteboard and digital materials instead of printed learning materials. The students were explored into the world under the transformation brought by industry revolution 4.0. The students are experiencing the digital age where they were familiar with the application of the advancement of Information Communications Technology (ICT) and Artificial Intelligence (AI). To deal with the challenges of 21st Century, one needs to be equipped with competent skill to perform well in the applications of robotics and computing. As a result, the topics of computational thinking (CT) has been widely discussed in the field of research and educations. In K-12 education, ISTE and CSTA (2011) defined CT as the problem-solving process by involving algorithms thinking, problem formulation, generalization, and pattern recognition. Likewise, the definition of computational thinking is regarded as the logical thoughts process of delivering algorithms (Child, 2015).

Further, computational thinking has frequently been associated in the problem-solving setting (Román-González, PérezGonzález, & Jiménez-Fernéndez, 2017). CT

has widely been associated with STEAM (Science, Technology, Engineering, Arts and Mathematics). Both has a corresponding relationship among others (Barr & Stephenson, 2011). The studies showed that CT skills could be developed with better understanding when the students were exposed to scientific and mathematics problem in a stimulated modelling environment (Brennan & Resnick, 2012; Basu et al., 2017). This helps the students to interpret the scientific and mathematics concepts to understand the real-world problem. Thus, as can be seen that a STEAM classroom could be executed with mathematics modelling tasks to access the CT skills.

The mathematics modelling is closely related to application of mathematics. The model or the modelling process itself served the role of integrating the element of real-world problem which is accurate and true to life. It helps the learners to comprehend and understand mathematics with the modelling tasks that is relevant to real life issues (Kaur & Dindyal, 2010). Mathematics modelling enables the possibility to transfer realistic content to mathematics to drive for the usage and conceptualisation of applied mathematics (Jablonka, 2007). It is vital to connect the teaching and learning of the classroom to the real life setting where this relevance raises the enthusiasm of the students to apply mathematics concept and application daily life (Gainsburg, 2008). The concept and application of modelling has been presented as the pedagogy to enhance the significance and validity of STEM subjects with the real-world issues (Banks & Barlex, 2014). A capable problem solver can interact and engross between the components of real world and mathematics with the intention of achieving the stated goal (Blum et al., 2002). Ottesen (2001) stated that mathematics modelling could be leveraged as the tool to explore about mathematics and to develop mathematics thinking. Blum et al. (2007) stated that applied mathematics as well as modelling were being associated to illustrate the meaningful relationships between the area of mathematics and the extra-mathematics component.

Mathematics modelling was being highlighted in the national education curriculum of Germany ever since the year 2003 (Greefrath, 2016). The education syllabus requires the students to apply modelling to solve real life problem. In addition, Kaiser & Sriraman (2006) illustrated the specific modelling as modelling in the context of education by establishing the mathematics thinking in the context of the real-world

setting. The efforts in relating these 2 factors shall be emphasised for improved teaching and learning of mathematics. In the United States, there were many studies indicated that mathematics modelling ought to be emphasised in the K-12 education to assist the students in gaining experiences to solve real world problem with mathematics thinking (Asempapa, 2018).

It is frequently being discussed the significance of bringing the realistic contents to the K-12 mathematics classroom in united states (National Council of Teachers of Mathematics [NCTM], 2000). Hoyles et al. (2002) illustrated that the company preferred to hire young graduates with better mathematics fluency to be able to handle complicated and dynamic systems in the office. As the technology advancements of the world drives the establishment of various complicated and new arising structures, it is vital to study and apply different models to understand the complex system by building the competency in leveraging on mathematics modelling (Hmelo, Holton, & Kolodner, 2000). Modelling was treated as a device for the young leaners to identify the problems happening around their communities thus analysing the situations via mathematics framework (Mukhopadhyay & Greer, 2001).

1.2 Background Problem

TIMSS would be conducted every four years to improve each country education system to produce competent global citizen. Malaysia had representatives from the 8th grade (Form 2) students in TIMSS assessment since the year 1999. Below is the table that indicates their performance as compared to the international average score from year 1999 till year 2015.

