WATER WAVE PROPAGATION INTERACTION PATTERNS IN FORCED KORTEWEG-DE VRIES USING HOMOTOPY ANALYSIS METHOD

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DEDICATION

This thesis is dedicated to

my wife, Mathuri,

my son, Viven Daniel,

my kind parents, siblings, and friends

for their

Sacrifices, Love and Encouragement.

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ABSTRACT

Water waves phenomenon with a forcing disturbance causing unsteady waves is considered as a complicated phenomenon. In this research, forced Korteweg-de Vries (fKdV) equation is found to explain the behaviour of unsteady waves over underwater obstacles. The forcing terms in fKdV non-linear equation are modelled and the approximate analytical solutions are found using Homotopy Analysis Method (HAM). Specifically, standard fKdV equation for three different choices of forcing term such as quadratic, sinusoidal and exponential are studied in this research. The ability of HAM in solving non-integrable soliton-type fKdV models are validated using Hirota's Method with reference to Jun-Xiao and Bo-Ling works in 2009. The relationship between forcing term in fKdV equation and bottom topography with specific critical flow of an ocean are also investigated. Transcritical flow over a hole and a bump are examined using nonlinear shallow water fKdV equation. It is found that multi solitary waves exist and maximum elevation of waves occurs at the deepest hole of the seabed. The water wave exhibits solitary pattern when it flows over sloping region of a hole but no distinctive pattern on flattened based seabed. The transcritical flow over a bump consequently generates upstream and downstream flows. Meanwhile, flow over a flatten bump shows no activity on the flat part of bottom topography but the waves exhibit multi solitary interactions over positive and negative sloping region bump. Furthermore, water wave propagation interaction patterns over a moving bump is explored and it is found that the flow of water waves become subcritical and supercritical based on the critical parameter in the fKdV equation. Three different sloping shapes of Gaussian bump are analyzed as underwater disturbances. If the forcing slope is steep, then it triggers a high amplitude peaked waves. The water wave propagation interaction patterns are also observed when it travels over a flat bottom to inclination plane. In particular, at different degree of inclinations, water wave interaction patterns show higher amplitude at higher steeper planes. In summary, this study shows that steeper sloping underwater topography and types of criticality flow determine the nonlinearity of water wave propagation interaction pattern when it travels over some certain underwater topography.

ABSTRAK

Fenomena ombak air dengan gangguan paksaan menyebabkan gelombang yang tak mantap dianggap sebagai fenomena yang rumit. Dalam kajian ini, persamaan Korteweg-de Vries (fKdV) paksa didapati menerangkan kelakuan gelombang yang tak mantap dengan halangan bawah air. Istilah memaksa dalam persamaan tak linear fKdV dimodelkan dan anggaran penyelesaian analitikal telah didapati dengan menggunakan Kaedah Analisis Homotopy (HAM). Khususnya, persamaan fKdV piawai bagi tiga pilihan paksaan seperti kuadratik, sinusoidal dan eksponen telah dikaji. Keupayaan HAM dalam menyelesaikan model fKdV bukan terkamir jenis soliton disahkan dengan menggunakan Kaedah Hirota serta merujuk kepada kerja Jun-Xiao dan Bo-Ling pada tahun 2009. Hubungan antara istilah paksa dalam persamaan fKdV dan topografi bawah laut dengan aliran kritikal yang khusus juga diselidiki. Aliran transkritikal ke atas lubang dan benjolan telah dikaji menggunakan persamaan air cetek tak linear fKdV. Didapati bahawa berbilang gelombang tunggal wujud dan ketinggian maksimum gelombang berlaku pada lubang terdalam di dasar laut. Gelombang air menyerupai gelombang tunggal wujud sewaktu ia mengalir ke atas lubang dasar laut tetapi tiada corak gelombang pada dasar laut yang rata. Aliran transkritikal ke atas benjolan menyebabkan berlaku aliran hulu dan hilir sewaktu ia mengalir. Manakala, pada dasar topografi yang rata, aliran tidak menunjukkan sebarang aktiviti tetapi interaksi berbilang gelombang tunggal wujud pada kawasan rantau cerun benjolan yang positif dan negatif. Tambahan lagi, corak interaksi penyebaran gelombang air di atas benjolan bergerak diselidik dan didapati aliran gelombang air menjadi subkritikal dan superkritikal berdasarkan parameter kritikal di dalam persamaan fKdV. Tiga bentuk cerun yang berbeza dari benjolan Gaussian dianalisis sebagai gangguan bawah air. Sekiranya cerun adalah curam, maka ia mencetuskan gelombang puncak amplitud yang tinggi. Corak interaksi penyebaran gelombang air juga diperhatikan sewaktu ia bergerak dari dasar rata ke bahagian cerun. Terutamanya, pada darjah cerun yang berbeza, corak interaksi gelombang air menunjukkan amplitud tinggi pada cerun yang terlalu curam. Kesimpulannya, kajian ini menunjukkan cerun yang curam topografi bawah air dan jenis aliran kritikal air menentukan ketidaklinearan corak interaksi penyebaran gelombang air sewaktu ia bergerak di atas beberapa topografi bawah air tertentu.