Table 1.1: Comparison of the 8th grade Malaysian and an International Average Score

Year	Malaysia	International Average Score
1999	519	487
2003	508	467
2007	474	450
2011	440	467
2015	465	473

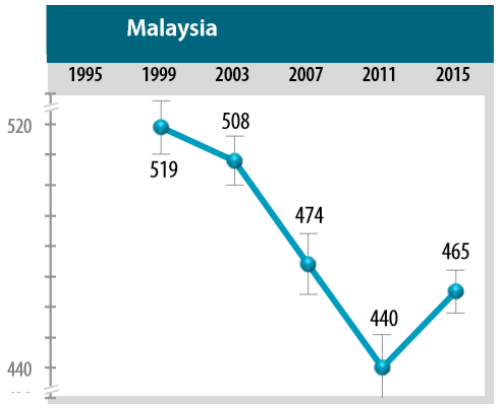

Figure 1.1: Trends of 8th Grade Malaysian' Mathematics Achievement (1999-2015)

From Figure 1.1, it is noticeable that Malaysia had performed poorly ever since taking part in this large-scale survey from the year 1999. Our average score in TIMSS has been dropping significantly from year 1999 till year 2011. It was about 79 marks difference in these 12 years. In TIMSS 2015, Malaysia achieved about 465 marks which is in between low international benchmark (400) and intermediate international benchmark (475). It is to imply that almost all students can utilize fundamental mathematics concepts in various scenarios (Mullis et., 2016). They can resolve issues related to the topic of negative numbers, decimals, percentage, and proportions in the context of straightforward question. On the contrary, it is suggested that Malaysian students failed to interpret data from various graphs. They failed to apply reasoning and generalizations in solving complex problem as shown in Figure 1.2 and Figure 1.3.

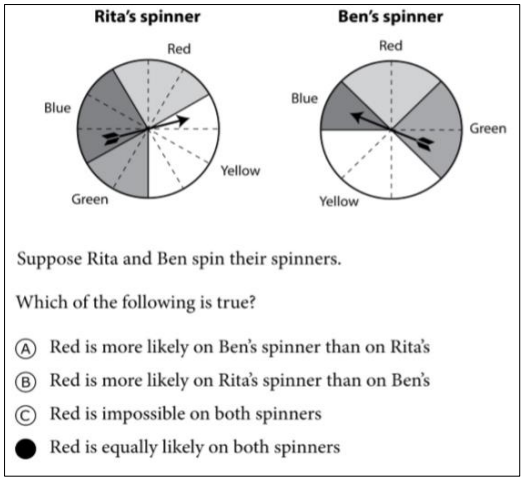

Figure 1.2: Example of High International Benchmark Question

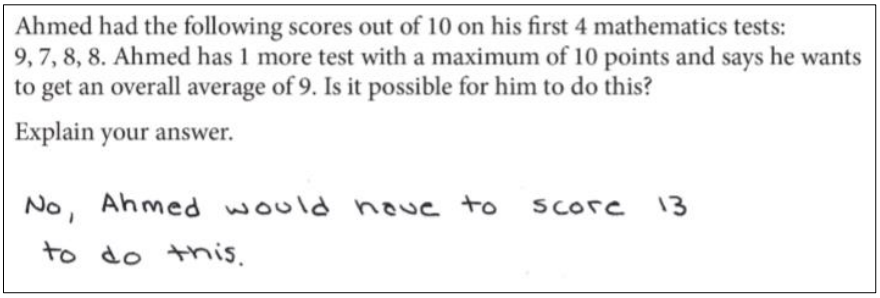

Figure 1.3: Example of Advanced International Benchmark Question

With that, the Malaysia Examination Board (LPM) had decided to instil more higher order thinking skill (HOTS) questions in one of the national assessments by transforming the Lower Secondary Assessment (PMR) to Form 3 Assessment (PT3) in year 2014. There will be more HOTS questions in PT3 as compared to PMR. This is due to a lack of proper assessment to examine the learners' higher order thinking. Most of the assessment materials are in the form of routine task where it requires minimum problem-solving skill. Nonetheless, in TIMSS 2015, the average marks obtained by Malaysian, which is 465, is still below the intermediate benchmark.

Dorothy (2017) claimed that the teachers prefer traditional teaching methods which is feed and spoon approach, where the learners could only involve in passive learning. The time constraints and lack of competency of the teachers could lead to these issues (Nagappan, 2001). Furthermore, the teachers must rush to complete the syllabus to cater the needs of examination-oriented culture learning environment (Wun, 2017). Further, while solving problems in the school, the students used the same methods to check back their solutions. Some even faced difficulties in interpretations of the problems (Bryant, 2006). Moreover, the learners tend to jump straight into the calculation part of the problem without much considerations on planning part (Faridah, 2004). To sum it up, the students do not own the conducive learning environment that provokes higher order thinking in solving problem. Further, it was the role of the educators to prepare suitable learning platform to encourage the students to deal with complicated mathematics problem successfully. The students were not trained to think computationally to solve complicated questions in mathematics. Correspondingly, problem solving has widely been discussed in the context of computational thinking (Korkmaz, Çakir, & Özden, 2017). In addition, Buteau, Gadanidis, Lovric & Muller

(2017) also agreed that CT activities enable the students to widen their perception on execution to mathematics problem solving.

On the contrary, the previous research showed that there were still many students that are not able to master computational skill (Papadopoulos & Tegos, 2012). Most of the students are only taught the basic mathematics skills with the use of computer application namely spreadsheet (Microsoft Excel). This is because most of the teachers are not exposed to the usage and application of computational thinking. Likewise, Sanford and Naidu (2016) argued that it is a must to obtain the trained teachers for the preparation and instruction of potential computational thinking education. Fields, Lui and Kafai (2019) stated that the students learnt and performed better when the teachers display and showcase their own computational thinking processes and errors during the teaching instruction. However, there was no guideline provided by the Ministry of Education (MOE) on the evaluation standard of the computational thinking skill. Thus, it is challenging for the educators to assess the level of the computational thinking of the students.

In the school, in teaching and learning, the teachers desired to focus more on the standard procedure or conventional approach in delivering the subject of science rather than to guide the students to discover the 'realistic' content of science (Clement, 2000). This suggest that the similar scenarios happened in the context of mathematics as well. The students were trained to solve word problems on textbook only. Even though some of the word problems in textbooks includes real life scenarios, the problem-solving process require general key steps in doing the calculation. It does not push the learners to go out of the comfort zone to relate all the considered variables in structuring the problem-solving thought process. Nonetheless, the general problem only allows the students to utilise the known variables and implement their workings in a safe environment (English & Lesh, 2003).

Most of the problem setting on the reference books are created in a way that overlook the features of real-world scenarios. Most of the learners were trained to solve the problem without much considerations on inferencing the genuine setting of the world (Boaler, 2002). Most of the times, the learners intended to ignore the practical

considerations on the components of the real-life situations when they were assigned with the mathematics problem (Dewolf et al., 2014). This is due to the students only trained in the classroom to solve conventional problem with the approaches and concepts introduced in the mathematics syllabuses.

Unlike Malaysia, Singapore has been introducing the concept of mathematics modelling and performing well in the assessment in TIMSS. The Ministry of Education in Singapore stated mathematics problem solving as the core element in Singapore's Mathematics Framework. There are five pillars in supporting the core element, which is "Concept", "Process", "Metacognition", "Attitudes" and "Skills". The work problems that is introduced in the textbooks could not be solved by using direct calculations only. It is a way more complex and it requires more critical thinking to be leveraged to work on the steps. Several non-routines, open-ended and real-world problem were introduced in the textbook (Andy, 2010). One of the most famous approach being taught in Singapore teaching and learning would be "Modelling". It helps the students to visualise the relationships between the elements. Furthermore, Singapore believes in students being positive in problem solving helps in enhancing their metacognitive skills. In terms of assessment, the usage of multiple-choice questions was being reduced to encourage the learners to "talk" in mathematics about their thought process and reasoning to work out the problem (Yeap & Kaur, 2008).