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LIST OF ABBREVIATIONS

HAM	-	Homotopy Analysis Method
KdV	-	Korteweg-de Vries
fKdV	-	Forced Korteweg-de Vries
2D	-	Two-Dimensional
3D	-	Three-Dimensional
OHAM	-	Optimal Homotopy Analysis Method
HPM	-	Homotopy Perturbation Method
VIM	-	Variational Iteration Method
ADM	-	Adomian Decomposition Method

LIST OF SYMBOLS

$\mathcal{N}$	-	Nonlinear operator
ℓ	-	Auxiliary linear operator
q	-	Embedding parameter
c_0	-	Convergence parameter
R_c	-	Effective region
K, U	-	Homotopic function
u_0	-	HAM approximation function
r	-	Vector of all spatial independent variables
τ	-	Function
t	-	Time
η, φ	-	Water wave elevation
λ	-	Critical parameter

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CHAPTER 1

INTRODUCTION

1.1 Introduction

The nonlinear phenomena such as flow of water waves over an obstacle play a vital role in the field of fluid dynamics and wave propagation which is a branch of applied mathematics and physics. In the past decades, many researchers have worked on water waves flow over an obstacle. The initial work on water waves over flat bottom was started by John Scott Russell in 1834. The new type of long stationary waves was called cnoidal waves by Korteweg and de Vries. Soon on later, it was concluded as the soliton by Zabusky and Kruskal in 1965 (Hazewinkel et al, 1995).

Interesting and fascinating characteristics of the theory of the solitons lies in the branch of pure and applied mathematics, such as nonlinear partialdifferential equations, differential geometry, topology and analysis. In addition to that, soliton had multitude of applications in physics, for example, in hydrodynamics, field theory, and fluid mechanics. In specifically, Korteweg-de Vries (KdV) equation had paved the way for the development of soliton theory which is fully integrable using analytical methods.

Recently, the study of solitary waves due to the movement of underwater obstacles has received much attention by researchers since the initial experiments of Huang et al (1982) and the notable numerical findings of Wu & Wu (1982). The development of research on KdV equation and waves over obstacle make a way for the new remarkable equation called forced Korteweg-de Vries (fKdV). The investigation on fKdV is intensive and fKdV equation is able to be solved by numerical methods (Zhang & Chwang, 1996). Up to now, to the best of our knowledge, the solutions of fKdV equation can only be obtained by numerical or perturbation techniques (Jun-Xiao and Bo-Ling, 2009). Recently, the analytical solution of fKdV equation with a certain form of forcing term have been solved by Jun-Xiao and Bo-Ling in 2009, using Hirota bilinear method.

It is essential to solve fKdV as it had multitude of applications in the real world problems such as submarine landslide, earthquake modeling, tsunami waves, and nonlinear optics.

1.2 Motivation of Study

This research is motivated by the unpredictable water wave propagation interaction patterns when it moves over the uneven bottom topography. It is known that waves propagate in the solitary manner from upstream to downstream but existence of eternal friction leads to a chaotic wave profile. The simplest form of shallow water equations would be forced KdV equation where it could explain the water wave behaviour over the uneven bottom topography. Many researches have been done through experimental study and extensive work but this research intended to explore water wave interaction patterns using fKdV equation. HAM is an analytic approximation method to solve highly non-linear problems for a certain range of spatial distance. Solving fKdV models through HAM could further understand the water wave interaction patterns when it moves over the uneven bottom topography.