1.3 Statement of the Problem

Ever since Malaysia took part in TIMSS in 1999, our performance in 8^{th} grade mathematics has been underperforming. From average score of 519 in year 1999 till 465 in year 2015, it indicates that our students are not capable in solving HOTS or complicated questions that requires more than one step solution. In 2014, MOE has been working on transforming the national assessment from PMR to PT3 format to establish more subjective questions that is based on HOTS. Conversely, the students are still performing poorly in TIMSS 2015. There are many research stated that there is no conducive and inspiring learning environment that promotes the students in solving HOTS questions.

In some studies, it is stated that mathematics problem solving has often been portrayed in the terms of computational thinking. By improving the students' computational thinking, they learnt to recognise the pattern and establish an algorithm in solving problem. This is very similar to the steps of problem-solving model. There were studies indicated that computational thinking and mathematics problem-solving skills could be developed in the context of STEM subjects. It illustrated that the learners' mathematics problem-solving skills could be enhanced by cultivating computational thinking skills in the stimulated environment. The environment could be the modelling electing activities if the learners were given appropriate scaffolding and guidance from the educators. According to the previous researchers, mathematics modelling activities could contribute to the development of problem-solving skill by constructing a model via generalising the problem setting. In fact, the research studies about the development of computational thinking skill and mathematics problem-solving via mathematics modelling activities in the context of national syllabus of mathematics of Malaysia has yet to be investigated. Thus, it is significant to conduct this research to investigate the development of computational thinking skill and mathematics problem-solving competency of secondary students through modelling-based activities.

1.4 Research Objectives

The objectives of the study are to investigate

i. the development of the computational skill of secondary students in the subject of mathematics based on mathematics modelling-based activities
ii. the development of mathematics problem-solving skill of secondary students based on mathematics modelling-based activities

1.5 Research Questions

Based on the research objectives, the research questions were identified and introduced as below:

i. Do the secondary students develop computational skill through mathematics modelling activities?
    a. decomposition
    b. abstraction
    c. pattern recognition
    d. algorithms
    e. logical reasoning
    f. evaluation

ii. Do the secondary students enhance their mathematics problem-solving skill through mathematics modelling activities?
    a. understanding the problem
    b. devise the plan
    c. carrying out the plan
    d. looking back

1.6 Conceptual Framework

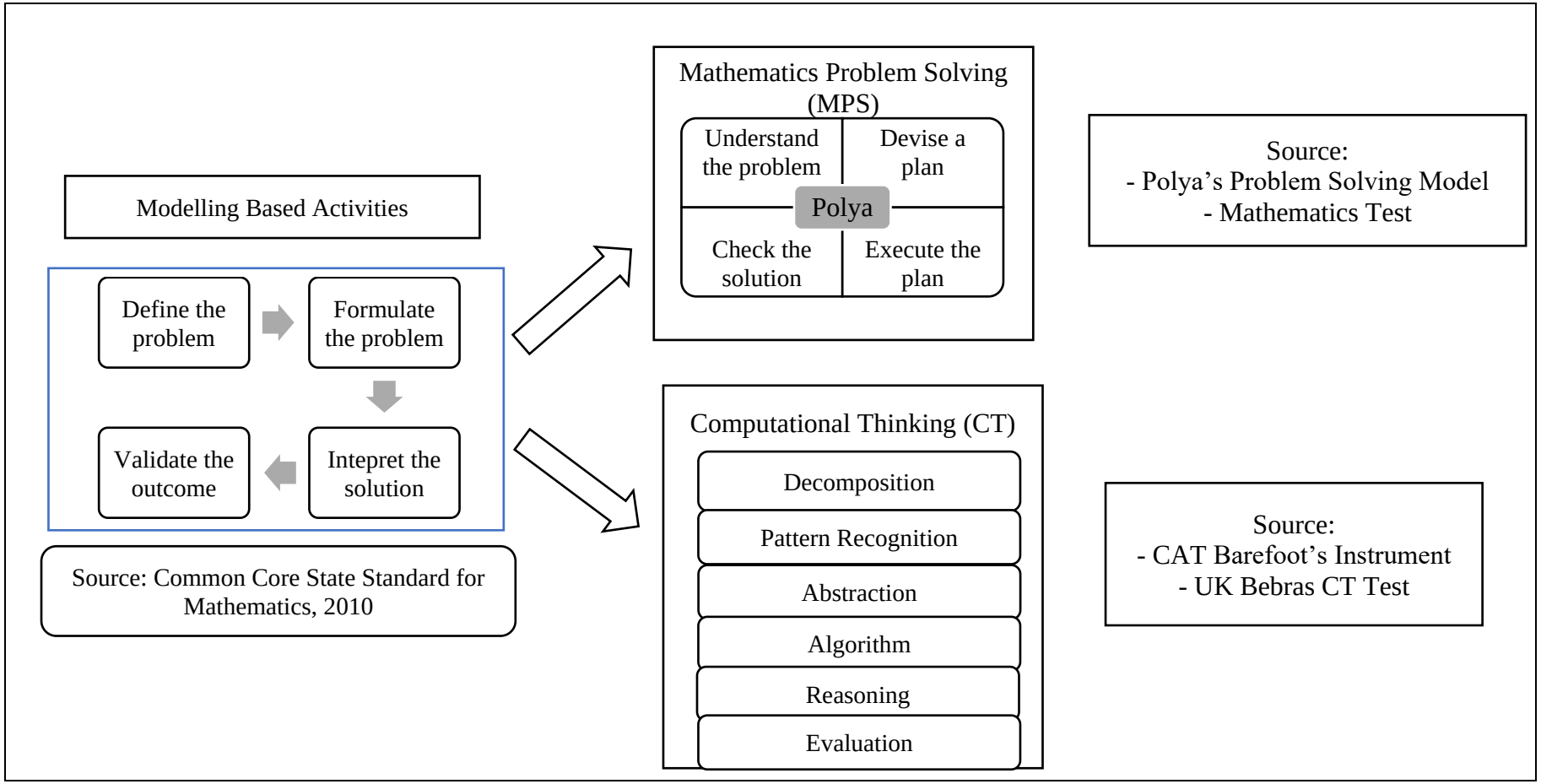

Figure 1.4: Conceptual Framework

Figure 1.4 indicates the conceptual framework of the study. A sociocultural perception of awareness, developed in contact with others based on Vygotsky proposed framework (Harré, R.,2012). Others could be referred as the approaches used to recognize and support the logical thought process of human being (Borba & Villareal, 2005). Thus, mathematics modelling eliciting activities would be served as the learning platform based on socio cognitive theory. There were several refined modelling models in the research field namely Weigand &Weller (1998), Swetz & Hartler (1991), Blomhoj & Jensen (2013), Blum & Leiq (2007) and Common Core State Standard for Mathematics (2010). Among the models of mathematics modelling, the one proposed by the Common Core State Standard for Mathematics in the year of 2010 shall be adopted at the model of this study. This was due to the stated model refined and categorise the modelling process including the understand the problem, formulate the problem, interpret the resolution, and validate the outcome. These stated processes were more action oriented and easy to be interpreted for the descriptions of modelling process. The effects of participating the modelling activities for the duration of six weeks on the development of CT and MPS of secondary students ought to be recorded and analysed for more discussion.

A pre-test and post-test would be conducted to investigate the changes on the mathematics problem-solving skills of the students after going through the process of mathematics modelling activities. As for pre-test and post-test, ten word problems based on the topics of mathematics in the context of *Sijil Pengajian Malaysia* (SPM) shall be designed to cater the needs of examining the mathematics problem solving (MPS) of students. The topics included were mainly form 4 and form 5 syllabuses of additional mathematics based on the *Kurikulum Bersepadu Sekolah Menangah* (KBSM) framework. This is due to the target audiences selected in this experimental research were seven form 5 students from the science stream with the background knowledge of additional mathematics and physics which were suitable and fit to handle the mathematics modelling task which required high cognitive thinking ability. The instrument used in the pre-test and post-test was the model proposed by Polya on mathematics problem solving. The rubric of the model was created based on the following stages, namely understand the problem, devise a plan, execute the plan, and check the solution. There were several mathematics problem-solving models studied

in the education field including Polya, Alan Schoenfeld, John Mason and Lester. Polya's model ought to be adapted as the instrument of the MPS skill due to its popularity and frequencies of usage the models in the research world. The stated processes were solid and recommended in other research studies.