1.3 Problem Statement

KdV equation is a homogenous equation and it can be solved analytically. It is known that the outcome of KdV equation is a solitary wave. However, fKdV is non homogenous and highly non-linear, which is difficult to be solve analytically. Up to now, it has only been solved by numerical or perturbation method. Therefore, it is important to have another method to solve fKdV equation. As the solution of fKdV equation is found to exhibit water wave propagation interaction patterns over uneven bottom topography hence, unpredictable water wave interaction patterns could reveal the characteristics of water waves. Since no analytical solution of fKdV was available; more methods of solving fKdV is an advantage in the nonlinear wave theory.

1.4 Research Objectives

The objectives of the research are:

(a) to obtain analytic approximate solution for third order forced Korteweg-de Vries equation (fKdV) with variants of forcing term via HAM,

(b) to identify water wave propagation interaction patterns at different critical flow when it travels over certain uneven bottom topography,

(c) to determine the factors that trigger nonlinear interaction patterns of water wave propagation when it travels over certain uneven bottom topography.

1.5 Research Scope

In this work, the sub-class of Navier Stokes equation which is the nonlinear shallow water equation and in more simplified version, forced KdV were studied. Only third order forced KdV equation was investigated. Approximate analytical solutions for the third order of fKdV is generated via HAM and the study is constrained by the following assumptions.

(a) The fluid is assumed to be inviscid, incompressible and two-dimensional and it deals with unidirectional flows.

(b) The system is shallow as the depth is much smaller than the horizontal scale of the fluid.

(c) There is only one boundary condition on the uneven bottom topography which is the geometric condition.

1.6 Significance of Study

In mathematics, this research will be a milestone in obtaining an approximation solution for non-homogenous fKdV equations via homotopy analysis method (HAM). HAM solution is a summation of an infinite series in which the series converges rapidly to the exact solution. The HAM has advantages compared to numerical methods as it does not involve discretization of variables and free from rounding off errors (Meenatchi and Kaliyappan, 2017). It is found that HAM could obtain convergent series solutions that agrees with exact solution whereby Homotopy Perturbation Method (HPM) and Variational Iteration Method (VIM) found the solution diverges (Liang and Jeffrey, 2009). The strength and advantage of HAM is its convergent parameter as it provides a convenient way to ensure the convergent series solution. This proved, HAM is the better method compared to an existing perturbation method such as HPM, IVM and Adomian Decomposition Method (ADM). Thus, HAM able to solve fKdV equations and provide an accurate approximation solution or near exact solution as up to now, as no exact solutions found in the fKdV models for a certain range of the spatial distance.

In real life, the study enables the geophysicists, geologists, engineers and water waves researchers to comprehend the characteristics of the water waves when it flows over uneven geometry. It is important as the geometry of wave profile is not symmetric as it travels over the underwater obstacles. It is necessary to distinguish the types of water flow and the effects of different forms of uneven obstacles to the water wave generation and propagation. The interaction patterns of water wave propagation with some disturbances could give vital information on amplitudes, types of waves and the wavelength of sea water. Water waves propagation interaction patterns over uneven obstacle at different types of criticality flow will also be able to elaborate the nonlinearity and dispersion of the wave. Obviously, in the real world, it is costly to build and perform an experimental study, therefore, mathematical model is a simpler way to describe the phenomenon of water waves with some disturbances and indirectly, to reduce the impacts and damages to human by natural disaster.

1.7 Thesis Outline

This study includes seven chapters including of this chapter as introduction. Chapter 2 is literature review on fluid flow over an obstacle using forced Korteweg de-Vries equation (fKdV). FKdV equation is a simplified version of shallow water equations which derived from Navier Stokes equation. The chapter also discusses on recent studies on water flow over obstacles.