As for CT, the instrument and measurement would be conducted in assessing the development of computational skill based on the model by CAS Barefoot Team in 2014. There are six indicators namely decomposition, abstraction, pattern recognition, algorithms, reasoning, and evaluation. These indicators would be adapted as the guidelines in creating the rubrics. Further, the CT tests were created and adapted from the source of UK Bebras CT Challenge, which was an online CT competition for students around the world. Teachers ought to prepare teaching and learning activities that provide a platform for students to actively contribute to the development of computational thinking and problem-solving skill via modelling. Learning constructivism is a learning that depend upon action and connects to the experience of real-life students. Learning would happen by learning on the basis of their learners' own experience. The constructivism theory will support the opinion where the students learn best when they are required to think intensively. For instance, the hands-on activities could be conducted to reinforce the learning impact based on this learning model. In this study, students can engage actively in teaching and learning using modelling to solve the genuine problem with the modelling tasks.

1.7 Importance of the Research

This research would bring significant importance to the several parties that involved in the planning and implementation of teaching and learning of education in Malaysia.

1.7.1 Importance to the Ministry of Education (MOE)

This research is essential to study the development of mathematics problem-solving skills of the secondary students via modelling-based activities in Malaysia.

Even though to think mathematically is the core element of national syllabuses of Malaysia, the students still performed poorly ever since participating the international assessment. The students failed to handle high order thinking questions which required them to analyse and solve complicated real-life problem. It is believed that modelling and computational thinking has been frequently discussed and emphasised in the research world. Nonetheless, as for the application of modelling and CT has yet to be fully executed in the teaching and learning of mathematics. Likewise, modelling, and computational thinking were both the concepts that has been introduced in the field of education in Malaysia.

However, there were not many resources to be relied on to better prepare the teachers to understand the concepts and hence to promote the modelling activities as the classroom instructions. In this research, the researcher ought to observe and show the steps and ways in the development of the level of the computational skills and modelling. Ultimately, MOE could introduce the model or guideline of the computational thinking skills and modelling to the teachers in evaluating the different levels of the competencies among the students. Thus, a good teaching implementation & facilitation (PdPc) of mathematics problem-solving practise shall be conducted to solve problem.

1.7.2 Importance to the Teachers' Training Division

Leveraging computational thinking and mathematics modelling to solve mathematics problem is a new concept in the industry of education. Therefore, there would be teachers' training on computational thinking and modelling conducted to prepare the teacher in teaching and learning of the element of computational thinking and modelling. As a result, the research could serve as a guideline for the organizers to plan and facilitate the training that ensure the lesson of modelling activities in the school to be carried out smoothly. The teachers would be more interested and confident in delivering the modelling-based activities after exploring the teachers' trainings.

1.7.3 Importance to the Mathematics Teachers

By this research, the mathematics teachers can understand well about the steps and advantages in conducting modelling activities to provoke computational thinking, hence solving mathematics problem. Further, the teachers ought to keep themselves updated with the latest approaches and technologies about modelling activities in order to conduct the lesson that best benefits the students. The findings of the study could contribute as the source of teaching and learning of mathematics modelling activities as the classroom instruction. Moreover, the rubrics and tests used in the study could be regarded as the guideline in assessing the students' CT skills and MPS competency via modelling activities.

1.7.4 Importance to the Students

By exploring to modelling process, the students are expected to be trained on thinking computationally. Hence, the students can also deepen their understanding of abstract mathematics concepts. If the results of this study show that Scratch programming can help improve students' computational thinking skills, teachers can use this software regularly. With this, students will have the opportunity to always use modelling approaches. Directly, students' computational thinking and mathematics problem solving skills will be increased. Therefore, students will be able to solve a variety of complex problems in the future.

1.8 Operational Definition

This section describes the definition of some terms used in this study to prevent any misrepresentation or misunderstanding. The reviewer expects the reader to be able to understand this review appropriately. Among the terms used are as follows:

1.8.1 Computational Thinking

The term of “computational thinking” was firstly introduced and frequently referred by Wing (2006) that computational thinking is about the thought process following several prefixed procedures. This is to imply that one needs to think in the structure where certain steps need to be fulfilled throughout the thinking process. Admittedly, Wing (2011) revised the definition of computational thinking as devising resolutions to the problems that could be conducted by information-processing agent. It suggested that the agent process the data effectively to resolve the issue. On the contrary, Denning (2009) argued that computational thinking shall be described as algorithmic thinking where there should be intellectual process between the input and output of the mental alignment. He stated that one needs to think in the form of “algorithmic” to connect and turn the issues(input) into the resolution(output). Conversely, computational thinking should be regarded as the intellectual process for extraction of problems and the formation of computerized results (Yadav et al., 2014). It suggests that by thinking mentally, the user shall get to the core of the problem thus apply the automatable solutions. Overall, most of the studies agreed that computational thinking involving resolving the problem by utilizing the cognitive skill. In this study, CT shall be defined as the cognitive thinking with algorithms to process the information to solve problem like a computer scientist.

1.8.1.1 Decomposition

Decomposition is about breaking the problems into smaller and manageable components hence solving it one by one. The method of decomposition was applied by splitting the questions into smaller parts. In this study, decomposition is defined as the process of dividing the core object into several smaller subjects to be handled properly.

1.8.1.2 Pattern Recognition

By recognizing how alike issues has been resolved earlier, each problem could be observed and solved independently by following certain form of pattern. In this study, pattern recognition was regarded as the ability of identifying the certain forms of patterns based on the observations and analysis on the problem.

1.8.1.3 Abstraction

Abstraction enables the learners to see and analyse the problem by identifying the similarities and differences of it. It is a process of heading towards direct resolution by eliminating the that unwanted components thus making it more comprehensible to realize. Wing (2008) listed related cases offered for various subjects to let the theories to be easily recognized as below:

i. Mathematics – word problem with storyline context were designed and delivered namely water filling rates of containers, floor areas to be tiled, trajectory of the projection of a shooting arrow, resultant vectors of one swimming across the river and recognizing the plans and elevation of three-dimensional objects are significant example of the context of the exercise. It required the leaners to reduce the undesirable and unrelated elements and capture the desirable and wanted information to indicate their resolution in terms of the concepts of rate of change, geometry, and algebra.

ii. Geography – countless viewpoints of actual geography were not included in the particular maps in the education context namely topographic and travel map to access the associated information for their education purposes.

iii. History – various actual world events were eliminated in the textbook nowadays. This is to emphasize selected regional histories and personal profiles as per world history.

In this study, abstraction was defined as the process of removing unwanted components and obtaining relevant information to be proceed in solving real life problem.

1.8.1.4 Algorithm

Algorithmic is about constructing your thought process in arranged sequence to solve all component issues in order to resolve the ultimate and fundamental problem. In this study, algorithm is about building cognitive process in an ordered progression.

1.8.1.5 Reasoning

There must a premise followed by a logical conclusion that implies the condition given. There were two types of reasoning which are deductive and inductive reasoning. In this study, reasoning referred as the competency to 'talk' about mathematics conclusion based on mathematics calculation in the modelling process.

1.8.1.6 Evaluation

Evaluation is about making decision deems reasonable and truthful in complicated and difficult circumstances based on past experiences. A child could realize that $1 + 3$ is the same as $3 + 1$ in an example of reflection. It takes evaluation skill of the children to come upon this conclusion. In this study, evaluation was a skill to be appllied to validate and endorse the resolution proposed.

1.8.2 Mathematics Problem Solving

According to Henderson & Pingry (1953), problem solving is related to setting a goal and there were many obstacles along the journey of achieving the target. Goldstein and Levin (1987) argued that it takes high level of mental process to solve problem. Problem solving in mathematics had been emphasized for few decades in the aspects of teaching and learning in mathematics. For this reason, there were many studies being conducted to investigate about the elements of problem solving in Mathematics. It can be concluded that there were 3 stages along the progress of scientific studies of mathematics problem solving, namely "The Heuristics", "The Creative" and "The Digital". In this study, MPS was defined as the competency to resolve mathematics word problems by leveraging on problem solving models. Further, to better understand about the idea of mathematics problem solving, we could agree that there is a need to resolve an issue by establishing several methods in hitting the goal.