Research methodology based on basic idea of Homotopy Analysis Method (HAM) is elaborated in chapter 3. The ability and flexibility of HAM is discussed thoroughly. The effectiveness and convergence interval of solution in HAM is explored in order to achieve a valid and suitable analytical solution.

In Chapter 4, standard fKdV equations are solved using Homotopy Analysis Method (HAM). Some examples of forcing terms are employed to analyze the behaviours of the HAM solutions for the different fKdV equations. The chosen forcing terms are quadratic forcing, sinusoidal forcing and exponential forcing. For validation, HAM solution is compared with the analytical soliton-type solution of fKdV equation as derived by Jun-Xiao and Bo-Ling (2009).

Chapter 5 investigates transcritical flow of water waves over a localized obstacle using fKdV equation. The relationship between the forcing term in forced KdV and seabed topography is defined over here. An approximate analytical solution of fKdV with a certain forcing term representing the seabed topography is solved and waves profile over the forcing region is discussed. Four different types of bottom topography are chosen as the forcing term which are hole, inverse bowl shape hole, Gaussian bump and flatten Gaussian bump. The effect of forcing term on waves characteristics are finally explained and compared with available existing result.

Chapter 6 explored critical flow of water waves over a moving obstacle using fKdV equation. This chapter deals with two main shapes of bottom topography. One is moving uneven bump and the moving sloping plane. Three different shapes of Gaussian bump are explored as forcing term in fKdV and the effects of the shapes of bump to waves profile are investigated. In second part, waves profile over from flat to sloping region is explored. Three different angles of sloping region are chosen as uneven bottom topography. The investigation reveals the effect of steeper slope towards wave profile and role of critical parameter in fKdV equation. Nonlinearity and dispersion of waves described throughout all the forcing regions.

Finally, Chapter 7 presents the conclusion, contributions of the current research and future work of this research.

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Appendix A

LIST OF PUBLICATIONS AND CONFERENCES

1. David, V. D., Nazari, M., Barati, V., Salah, F., & Abdul Aziz, Z. (2013). Approximate analytical solution for the forced Korteweg-de Vries equation. *Journal of Applied Mathematics*, *2013*. Volume 2013, Article ID 795818, 9 pages; (Indexed: Scopus, ISI:0.834)

2. David, V. D., Aziz, Z. A., & Salah, F. (2016). Analytical approximate solution for the forced Korteweg-de Vries (fKdV) on critical flow over a hole using homotopy analytical method. *Jurnal Teknologi*, 78(3-2), 107-112. (Indexed: Scopus)

3. David, V. D., Bahar, A., & Aziz, Z. A. (2018). Transcritical Flow Over a Bump using Forced Korteweg-de Vries Equation. *MATEMATIKA*, 34(3), 179-187. (Indexed: ECSI)

4. Barati, V., Nazari, M., David, V. D., & Aziz, Z. A. (2014). A New Homotopy Analysis Method for Approximating the Analytic Solution of KdV Equation. *Research Journal of Applied Sciences, Engineering and Technology*, *7*(4), 826-831. (Indexed: Scopus)

5. Nazari, M., Barati, V., David, V. D., Salah, F., & Aziza, Z. A. (2014). Approximate Analytical Solutions of KdV and Burgers' Equations via HAM and nHAM. *Jurnal Teknologi (Sciences & Engineering)* 67:1, 77-83 (Indexed: Scopus)

6. David, V.D., & Aziz, Z. A. Flow over an obstacle using Homotopy Analysis Method. Proceedings of International Conference on Mathematics, Statistics and Computing Technology (ICMSCT2017) October 16-17 2017, Malaysia. (Conference Proceeding)

7. David, V. D., Aziz, Z. A., & Salah, F. (2016). Analytical approximate solution for the forced Korteweg-de Vries (fKdV) on critical flow over a hole using homotopy analytical method. Proceeding of 3rd International Science Postgraduate Conference 2015 (ISPC2015) February 24-26 2015 (Conference Proceeding)

8. Nazari, M., David, V. D., & Aziz, Z. A. Approximate Analytic Solution for Constant Accelerated Flow of third grade Fluid in a Rotating Frame. International Conference on Advances in Engineering and Technology (ICAET 2014) March 29-30, 2014 Singapore. (Conference Proceeding)