1.8.3 Mathematics Problem-Solving Model

There are several problem-solving models could be used to evaluate the level of problem-solving skill of the students in this study. Alan Schoenfeld (1985) proposed another problem-solving model which comprised of reading analysis, exploration planning, implementation, and verification. In the same year, John Mason (1985) also introduced another model that included getting start, getting involved, mulling, keeping going, insight, being sceptical, contemplating. In the later year, Lester (2013) also improvised similar model namely entry, analyse, attack and review. In this study, the model used would be Polya model due to its vast and fundamental practise in the application of academics and research.

Polya (1957) stated that problem solving should follow the following stages namely understanding the problem, devising a plan, carrying out the plan and looking back as described below:

i. Understanding the problem – the students were required to first understand fully about the issues by eliminating the unwanted information and focus on relevant data for observation.

ii. Devising a plan – the best fit plan shall be established to resolve the issue.

iii. Carrying out the plan – proceed the plan with necessary skills for the appropriate tactics.

iv. Looking back – different strategies shall be implemented to check and observe the solution.

Class 5A wanted to choose for sports club for the curriculum activities. $\frac{1}{3}$ of the class wanted to go for basketball club, $\frac{1}{4}$ of the class intended to go for badminton, $\frac{1}{5}$ of the class wish to join the football club and the rest of the students aim for swimming club. What fraction of the class are interested in swimming club?

Step 1: circle the relevant information.

Step 2:

$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	?

Step 3: $1 - \frac{1}{3} - \frac{1}{4} - \frac{2}{5} = \frac{13}{60}$

Step 4: $\frac{13}{60} + \frac{2}{5} + \frac{1}{4} + \frac{1}{3} = \frac{60}{60} = 1$

Figure 1.5: Example of Mathematics Question with Polya Problem Solving Model

1.8.4 Mathematics Modelling

Mathematics modelling shall be regarded as the process of solving real life issues by understanding the problem context, making statements, deriving calculation, and endorsing the outcomes (Pollak, 2003). There were significant differences on the ways leveraging on mathematics modelling, which are applying modelling as the 'content' and using modelling as 'tool' (Julie, 2002). In this study, modelling would be used as the tool or vehicle to enable the students to develop mathematics thinking and solve the genuine problem.

1.9 Summary

This chapter has discussed about the introduction of the study and the background of the problem being investigated. Hence, the statement of the problem has been identified and listed out. This study also included the research objectives and research questions based on the problem statement. This aimed to provide the direction on where the study heading to within the framework stated. Further, the conceptual framework that will be implemented is also introduced and comprised to provide the overview and comprehensive models on related variables to be accessed. Moreover, the importance of the study is also stated to determine the significance of the planning and implementation of the research study. Likewise, this chapter also included the section of operational definition to distinguish and justify the term and its' definition based on various previous studies. Since the same term used in different studies were viewed from various perspectives and angles hence the definition used in this study must be determined and validated.

Malaysia students faced challenges in handling complicated and dynamic problem in international assessment of mathematics. As a result, the students have been performing poorly ever since participating PISA and TIMSS. Their mathematics problem skill is low, and the leaners seems to have difficulties to use higher order thinking to analyse and resolve the mathematics problem. Computational thinking and modelling have been discussed very frequently along with the topic of mathematics problem solving. Since the components of computational thinking is similar to the modelling process, therefore it is essential to conduct the modelling eliciting activities to assess the students computational thinking. Further, the development of mathematics problem-solving skill throughout the modelling tasks has to be investigated. This aimed to examine the effectiveness of modelling activities to be conducted in the classroom to instil computational thinking and mathematics problem-solving skill. Thus, the learners would be able to construct a model to generalise different real-world problem setting. Equally, the students were equipped with critical and creative thinking to solve 21st century problem.

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